

A generic housing grammar for the generation of different housing languages

A generic housing shape grammar for Palladian villas, Prairie and Malagueira houses

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This thesis is dedicated to my family and to Jerry.

London, January 2018

Deborah Benros

I Deborah Benrós, confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the thesis.

Deborah Benrós

January 2018

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Abstract

Shape grammars have traditionally described a design language and replicated it using a procedure. In the majority of existing studies, one language corresponded to one grammar and vice versa; the generative procedure was univocal and language specific.

Generic grammars, which are capable of describing multiple design languages, potentially allow greater flexibility and help describe not only languages but relationships between languages. This study proposes a generic housing process based on a parametric shape grammar, and uses this to investigate relationships between several grammars or families of designs.

A study case of three single housing grammars was selected using the Palladian villas, Prairie and Malagueira houses. Specific parameterisation confers the sense of style required to define a language. From the generated corpora two methods were exercised to explore two research questions:

1. A qualitative method tested how the parametric space of a shape grammar corresponded with our intuition of similarities and differences amongst designs. This was performed using a set of questionnaires posed to both laymen and expert observers.
2. A quantitative method was used to test how well the parametric space of a shape grammar coincided with the design space expressed by the different corpora. Principal Components Analysis was used to inform if the set of parameters used to design the solutions would group into clusters.

Results indicate that the expected relationships between individual designs are captured by the generic grammar. The design solutions generated by the generic grammar were also naturally perceived by observers and clustering was identified amongst language related design solutions.

A tool such as a generic shape grammar captures the principles of design as described by the generative shape rules and its parameterisation, which can be used in academia, practice or analysis to explore design.

Keywords: generic shape grammar, rule-based system, customised housing

Contents:

0.	Abstract	0
1.	Introduction	1
1.1	Evidence:	1
1.2	State of the art and literature review:	9
1.3	Case study:	12
1.4	Research	13
1.5	Assumptions	15
1.6	Research questions:	17
1.7	Hypothesis	18
1.8	Experiments	19
1.9	Experimental Evidence:	21
1.10	Analysis of data and experimental evidence results	22
1.11	Chapter review	23
2.	State of the art, preliminary concepts and literature review:	27
2.1	Noam Chomsky's syntactic structures and universal grammar	28
2.2	Introduction to shape grammars	33
2.2.1	Shape grammar notions	36
2.2.2	Rule Schemas	44
2.2.3	Embedding	51
2.3	Shape grammar typology and analysis	53
2.4	Automation in shape grammars: the car industry, façade detection	63
2.5	Previously inferred shape grammars:	
	Palladian, Prairie House and Malagueira	65
2.6	Shape exploration	69
2.7	Chapter reflection	73
3.	Case study: three architectural housing grammars	77
3.1	The Palladian Villa grammar by Stiny and Mitchell	78
3.1.1	Grammar type and classification	78
3.1.2	Grammatical Structure	81
3.1.3	Rule System	84
3.2	The Prairie House grammar	89
3.2.1	Grammar type and classification	89
3.2.2	Grammatical structure	89
3.2.3	Rule System	92
3.3	The Malagueira House Grammar	98

3.3.1	Grammar type and classification	98
3.3.2	Grammatical Structure	101
3.3.3	Rule System	103
3.4	Comparison of the case study shape grammars	108
3.4.1	A shape grammar comparative study	108
3.4.2	Comparison and results	110
3.5	Chapter Reflection	114
4.	Generic grammar rules	117
4.1	Introductory Notions	118
4.2	Rules Schemas	120
4.3.	Four generic shape rules:	120
4.3.1	Shape Addition:	121
4.3.2	Shape parallel copy or Offset	123
4.3.3	Shape Subtraction	125
4.4	Other additive generic rules	129
4.4.1	Copy generic rule	129
4.4.2	Move Generic rule	131
4.4.3	Scale Generic rule	133
4.4.4	Rotate Generic rule	135
4.4.5	Shape subdivision generic rule	136
4.5	Results and reflexions from generic rules	139
5.	Concept of Housing Function	141
5.1	Shape and Function	141
5.1.1	Palladian villas	143
5.1.2	Prairie Houses	151
5.1.3	Malagueira houses	156
5.2	Shape and Function findings	158
5.3	Preliminary results and conclusions	172
6.	An Alternative Grammar	179
6.1	Motivation	179
6.2	Original Palladian Villas Grammar vs. Alternative Grammar	187
6.3	Alternative Grammar type and classification	192
6.4	Rule System	194
6.4.1.	Wall placement:	195

6.4.2.	Door placement:	198
6.4.3.	Window placement:	200
6.4.4.	Staircases:	202
6.4.5.	Loggias:	203
6.4.6.	Colonnade:	203
6.5	Palladian Function Rules	205
6.6	Villa Derivation	222
6.7	New Designs	225
6.8	Computer implementation	234
6.9	Chapter results and conclusions	235
7.	Generic Grammar	239
7.1	Generic shape grammar methodology	241
7.2	Shape Grammar formulation	247
7.3	Generation of solutions and the derivation process	253
7.3.1	Derivation of Palladian villas	255
7.3.2	Prairie House derivation	256
7.3.3	Malagueira single-family house derivations	265
7.4	Generic grammar syntax	267
7.5	Generic Grammar structure	270
7.6	Generic grammar validation	291
8.	Hybrid designed by a generic shape grammar	295
8.1	A forth grammar generated by the generic shape grammar	295
8.2	Design hybrids amongst housing designs	304
8.3	Design hybrids within different shape grammars	316
8.4	Results	318
8.5	Hybrid questionnaires and user perception of design	337

9.	Validation	341
9.1	Case study	
	5 Palladian villas, 5 Prairie and 5 Malagueira houses	343
9.2	Methodology used	345
9.3	Comparison and analysis	356
9.4	Results	359
10.	Conclusion	367
10.1	Objectives, research hypothesis and methodology used	367
10.2	Findings and analysis of results	374
10.3	Conclusions and evaluation of results	378
10.4	Final reflections	381
11.	Bibliography	383
12.	Figure index	403
12.	Table index	415

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1. Introduction

1.1 Evidence:

The shape grammar theory was developed to illustrate and recreate the generic procedure designers use to create design solutions. Often designers develop a language which allows immediate signature recognition. This was successfully captured by shape grammars along the last three decades.

The first successful architectural grammar was the Palladian villas by Stiny and Mitchell (Stiny & Mitchell 1978). This allowed for the design of existing villas as originally designed by Palladio and for new villas that fit within the design criteria.

Most architectural shape grammars were created focusing on single housing, churches, aristocratic villas, gardens and city fabric, however to this point the main concern with most grammarians is to develop new grammars to describe existing design languages.

Often this can be described by a univocal relation – to one grammar corresponds one corpus – and so far that is what literature shows. Grammarians focused their research on a specific design language developed by an acclaimed architect with a distinct vision and from an array of several existing designs they try to extract a rule system (inference process). The rule system is based on a design procedure. Design rules carefully ordered that allow the design of the corpus of solutions that inspired it. This also allows a corpus of new solutions that can be perceived as part of the language. Once the grammar is proved successful in describing this language the task is accomplished and new research will focus in new grammars.

Table 1.1 illustrates the consulted literature that corroborates the assumption that to one grammar corresponds one language. From the contents table provided is shown that research has focused up to now into illustrating new grammars or analysing existing ones.

Table 1. 1: Shape grammar research contents table

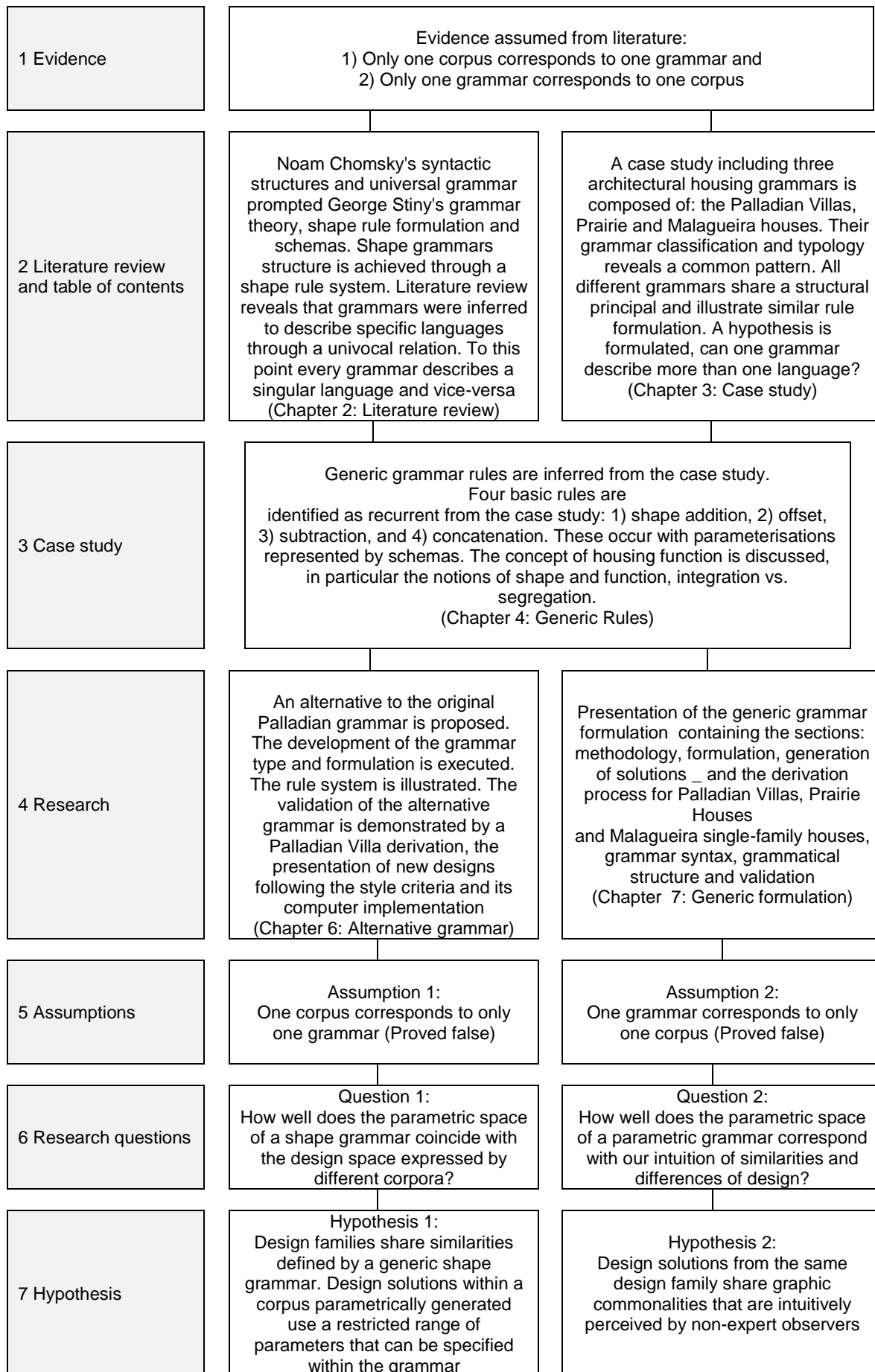
Research field	Bibliographic information
New grammar	1. Buelinckx, H., 1993. Wren's language of City church designs: a formal generative classification. <i>Environment and Planning B: Planning and Design</i> , 20(6), pp.645 – 676
Computer implementation of grammars	2. Chase, C; "Shapes and shape grammars: from mathematical model to computer implementation," <i>Environment and Planning B: Planning and Design</i> 16, no. 2 (1989): 215 – 242.
New grammar	3. Chiou; S. and Krishnamurti, R.; "Example Taiwanese traditional houses," <i>Environment and Planning B: Planning and Design</i> 23, no. 2 (1996): 191 – 216.
New grammar	4. Chiou; S. and Krishnamurti, R.; "The grammar of Taiwanese traditional vernacular dwellings," <i>Environment and Planning B: Planning and Design</i> 22, no. 6 (1995): 689 – 720.
New grammar	5. Downing; F. and Flemming, U.; "The bungalows of Buffalo," <i>Environment and Planning B: Planning and Design</i> 8, no. 3 (1981): 269 – 293.
New grammar	6. Duarte, J. P.; "A discursive grammar for customizing mass housing: the case of Siza's houses at Malagueira," <i>Automation in Construction</i> 14, no. 2 (March 2005): 265-275.
New grammar	7. Duarte, J. P.; "Towards the mass customization of housing: the grammar of Siza's houses at Malagueira," <i>Environment and Planning B: Planning and Design</i> 32, no. 3 (2005): 347 – 380.
New grammar	8. Duarte, J. P. Et Al., "An Urban Grammar For The Medina Of Marrakech," in <i>Design Computing and Cognition '06</i> , 2006, 483-502, http://dx.doi.org/10.1007/978-1-4020-5131-9_25 .
Shape grammars Design exploration	9. Earl, C.F.; "Creating design worlds," <i>Environment and Planning B: Planning and Design</i> 13, no. 2 (1986): 177 – 188.
Shape and grammars	10. Earl, C.F.; "Shape boundaries," <i>Environment and Planning B: Planning and Design</i> 24, no. 5 (1997): 669 – 687.
Grammar research	11. Economou, A.; "The symmetry lessons from Froebel building gifts," <i>Environment and Planning B: Planning and Design</i> 26, no. 1, <i>Environment and Planning B: Planning and Design</i> (1999): 75-90.
Grammar research	12. Fleisher, A.; "grammatical architecture?," <i>Environment and Planning B: Planning and Design</i> 19, no. 2 (1992): 221 – 226.
New grammar	13. Flemming, U., 1987. More than a sum of parts: the grammar of Queen Anne houses. 14, pp.323–350
Shapes and typologies	14. Krstic, D. "Approximating Shapes with Topologies," in <i>Design Computing and Cognition '08</i> , 2008, 81-100, http://dx.doi.org/10.1007/978-1-4020-8728-8_5 .
Style representation	15. Hanna, S.; "Representing style by feature space archetypes: description and emulation of spatial styles in an architectural context," <i>Proceedings paper</i> , August 30, 2006, http://eprints.ucl.ac.uk/3285/ .
Representation of style	16. Hanna, S.; "Automated Representation of Style by Feature Space Archetypes: Distinguishing Spatial Styles from Generative Rules," <i>International Journal of Architectural Computing</i> 5, no. 1 (January 1, 2007): 2-23.

Grammar research	17. Lim, S. et al., "Categorisation of Designs According to Preference Values for Shape Rules," in <i>Design Computing and Cognition '08</i> , 2008, 41-60, http://dx.doi.org/10.1007/978-1-4020-8728-8_3 .
Grammar research	18. Knight, T. W.; "Comparing designs," <i>Environment and Planning B: Planning and Design</i> 15, no. 1 (1988): 73 – 110.
New grammar	19. Knight, T. W.; "Mughul gardens revisited," <i>Environment and Planning B: Planning and Design</i> 17, no. 1 (1990): 73 – 84.
Grammar research	20. Knight, T. W.; "Shape grammars: six types," <i>Environment and Planning B: Planning and Design</i> 26, no. 1 (1999): 15 – 31.
New grammar	21. Knight, T. W.; "Transformations of De Stijl art: the paintings of Georges Vantongerloo and Fritz Glarner," <i>Environment and Planning B: Planning and Design</i> 16, no. 1 (1989): 51 – 98.
Design language research	22. Knight, T. W.; "Languages of designs: from known to new," <i>Environment and Planning B: Planning and Design</i> 8, no. 2 (1981): 213 – 238.
Grammar research	23. 4Knight, T. W.; "The forty-one steps," <i>Environment and Planning B: Planning and Design</i> 8, no. 1 (1981): 97 – 114.
Grammar research	24. Knight, T. W.; "Transformations of languages of designs: part 1," <i>Environment and Planning B: Planning and Design</i> 10, no. 2 (1983): 125 – 128.
Grammar research	25. Knight, T. W.; "Transformations of languages of designs: part 2," <i>Environment and Planning B: Planning and Design</i> 10, no. 2 (1983): 129 – 154.
Grammar research	26. Knight, T. W.; "Transformations of languages of designs: part 3," <i>Environment and Planning B: Planning and Design</i> 10, no. 2 (1983): 155 – 177.
New grammar	27. Knight, T. W.; "The generation of Hepplewhite-style chair-back designs," <i>Environment and Planning B: Planning and Design</i> 7, no. 2 (1980): 227 – 238.
New grammar	28. Koning, H. and Eizenberg, J.; "The language of the prairie: Frank Lloyd Wright's prairie houses," <i>Environment and Planning B: Planning and Design</i> 8, no. 3 (1981): 295 – 323.
Grammar research	29. Krishnamurti, R.; "The construction of shapes," <i>Environment and Planning B: Planning and Design</i> 8, no. 1 (1981): 5-40.
Grammar research	30. March, L. and Stiny, G.; "Spatial systems in architecture and design: some history and logic," <i>Environment and Planning B: Planning and Design</i> 12, no. 1, <i>Environment and Planning B: Planning and Design</i> (1985): 31-53.
Computational model for precedents	31. Oxman, R; "Precedents in design: a computational model for the organization of precedent knowledge," <i>Design Studies</i> 15, no. 2 (April 1994): 141-157. George Stiny and James Gips, <i>Algorithmic aesthetics: computer models for criticism and design in the arts</i> (University of California Press, 1978).

Design theory	32. Oxman, R; "Theory and design in the first digital age," <i>Design Studies</i> 27, no. 3 (May 2006): 229-265.
Grammar research	33. Osborn, S., Cagan, J. and Boatwright, P.; "Automating the Creation of Shape Grammar Rules," in <i>Design Computing and Cognition '08</i> , 2008, 3-22, http://dx.doi.org/10.1007/978-1-4020-8728-8_1 .
New grammar	34. Stiny, G.; "Ice-ray: a note on the generation of Chinese lattice designs," <i>Environment and Planning B: Planning and Design</i> 4, no. 1 (1977): 89 – 98.
Grammar research	35. Stiny, G. and Mitchell, W.J.; "Counting Palladian plans," <i>Environment and Planning B: Planning and Design</i> 5, no. 2, <i>Environment and Planning B: Planning and Design</i> (1978): 189-198.
Grammar theory	36. Stiny, G. and March, L., "Design machines," <i>Environment and Planning B: Planning and Design</i> 8, no. 3 (1981): 245 – 255.
New grammar	37. Stiny, G. and Mitchell, W.J.; "The grammar of paradise: on the generation of Mughul gardens," <i>Environment and Planning B: Planning and Design</i> 7, no. 2 (1980): 209 – 226.
New grammar	38. Stiny, G. and Mitchell, W.J.; "The Palladian grammar," <i>Environment and Planning B: Planning and Design</i> 5, no. 1 (1978): 5 – 18.
Grammar research	39. Stiny, G.; "Boolean algebras for shapes and individuals," <i>Environment and Planning B: Planning and Design</i> 20, no. 3 (1993): 359 – 362.
Grammar research	40. Stiny, G.; "Composition counts: $A + E = AE$," <i>Environment and Planning B: Planning and Design</i> 14, no. 2 (1987): 167 – 182.
Grammar research	41. Stiny, G.; "Introduction to shape and shape grammars," <i>Environment and Planning B: Planning and Design</i> 7, no. 3 (1980): 343 – 351.
New grammar	42. Stiny, G.; "Kindergarten grammars: designing with Froebel's building gifts," <i>Environment and Planning B: Planning and Design</i> 7, no. 4 (1980): 409 – 462.
Grammar research	43. Stiny, G.; "What is a design?," <i>Environment and Planning B: Planning and Design</i> 17, no. 1 (1990): 97 – 103.
Grammar implementation	44. Tapia, P., "A visual implementation of a shape grammar system," <i>Environment and Planning B: Planning and Design</i> 26, no. 1 (1999): 59 – 73.
Pattern recognition	45. Wang, H.; "Proving theorems by pattern recognition I," <i>Communication. ACM</i> 3, no. 4 (1960): 220-234.
Grammar research	46. Yoav I. Parish, H. and Müller, P.; "Procedural modelling of cities," in <i>Proceedings of the 28th annual conference on Computer graphics and interactive techniques</i> (ACM, 2001), 301-308, http://portal.acm.org/citation.cfm?id=383259.383292 .

This provides evidence to the assumption stated. The contents table 1.1 was categorized as:

1. New grammar
2. Research on shape grammar
3. Computer implementations of grammars
4. Grammars exploration
5. Grammar theory
6. Pattern recognition
7. Design theory
8. Style representation



8 Experiments	<p>Validation of the generic grammar:</p> <ol style="list-style-type: none"> 1) recreating existing designs using the grammar 2) grammar and recreating new solutions using the same formulation similarities with original family 3) analysing the corpus of results using a quantitative method to test a set of numeric values for different house variables measured bi-dimensionally on the floor plan. Statistical processes are used such as Principal Component Analysis to plot the results with Cartesian representation, reducing high dimensional data (Chapter 8: Validation) 	<p>To test the perception of familiarities in design an experiment was proposed. A hybrid formulation was created using two parent languages previously explored. This hybrid grammar is presented as a sub-grammar of the generic housing shape grammar, showcasing the same set of rules but with a particular parameterization suitable for the hybrid style. The hybrid study points towards the boundaries of design language and the implications amongst different languages. Independent subjects are asked to identify language similarities in a range of corpus solutions. The solutions point towards a graphical identification of a family of results (Chapter 9: Hybrid forms)</p>
9 Experimental Evidence	<p>PCA RESULTS:</p> <ol style="list-style-type: none"> 1. Design families share language similarities. A restricted range of parameters allow for specific design features within a shape grammar. Clustering is perceived illustrating a corpus space for each language 	<p>QUESTIONNAIRE RESULTS</p> <ol style="list-style-type: none"> 2. Design solutions within a design family share graphic commonalities that are intuitively perceived. A language space is perceived by subjects
10 Results and conclusions	<p>The generation of design families can be triggered by the use of a restricted set of parameters exclusive to a specific language. The parameter space reveals the expected similarities and differences between examples. PCA provides a tool to visualise the 'cluster' and 'space of results' amongst a design family. Individual design solutions tend to cluster amongst a space of results according to the parameters used to generate them. Hybrids fall within intermediate spaces of the families that generated them. Similarities are parametrically and graphically illustrated. Style features and commonalities can be objectively measured and compared. Similarities are also empirically perceived. (Chapter 10: Conclusion)</p>	

Table 1. 2: Thesis argument structure

From the bibliography consulted, 15 titles refer to new shape grammar proposals. All new grammars implicitly describe a new shape grammar to describe a language. None claims that their solution will solely provide a system to recreate the design language. However, to the extent of our knowledge, no two grammars were published focusing on alternative grammars with a complete rule set.

Evidence suggests that one grammar enables the description of one corpus. This corpus is composed of design solutions that share genetic communalities. This corpus behaves as a design family that share distinctive and identifiable characteristics. In architecture the set of characteristics can be perceived through the graphical representation of each solution. This materializes into geometry, proportion, spatial organization, façade composition and so on. Often shape grammars corpus describe firstly existing solutions previously designed by the original designer, secondly a set of new solutions that follow the original paradigm and lastly solutions that are not intuitively perceived as fitting within the family characteristics. Nevertheless one grammar describes one corpus as the evidence suggests.

Assumption 1:

To one corpus of solutions corresponds one grammar

From the diverse set of architectural corpus that inspired the grammars showcased in table 1.1 to each corpus of existing houses corresponded one grammar. This is a fact. To this point no one claimed otherwise, however no one dedicated to create any alternative grammar methodology to describe a given corpus. Hensey (Hersey, G., 1992) did develop an alternative methodology based on an expert rule system with computer implementation to design Palladian villas; however, this was accompanied by harsh criticism and distance from grammar research. From the bibliography available allow us to conclude that one corpus corresponds to one grammar.

Assumption 2:

To one grammar corresponds one corpus

‘Each shape grammar defines a language of designs’ (Stiny and March, 1980b) and therefore one corpus.

From the extent of published work showcased in Table 1.1 never a corpus of solutions corresponds to more than one grammar. It seems like an obvious affirmation but to one corpus corresponds one grammar. To the extent of our knowledge no design solution so far was prompt by two distinct shape grammars. The Malagueira houses are generated exclusively by the Malagueira grammar. No other previously inferred grammar could design Siza’s Malagueira houses. Generically a specific corpus of solutions will be generated by a grammar that suits its particularities and is tailored by its design family traits. The omission of examples that corroborate the hypothesis that a corpus can be generated by more than one grammar in literature seems to confirm the second assumption:

1.2 State of the art and literature review:

Literature on shape grammars further consubstantiates the notion of 'one grammar one language'. From the first publications on the notion of shape grammar theory the idea and purpose of grammars was well defined. Shape grammars are formulations composed of shape rules that effectively describe design languages. 'The shape grammar formalism allows for algorithms to be defined directly in terms of labelled shapes and parameterized labelled shapes. Each algorithm defines a language of shapes.' (Stiny 1980a).

As table 1.1 demonstrates many were the new grammars developed focusing on different design corpus:

- a. Palladian villas (Stiny and Mitchell 1978)
- b. Wren city churches (Buelinckx 1993)
- c. Queen Anne house (Flemming 1987)
- d. Buffalo bungalow (Downing & Flemming 1981)
- e. Japanese tea house
- f. Taiwanese house
- g. Prairie house (Koning & Eizenberg 1981)
- h. The De Stijl paintings (Knight 1989)
- i. Hepplewhite chair grammars (Knight 1980)
- j. Siza's Malagueira houses (Duarte 2001)

Each one of these grammars provided one independent corpus demonstrating Assumption 1. The state of the art in shape grammar research conducted to a series of issues that remain unexplored. Some of these are linked to the criticism to the grammar theory mentioned.

The concept of a universal grammar was developed by the linguist Noam Chomsky in 'Syntactic structures' (Chomsky 1957), a work which explored the idea of an innate grammar formulation that obeyed structural rules common to most languages. Chomsky claimed that language was 'a set (finite or infinite) of sentences, each finite in length and constructed out of a finite set of elements' and that 'syntax is the study of the principles and processes by which sentences are constructed in particular languages. Syntactic investigation of a given language has as its goal the construction of a grammar that can be viewed as a device of some sort for producing the sentences of the language under analysis.' (Chomsky 1957)

Additionally, he focused on the generative power of language grammars. Each language, with its individual finite vocabulary and finite set of grammatical rules, enables almost infinite combinations of feasible sentences to be generated.

'Hence, a generative grammar must be a system of rules that can iterate to generate an indefinitely large number of structures. This system of rules can be analysed into the three major components of a generative grammar: the syntactic, phonological, and semantic components. (Chomsky 1965)

This inspired the work of George Stiny on shape grammar theory. Shape grammar theory is based on the concept that what is informally called a design style is, in fact, part of a design language. Like languages, a design language has a set of specific vocabulary, or shapes, and grammar rules. These rules dictate how shapes are combined to form spatial relations, and are analogous to the way in which word order and sentence construction is organised.

‘Languages of design defined by shape grammars are based on specific spatial relations’ (Stiny & March 1981).

The concept of shape grammars explains how a family of designs can express similarities and form a cohesive group of solutions.

Shape grammars are a rule-based system composed of graphically arranged shape rules. These shape rules are illustrated using a before and after representation showing on the left-hand side the original shape or shapes, a transformation represented by an arrow, and on the right-hand side the result.

Shape grammars present various advantages: ‘Thinking about rules used to construct designs has several advantages: rules are usually much less complicated than the designs they produce, they can be framed in terms of simple relationships they produce’ Much alike language grammars shape rules as part of shape grammars, form a finite number within a set but capable of a multiple set of results. ‘Few rules can be used to construct a multiplicity of complicated designs’. Rules open up new avenues or directions for design with a given vocabulary, they increase the designer’s power of observation in both creative and selective senses.’ (Stiny 1980b)

Several shape grammars have since been developed to attest and validate the theory. The first application to architecture was the Palladian villas grammar (Stiny & Mitchell 1978). This constituted a comprehensive set of shape rules to recreate a set of original designs and a corpus of new solutions. Grammars encode several design qualities such as geometry, spatial distribution and ornamentation.

Since then, several other grammars have been inferred from architectural examples, such as the Wren City Church (Buelinckx 1993), Buffalo Bungalow (Downing & Flemming 1981), Taiwanese House (Chiou & Krishnamurti 1995), Prairie House (Koning & Eizenberg 1981) and Malagueira (Duarte 2001) grammars, in addition to some art-related grammars such as the De Stijl grammar (Knight 1989) or the Hepplewhite Chair grammar (Knight 1980).

Various different types of formulations have been proposed and identified, involving both top-down and bottom up approaches and addition, grid and subdivision processes. The nature of the grammars also ranged from restricted to additive or descriptive.

These were typified in Terry Knight’s ‘Six types of shape grammars’ (Knight 1999) namely:

- Deterministic
- Non-deterministic
- Basic
- Additive
- Restrictive
- Unrestrictive

Knight, besides the grammar classification, focused also on the utility of shape grammars, stating that 'For architectural design education, combined analytical synthetic applications may be more useful than either analysis or synthesis alone simply because they teach multiple skills in a coordinated way' (Knight 1999) '(...) rules are generally easier to understand than the designs they generate the consequences of changes made to rules are easier to understand and control' (Knight 1983). This means that shape grammars encompass a set of useful knowledge ready to be enabled in design.

Previous studies on floor plan optimisation were developed by Steadman and Mitchell. (Mitchell et al. 1976). In this work, rectangular floor plans were tested and optimised using a subdivision system and housing units or flats were generated from a rectangular boundary using a top-down approach. This boundary is later subdivided using up to eight subdivisions to create different compartments. To this is added an algorithm that allows for functional assignment and the adjacency system. The different functions are mapped and connections between spaces are assigned or avoided.

The rectangular floorplans study 'It seems particularly well suited for use in housing design where the limitations to a rectangular geometry and to eight spaces per storey may not be at all inappropriate, where adjacency, orientation, and dimensional requirements can usually be clearly specified, and where plan optimisation is often highly desirable' (Mitchell et al. 1976)

Their work also reveals a variety of design solutions calculated using the subdivision system, thus showcasing its generative potential.

In a similar manner, other studies using subdivision processes and rule-based systems led to the generation of Palladian villas using alternative methods to shape grammars, published in 'Possible Palladian villas' by Hernsey and Freedman. In this work, the authors focus on the concept of architectural aesthetics and musical harmonies as proportional principles. (Hernsey & Freedman 1997)

A series of subdivisions based on musical fractions are proposed as standard design patterns, in order to recreate the work of Palladio. The work manages to successfully recreate some of the original villas and explain most of the design principles such as the introduction of architectural elements and the design of the facades. However, the study fails to represent some of the most paradigmatic villas (such as villa Rotonda) and proves to be as accurate as it is inflexible. In addition, the new designs disappoint and no corpus of solutions is provided.

On the issue of style and archetypes, Hanna also proposes a new method to characterise and identify style: a 'similarly flexible method of representing style is proposed based on the idea of an archetype, to which real designs can be compared, and tested with examples of architectural plans. Unlike a fixed, symbolic representation, both the measurements of features that define a style and the selection of those features themselves can be performed by the machine, making it

able to generalise a definition automatically from a set of examples.’ (Hanna 2007) In this work, a selection of iconic buildings of the twentieth century are represented by analysis of their axial line and the results mapped onto a graph. Not surprisingly, buildings of a similar period seem to have stylistic affinities and appear close together in the graph, emphasising the idea that some designs might be related as a family of designs (informally called a style).

Duarte focused on studying the Malagueira House grammar. This work was important both in terms of the accuracy of the shape rules set produced and the inference process used. The corpus of existing design solutions was extensive, allowing for a good sampling process and, in addition, the original architect attested to the veracity of the rule set and validated some of the new design solutions generated by the grammar.

The grammar was later encoded into a computerised tool that allowed designs to be explored and facilitated use of the complex and detailed grammar produced.

1.3 Case study:

From the extensive body of work on generative shape grammars three were selected to acquire further evidence to test the assumption 1.

The three grammars selected responded to the following criteria:

- a. Shape grammars that enabled the generation of single housing
- b. Grammars with distinct grammar types
- c. Grammars with radically different shape rule sets
- d. Grammars that illustrated different corpora and language
- e. Grammars that were inferred based on an extended set of original designs
- f. Grammars that prompted the generation of an extended corpus of results

The selected grammars that complied with the above criteria were:

1. Palladian villas (Stiny & Mitchell 1978)
2. Prairie houses (Koning & Eizenberg 1981)
3. The Malagueira house grammar (Duarte 2001)

The above grammars were analysed and scrutinized in Chapter 3. This chapter describes the original set of designs the grammatical structure and the shape rule system for each one of the case study.

The Palladian villas devise a typical grid grammar to describe the highly symmetrical and orthogonal Palladian examples. The particular rules are demonstrated graphically as well as the corpus generated. The prairie houses inferred a grammar that relates directly from the intuitive process set up by the architect. The process can be described as an addition grammar and proposes the consequent addition of volumes stacked around the fireplace and chimney until the typical house is accomplished. The process is highly intuitive and generates the original houses

that allowed the grammar inference and a credible set of new solutions. Similarly, the Malagueira grammar bases its inference on a set of 200 original houses generated by Siza. This grammar uses a subdivision formulation to generate the single houses, a process driven by the intuitive generation process used by Siza. Once more this grammar also attains a corpus with two groups of designs, the original designs and the new set of solutions that follow the same design criteria and fit the genre.

1.4 Research

Observation indicated that parametric shape rules are efficient ways of representing design solutions whilst maintaining language consistency and providing variety.

This is the principle that served as director for the generic shape rules formulation. Chapter 4 uses the concepts explored in the previous chapter to extrapolate generic grammar rules. Four basic rules are identified as recurrent in the case study:

- 1) Shape Addition
- 2) Offset or parallel copy
- 3) Subtraction
- 4) Concatenation

These recur in several examples with similar graphic representations but specific parameterisations and are therefore represented by a schema or algebraic expression. They are intended to be used parametrically and can respond to a number of situations. Other useful (but less recurrent) generic rules are also proposed, including copy, move, scale and rotate and shape subdivision rules.

This generic grammar used the structure created for the alternative grammar, the set of generic rules identified in the generic study, a new set of (less frequently recurring) generic rules and a set of particular rules exclusive to each language. Within the rule set, a balance was achieved by using the same graphic shape rules but different parameterisations specific to each language.

A computerised tool was developed to incorporate and encode the generic grammar shape rules. This tool includes the formulation which, although targeting these specific design languages, could, with minor adjustments, generate random solutions.

The alternative Palladian grammar constituted the first step towards systemising a generic grammar. Developing this independent grammar formalism allowed for the generation of a consistent corpus of pre-existing solutions, solutions proposed by the original grammar and a completely new set of designs. This constitutes a novelty in shape grammar research and is the first step towards a generic grammar. The grammar has to be generic enough to accommodate several languages but specific to design the particular features of one language in order to avoid shallowness. In addition, the development of the new Palladian grammar allowed for the inclusion

of new items such as functions, and proposed new shape grammars and processes for designing house types not produced by the previous grammar or other rule-based systems.

The study of previously published grammars by grammarians such as Stiny, Knight, Koning, Eizenberg, Duarte and others resulted in a cross-comparison study which revealed important issues. In particular, it showed that despite the grammar type, structure or rule system chosen, most grammars follow similar structured formulations. This is also noticeable when grammars with different top-down or bottom-up approaches are compared. Most use shape rules that are graphically similar and seem to vary only in terms of the application of labels or parameterisation. Moreover, some of the most admired examples of houses in the corpora studied share similar features. Several of the studied authors and designers like Palladio, Frank Lloyd Wright and Siza famously favour square proportions and rectangular ratios of 2:3, both for room sizes and the external envelope footprint.

The most common (and sometimes recurrent) rules observed in all grammars are polygonal addition (mostly rectangular), boundary offset, polygonal subtraction and spatial concatenation or merging as evidence provided in the following sections will attest.

These were the main rules used in the generic rule set. Some transformation rules, also common in the cases studied, were illustrated, also including rules such as copy, rotation, move, scale and subdivision.

Most grammars observed in this study can be described using a tree diagram that illustrates their structure. This structure is found consistently in the examples observed and has a common distribution, despite the different nature of the grammars. The first stages involve addition operations, the main body varies according to whether a bottom-up or top-down approach is used and the ending is remarkably similar, featuring subtraction processes associated with the detailing stage. Regardless of the approach, the final stages are dedicated to refining the design and label deletion.

It is also evident that additive and grid grammars have more in common than subdivision grammars, although subdivision grammars seem to provide a greater level of flexibility.

These findings enabled the derivation process to be documented and effective housing designs to be produced.

Another important contribution is the proposal of a generic grammar that replicates three separate design languages. The aim of this is to propose a generic formulation which leads to the design of housing solutions on a high level and can generate specific architectural languages on a low level.

This generic grammar proposes a contribution in the design and research fields. This grammar can be used for design exploration, analysis of the languages represented and recreation of design solutions within the universe or corpus, with potential direct applications in art history and teaching.

A computerised tool that allows for the generation of design solutions and serves as a platform for a generic grammar. Computer tools for housing design are not, in themselves, novelties, since these have been proposed in previous works on shape grammars and applied rule systems for housing in the past (Duarte 2001) (Benros & Duarte 2009). Nevertheless, due to its restricted nature, a computer implemented shape grammar can be easily used by an inexperienced user and can also be converted into a less restricted grammar to allow for design exploration and design creation. This offers several benefits: it can be used as a computer assisted design tool to generate feasible workable design solutions that incorporate expert knowledge and to improve designs by using an expert design system. The results can be encouraging and diverse and the tool offers a contribution to work in the field of computer aided design, due to its generic parametric features.

The generic grammar methodology was based firstly on the analysis of three case studies, secondly on the extraction of shape rules from the examples and corpus of existing examples and thirdly on the composition of generic shape rules. This resulted in a generic formulation that not only encodes knowledge of the design concepts originally developed by the architects, but also spatial relationships, as well as a post reading by the experienced grammarians who encoded and derived an extensive set of shape rules to describe each grammar. This was then transformed, systematised and simplified into one single grammar. The singularity of this grammar set is related to the fact that it is not a conglomeration of shape rules or a simple merging of all the grammars observed in the case study.

1.5 Assumptions

The previously stated assumption 1: one corpus corresponds to one grammar is provisionally accepted since to this day exclusively one corpus per grammar was deployed. The consulted literature as presented in (table 1.1) corroborates the unstated claim.

The negative form of assumption 1 would be: one corpus of solutions corresponds to more than one grammar. Although neither have been previously discussed within the studied previous work, research conducted in specific grammars to describe specific corpus seems to indicate the falsity of assumption 1.

However, this has been discussed in academia even if scarcely published. Trials run in academic context with students have tested the shape rule inference process whilst introducing shape grammar theory. Experiments were made where students were asked to infer shape rules from given ceramic tiles patterns with telling results. Two different teams were confronted with the same ceramic tiles pattern and asked to draft a shape grammar that could generate the original pattern. The results illustrate that both teams used different shape rules and different processes to illustrate the same design solution. The two independent grammars enabled the same corpus in common and many others independently. A clear evidence of the theory that one corpus of solutions corresponds to more than one grammar. (Benros, Eloy, Duarte, 2012)

In addition, the study of the previously inferred grammars as described in Chapter 3 revealed that some grammars appear to be more efficient and intuitive to use, whilst others seem to use procedures and algorithms that although valid and successful are not necessarily expedite. This was one of the observations gathered within the case study and particularly observed in the Palladian grammar. With the knowledge acquired previously with the Workshop in Ceramic tiles patterns a new experiment was devised. This time the challenge was set to infer new design rules and elaborate a new procedure to generate the corpus, using the existing Palladian villas corpus. The process used was inspired in the Malagueira grammar and used this system to describe and prompt design. The grammar type used was the subdivision which revealed to be simpler and more economical. The process was shorter and the number of rules created limited.

The results showed that the existing Palladian villas could be replicated. The new grammar could also generate existing villas not originally included in Stiny and Mitchell's original grammar. A new set of solutions that fitted the design criteria was also created showcasing the efficiency of the new grammar.

This new grammar was entitled an alternative grammar to the Palladian villas grammar (Benros & al., 2012).

The experiment concluded that two independent grammars could replicate the same corpus and therefore to one corpus corresponds more than one grammar.

This proves false the assumption 1: to one corpus corresponds one grammar.

In fact:

To one corpus corresponds more than one grammar

The second assumption – one grammar one corpus– is commonly accepted and expressed in literature as mentioned: 'Each shape grammar defines a language of designs' (Stiny 1980). The case study selected is based on this assumption. The singularity of each language is exemplified by the singularity of the shape grammar formulations prepared to describe them. To our knowledge the Palladian grammar designs exclusively Palladian villas. The same applies for the Prairie houses and Malagueira grammars respectively. Nevertheless, the case study also revealed that many shape rules shared graphic commonalities across grammars. In fact, chapters 4 and 5 ventured creating generic shape and spatial rules. These generic rules offered a parametric formulation and a schema that allowed for a specific geometric operation to take place within a set of parameters that could be manipulated. Furthermore, chapter 7 dwells on the generic shape rules and offers a new generic formulation that allows for the generic design of more than one language whilst allowing for the parameterization to tailor language specific traits. This generic grammar aimed towards a formulation that could describe both Palladian, Prairie and Malagueira houses. These efforts prove Assumption 2 – one grammar one corpus – false.

Sub-sequentially:

To one grammar corresponds more than one corpus

1.6 Research questions:

Shape grammars have proved in the past to be successful illustrating a design language. Despite the criticism of the application of certain shape grammars is undoubted the success of shape grammars to recreate existing and new solutions. Parametric shape grammars in the past allowed the successful manipulation of the design language to recreate possible solutions that fit the criteria and opened room for new designs. Malagueira is a good example with randomly generated solutions being vetted by the original architect Siza himself (Duarte, 2001). When it comes to a generic grammar new questions arise. A generic grammar proposes a base formulation that allows 'one formulation fits all' not leaving much space for specificities of design. To attain a design solution that is credible, feasible and occupies a space within the design language, requires careful consideration. The same formulation allows for A, B and C to be designed whilst A, B and C are highly dissimilar. This is only possible through a parametric shape grammar formulation. Moreover, through a parametric formulation where specific intervals of variables are identified and restricted to each language. The parameterization has a direct impact into the design language expressed through each graphical representation of each solution. In the end, this parameterization impacts directly the house generation process. The different corpora generated using different parameters will illustrate it allowing us to pose the following question:

Question 1:

How well does the parametric space of a shape grammar coincide with the design space expressed by the different corpora?

This question is in the centre of the parametric shape grammar discussion. More successful the grammar, greater the coincidence between the design space and the different corpora. Ideally the design space is expressed clearly by the corpora. The overlap between the design space and the different corpora will help measuring how successful the grammar is.

If the design space could be measured either quantitatively or qualitatively the result might point towards a definitive answer.

On the other hand, intuition plays a part analysing design corpora. Is know-how and intuition that drives grammarians through grammar inference processes. There is a possibility that intuition can assess the success of shape grammars corpus within the parametric space of a grammar. If this is the case then the following question should be placed:

Question 2:

How well does the parametric space of a shape grammar correspond with our intuition of similarities and differences amongst designs?

The answer to this question should be addressed by a qualitative method of analysis where intuition could be assessed and accounted. How well an independent subject can perceive similarities and differences amongst a sample of corpora? Other questions could be posed, such as how does the parametric space of a grammar reflects into the corpora, can variations in parameters be perceived by subjects, or if an untrained eye could observe and identify similarities and differences.

1.7 Hypothesis

To address the questions above posed a set of hypotheses were formulated. Question 1 requires a hypothesis formulation that allows testing how well the parameter space can operate. This will be formulated as follows:

Hypothesis 1:

Design families share similarities defined by a generic shape grammar. Design solutions within a corpus parametrically generated use a restricted range of parameters that can be specified within the grammar

To put this hypothesis to the test one requires:

1. to exercise the parameter space of a given language within the grammar context
2. generate several design solutions both existing and new solutions as control items
3. use the several geometric and spatial relations within the solutions created to attest results
4. use objective analytical methods to quantify results

As for question 2 a different hypothesis needs formulation. How one perceives design language characteristics might seem subjective. Nevertheless, differently inferred grammars seem to allow for successful results despite the use of intuition. Similarly tests with different subjects seem to determine that usually subjects react consistently whilst identifying similarities, so the hypothesis could be formulated as:

Hypothesis 2:

Design solutions from the same design family share graphic commonalities that are intuitively perceived by non-expert observers.

To attest this hypothesis a qualitative method comes to play:

1. use the generic grammar to replicate existing solutions and generate hybrids
2. hybrids are generated from 2 parent languages using the parameters intervals that are specific to each language
3. select a sample of original designs of different languages and hybrids
4. group dissimilar examples with hybrids that share some commonalities
5. test the samples with an universe of both experts and non-experts

1.8 Experiments

To test how a parametric restricted set of values can specify grammar, a generic grammar was developed. This allow for experiment 1 to be put in place.

Experiment 1: (Chapter 8: Validation of Generic Grammar) composed of the following steps:

1. Recreating existing designs using the generic grammar
2. Recreating new solutions using the same formulation and sharing family characteristics
3. Analysing the corpus of results using a quantitative method to test the set of numeric values for different house variables measured bi-dimensionally in each floorplan. Statistical processes are used such as Principal Component Analysis to plot the results with Cartesian representation reducing high-dimensional data- Principal Components Analysis experiment
4. Illustrating the results into Cartesian representation using a chart where each house is shown occupying its space within the corpus group

The results showed a clear clustering effect for each house typology. Intuitively perceived similarities between different houses were easily illustrated by the space proximity occupied by each one.

Different analyses were performed using different geometric criteria. The evidence was plotted into charts that showcase the data tested. In this is patent a clear tendency:

- a. Malagueira occupies the bottom space close to the origin and its houses illustrate a strong clustering effect. This is not surprising since all illustrate the same plot proportions and area only varying slightly in implantation, spatial distribution and envelope
- b. Palladian villas occupy the top space of the chart also illustrating a tendency of clustering but less well defined than the previous example. The house sample used illustrated more variety in area, proportions and spatial distribution with very dissimilar house envelopes. Despite the clustering effect observed it is identified an overlap with Prairie houses

- c. Prairie houses occupy the centre of the chart with a well-defined cluster. This sits neatly between the Malagueira house cluster and the Palladian villas. One house from the sample sits in disarray amongst the Palladian villas. This house does share some similarities with the Palladian examples, showcasing floorplan symmetry, similar proportions and an almost self-contained floorplan

Experiment 2: (Chapter 9: Hybrid forms) Proposes a hybrid formulation inspired by two parent languages previously explored. This hybrid grammar is presented as a sub-grammar of the generic housing grammar, showcasing the same set of rules, although with a particular parameterization suitable for the hybrid style. The hybrid study points towards the boundaries of design language and the implications amongst different languages.

A case study of hybrids was generated and its floorplan illustrated. The hybrids created showcased values derived from the parameters allowed for its parent languages. A Palladian-Prairie house, a Prairie-Malagueira and a Malagueira-Palladian was created and illustrated.

Sets of questionnaires were given to a universe of expert and non-expert subjects. Each questionnaire posed the following questions to ascertain qualitative conditions:

1. Amongst a group of real houses and one hybrid to select one that held the greatest aesthetic value
2. Amongst a group of houses, the subject had to select one most likely to represent an existing house
3. Given an example select one house amongst a group that shared the greatest similarities with the first house

The findings were telling:

- a. Whilst enquired about the visual and spatial qualities of a given house, both experts and non-experts picked in majority hybrid solutions even when confronted with well identifiable real house plans
- b. Whilst confronted with hybrid and real solutions both experts and non-experts in majority picked the hybrid as the most likely to represent an existing house
- c. When confronted with different house sampling and asked to pick families both experts and non-experts grouped existing houses with hybrid examples rather than with the existing house representations

The hybrids might have triggered a level of familiarity to each subject. The hybrid bonds two parent languages which might increase its perceived familiarity. That familiarity might explain the choice in detriment to the original solution since we are usually drawn to things we know well and understand.

1.9 Experimental Evidence:

1.9.1 Experimental Evidence 1: PCA results:

Design families share language similarities. A restricted range of parameters allow for specific design features within a shape grammar as illustrated by the experimental evidence 1 the plotting of houses in a Cartesian representation using PCA. This experiment seems to consubstantiate hypothesis 1: Design families share similarities defined by a generic shape grammar. Design solutions within a corpus parametrically generated use a restricted range of parameters that can be specified within the grammar.

The experiment used a quantitative method to attain the results. The results can be measured and compared with another set of examples generated.

The results seem to prove that families of solutions originated from the same design language share characteristics that are measurable and quantifiable. An array of geometric data is then converted into numerical data and reduced for bi-dimensional representation. The bi-dimensional representation clearly shows clustering effects for the three languages studied which allowed to confirm the hypothesis that the similarities shared by a given family can be translated into the set of parameters that originated them in the first place and these can be plotted to define a clear space. That identifies a language and a specific grammar.

1.9.2 Experimental Evidence 2: Questionnaire on hybrid design solutions results:

Qualitative methods were used as experimental evidence to test hypothesis 2: Design solutions from the same design family share graphic commonalities that are intuitively perceived by expert and non-expert observers.

In fact, the experiment attested that design solutions within a design family share graphic commonalities that are intuitively perceived. When confronted with traces of family related subjects both experts and the non-experts can perceive similarities. These were even identified if only partially imbedded in the genetic code of a hybrid example. More so the questionnaire results identify that subjects can even perceive aesthetic qualities inherited from the parent languages into hybrid solutions which emphasises the importance of the family characteristics. 103 independent subjects from a universe of expert and non-expert users helped assessing the results.

This assists proving the hypothesis that within a design family the specific characteristics are illustrated in each solution and these can be intuitively perceived.

1.10 Analysis of data and experimental evidence results

'One corpus corresponds to one grammar' constituted assumption one and was derived from literature. This was proved false by the two experiments that took place in this research. Firstly, the result of the workshop on ceramic tiles patterns and secondly by the chapter 6 where an alternative grammar to the Palladian grammar allows one corpus to be illustrated simultaneously by two independent grammars.

'One grammar corresponds to one corpus' assumption 2 as claimed in literature and proved false by the generic grammar as illustrated in chapter 7.

Two new assumptions can be posed now: One corpus corresponds to one or more grammars and one grammar corresponds to one or more corpus. The second assumption is against the grain of research. Until now the grammar research focused with specific grammars to generate specific design families.

In this study, the generation of design families can be triggered by the use of a restricted set of parameters exclusive to a specific language. This methodology allows for one generic grammar to prompt the design of distinctive design families by parametric manipulation. Design corpora are no longer a result of parametric variation but of parametric manipulation with different results whilst using the same formulation.

The parameter space reveals the expected similarities and differences between examples.

Two methods were deployed to attest the generic grammar efficiency, a quantitative method to determine the clustering of the design space and a qualitative method to determine how well one can perceive the visual and spatial similarities amongst corpora.

PCA provided a tool to visualise 'clusters' or 'space results' amongst a design family. Individual design solutions tend to cluster amongst a space of results according to parameters used to generate them.

Questionnaires provided a tool to test if existing solutions and hybrids fell within intermediate spaces of the families that generate them. Similarities are parametrically and graphically illustrated. Style features and commonalities can be objectively measured and compared. Similarities were also empirically perceived. Similarities were empirically identified even by non-expert observations.

There is a pattern that can be identified between design families which allow for similar traits to be latent in the design.

A generic grammar provides an analysis tool for both quantitative and qualitative methods in design and academia.

1.11 Chapter review

This thesis is divided into 9 chapters, organised chronologically to reflect the methodology.

This opening chapter introduces the problem and the hypotheses, the methodology, and outlines the main contributions.

Chapter 1 describes the relevant concepts and preliminary notions focusing on the research question and main hypothesis.

Chapter 2 focuses on the literature review. In this chapter shape grammars are introduced as formalisms, its structure defined and shape rules explained. The introduction of shape grammar theory is underlined. Some historical examples of grammars are explored and discussed. The classification and typology of grammars is also shown. There is a particular emphasis on Noam Chomsky's work on syntactic structures, George Stiny's shape grammar theory and some preliminary notions. The chapter also aims to clarify some preliminary notions concerning shape grammars that will be useful for later discussions.

Chapter 3 is dedicated to the presentation of the case study, which includes three architectural housing grammars, namely the Palladian Villa, Prairie House and Malagueira House grammars. The grammar type and classification, grammatical structure and rule system for each grammar are studied and the chapter concludes with a comparative analysis of the three grammars

Chapter 4 uses the concepts explored in the previous chapter to extrapolate generic grammar rules. Four basic rules were identified as recurrent in the case study: shape addition, offset, subtraction, and concatenation. These recurred in several examples with similar graphic representations but specific parameterisations and were therefore represented by a schema or algebraic expression. They were intended to be used parametrically and could respond to a multitude of situations. Other useful (but less recurrent) generic rules were also proposed, including copy, move, scale and rotate and shape subdivision generic rules.

Chapter 5 addresses the concept of housing function and discusses shape and function. Generic functional rules are traced and particular functional rules presented for each language.

This is followed by Chapter 6 alternative formulation to the original Palladian shape grammar. It begins with a section explaining the reason for this alternative formulation, a comparison of the original and alternative Palladian Villa grammars and the development and presentation of the grammar's type, classification and formulation. The section that follows demonstrates the rule system and some of the particular features of the style, such as wall placement, door placement, windows, staircases, loggias and colonnades. The validation of the alternative grammar is

demonstrated by a Palladian Villa derivation, the presentation of new designs following the style criteria and the presentation of its computer implementation.

Chapter 7 is dedicated to the generic grammar formulation and contains the following sections: methodology, formulation, generation of solutions and derivation process for Palladian Villas, Prairie Houses and Malagueira single-family houses, grammar syntax, grammatical structure and validation. It concludes with the presentation of the computer implementation. This chapter proves false assumption 2: to one grammar corresponds one corpus.

Chapter 8 proposes a hybrid formulation inspired on two parent languages previously explored. This hybrid grammar is presented as a sub-grammar of the housing generic shape grammar, showcases the same set of rules but a particular parameterization cater to the hybrid style. This constitutes the qualitative experiment of hypothesis 2.

Chapter 9 focuses on the validation of the generic shape grammar. This validation is performed through: 1) the recreation of existing design solutions using the shape grammar formulation, 2) recreation of new solutions using the same formulation and reflecting on the familiarities with the parent language 3) analysing the corpus of results through a quantitative method by testing a set of numeric values for different house variables measured bi-dimensionally on the floorplan. This last validation method recurs to statistics processes such as Principal Component Analysis to plot the results into a Cartesian representation, reducing the high dimensional set of data into a bi-dimensional representation. This allows an objective and direct comparison of results.

The final chapter reflects and comments the results that can be drawn from this work, namely the process used and its results, the analysis of results and comments on the findings, expected applications, advantages, disadvantages, possible conflicts and future developments of the study.

2.	State of the art, preliminary concepts and literature review:	27
2.1	Noam Chomsky's syntactic structures and universal grammar	28
2.2	Introduction to shape grammars	33
2.2.1	Shape grammar notions	36
2.2.2	Rule Schemas	44
2.2.3	Embedding	51
2.3	Shape grammar typology and analysis	53
2.4	Automation in shape grammars: the car industry, façade detection	63
2.5	Previously inferred shape grammars:	
	Palladian, Prairie House and Malagueira	65
2.6	Shape exploration	69
2.7	Chapter reflection	73

2. State of the art, preliminary concepts and literature review:

The following chapter reflects on the state of the art on the shape grammar theory research. The chapter provides a comprehensive literature review throughout a selection of the material exposed in the table 1.1 as well as other literature relevant to the understanding of the shape grammar theory and its relevance to the practice of design.

Another topic discussed is the introduction of relevant nomenclature specific to the use of shape grammars. Most of these terms showcase exclusive meanings and applications within the use of grammars and its explanation became key to the true understanding of its mechanisms. The shape grammar methodology is also described (section 2.2) where the development of the shape grammar theory is discussed as first developed and applied by George Stiny.

The chapter starts with the universal grammar theory originally presented by the linguistic Noam Chomsky and how the concept was then utilised for design as appropriated by Stiny. This is then followed by shape grammar analysis and typology, where shape grammars are categorised and identified as per Terry Knight's reflection. Other recent developments in shape grammar automation and façade detection are also exposed.

Since the introduction of shape grammars, the way design languages are perceived allowed us to establish relationships between design solutions. These important relationships allow the observer to group objects together by the familiarities they share. When a clearly identifiable group of objects can be identified by its characteristics a language can be perceived. Shape grammars became tools to describe languages.

This literature review and the bibliographic list provided in 1.1 allows us to assume that 'to one grammar corresponds one corpus'. Most of the grammars described showcase a parametric nature and therefore generate a parametric space where all design solutions can be placed. The interaction between the parametric space generated and the design space described by the corpora is fundamental to the understanding of shape grammars.

The study of the shape grammar bibliography will provide clues to this particular relationship.

2.1 Noam Chomsky's syntactic structures and universal grammar

In 1957 Noam Chomsky, a Cambridge-based linguist and MIT faculty member, published 'Syntactic Structures' (Chomsky 1957), an exploration of world grammars and world languages in which he claimed the existence of what he termed a universal grammar. The idea occurred to him when observing the way small children applied language rules and words they had learnt to create sentences they had never heard before. This led him to the conclusion that we may possess an innate grammar from birth, pointing to a common ground, or universal grammar linking various languages.

His theory of linguistic structure was innovative in linguistics, but it could be also extended to other disciplines such as geometry, art history and architecture, as can be seen in the work of George Stiny and his theories of shape explored later in this thesis.

In 'Syntactic structures' Chomsky defines his theory of linguistic structure, starting by identifying the key components of grammar, syntax and semantics:

"Syntax is the study of the principles and processes by which sentences are constructed in particular languages".

It may be considered the set of rules that allows for the orderly arrangement of words and sentences:

"A grammar can be viewed as a device of some sort for producing the sentences of the language under analysis". Associated to this notion is the notion of linguistic level. Linguistic level is a set of descriptive devices that can be used to construct grammars, whereas a language constitutes a set of finite or infinite sentences. The notion of Semantics relates to meaning, purpose or reasoning. Semantics may have two meanings, one linguistic and one philosophical, namely "The study or science of meaning in language" or "The study of relationships between signs and symbols and what they represent it can also be called semasiology". To demonstrate this, the following sentence is presented:

"Pigs fly".

Although grammatically correct with accurate syntax, this sentence is false from a logical point of view and therefore has no semantic value.

The grammar of a language is a device that generates all the grammatical sequences and none of the ungrammatical ones. *"Any grammar will protect the finite and accidental corpus of presumed infinite utterances to a set of grammatical..."* One requirement for an operative grammar is the need to be finite. In addition a "Grammar is autonomous and independent of meaning".

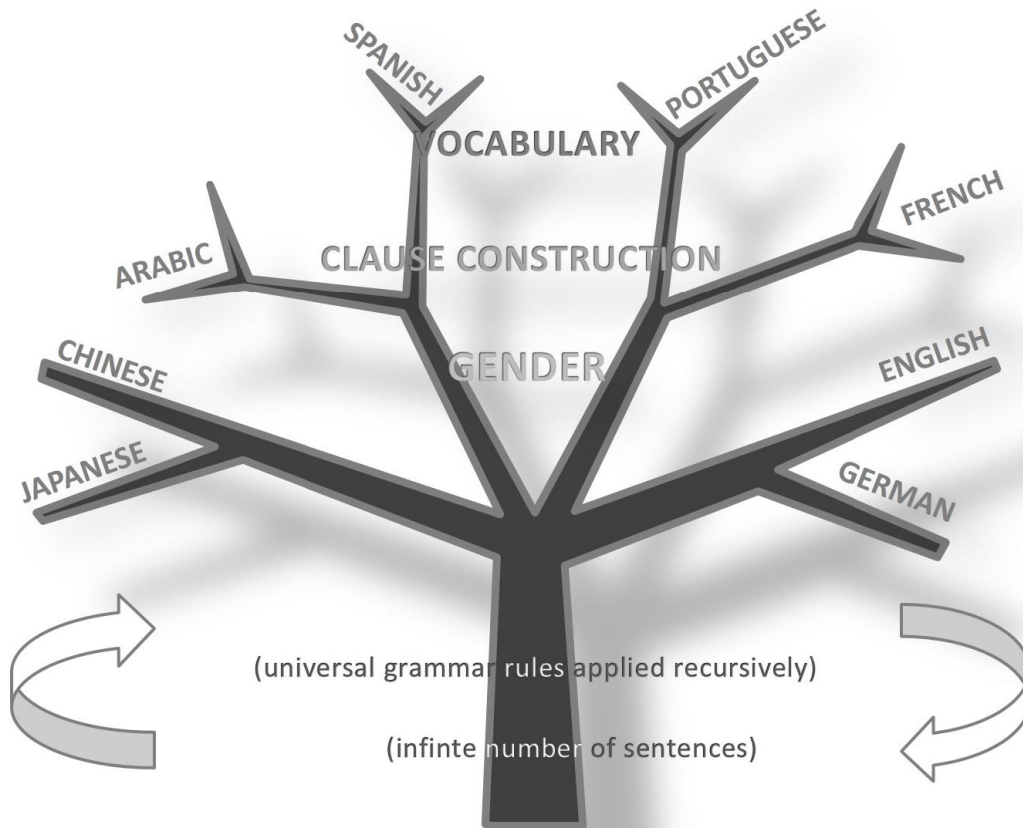


Figure 2. 1: Universal grammar tree diagram

The grammar formulation and its application requires each grammar to be defined by a set of strings and a finite set of instruction formulas (i.e.: $x \rightarrow y$) [E, F].

As illustrated in the diagram in Fig. 2.1, each branch of languages, from Germanics to Latin to Asian-based languages, is believed to group itself around a common structure. Whereas vocabulary is exclusive to a specific language, clause construction rules, such as those for gender and word order, might be similar in more than one language. However they all have some basic universal and innate rules in common and all allow for the construction of an infinite number of sentences from a limited set of grammar rules and lexicon set.

Hierarchically a grammar should with phrase structure or construction, transformational structure and morpho-phonetics. Phrase structure relates directly to each branch of language and to word order and clause construction, whereas transformational structure represents the stage that allows for computational mutation or grammar evolution in which grammar transformational rules can be applied. The final level, morphophonemic, encompasses both grammar morphology and phonetics, or the audible aspects of each language. Within each branch of languages pronunciation issues

vary according to phonological form, whereas semantic issues depend on the logical form. The way word order is dealt with in each particular language depends on what is called the surface grammar. Clause order and phrase construction are more specific than word order and are explained by what Chomsky called the deep grammar, using x-bar and n-bar theory to illustrate this, as shown in Figures 2.2 and 2.3.

Other hierarchies can be established to describe the complex formulations of grammars. On a superficial level vocabulary usually reflects a specific language or even dialect. This is usually specific to a language/dialect and the corpus can be extensive. One or more languages can share same, or similar, clause constructions. This is basically a means of representing how nouns, verbs, articles and prepositions can be arranged together and ordered to form clauses which, in turn, are ordered to create sentences. Gender can be considered at a higher level and, above this, word order. It is believed that a universal grammar stands at the top of this hierarchical structure, branching out into different languages, some of which share a common branch.

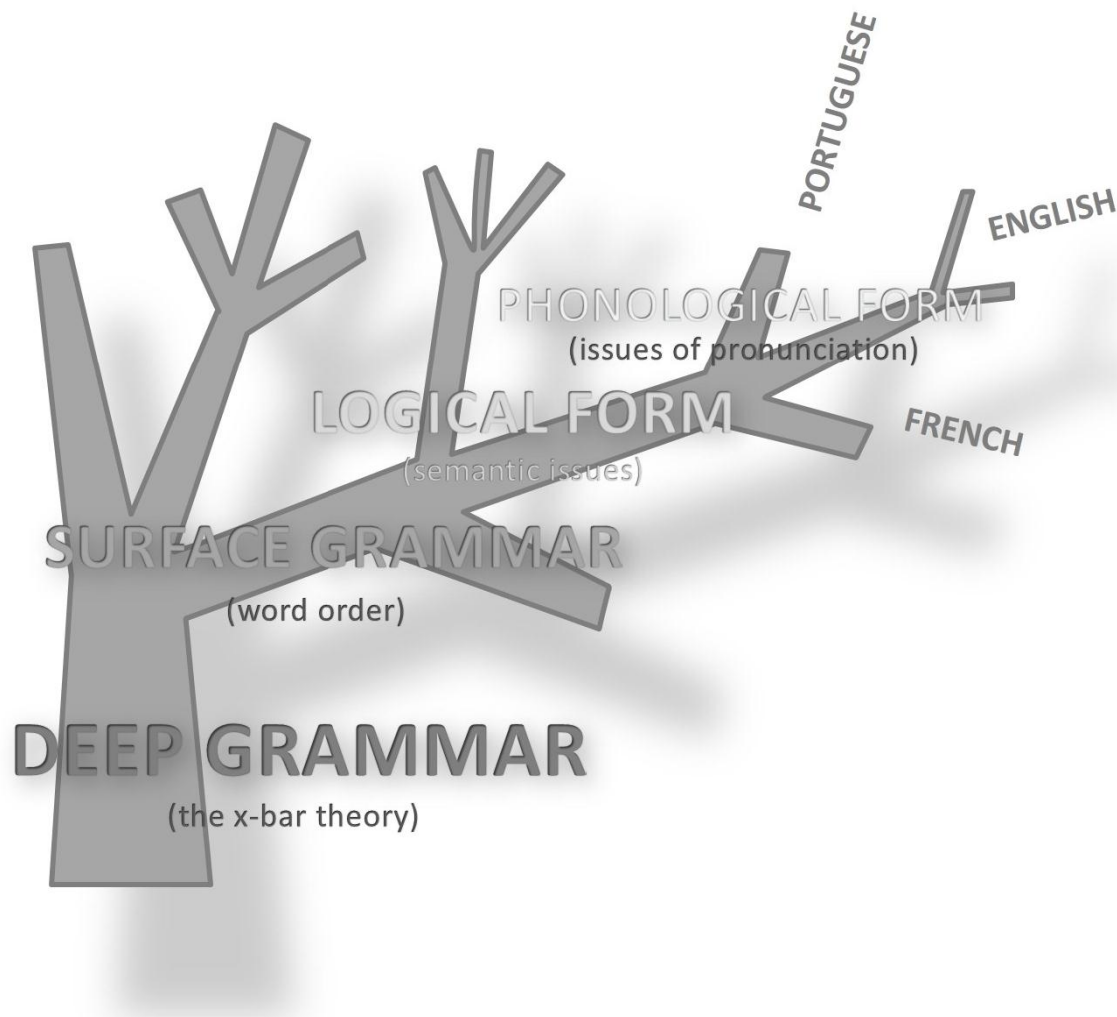


Figure 2. 2: Deep grammar x-bar theory

Deep grammars can also be illustrated with another analysis. Pronunciation issues are related to the first means of specifying each language. Even within a language, particular dialects or regional differences occur, contributing towards creating phonological form. Semantic issues, in turn, can be described as the logical form. Word order, or even clause order, can help describe the initial level of a surface grammar. The deep grammar seems to concur in most languages studied by Chomsky and this is significant. He illustrated this point with the x-bar theory, which provides a graphic illustration of how clauses are put together and arranged in many languages to create sentences. A clause containing a noun and an article is called a noun clause and is represented by (n+p-bar). In the example below this is transcribed as 'The clock'. It can be followed by a preposition clause (p+n-bar) 'in' and completed with another noun clause, 'corner' (n).

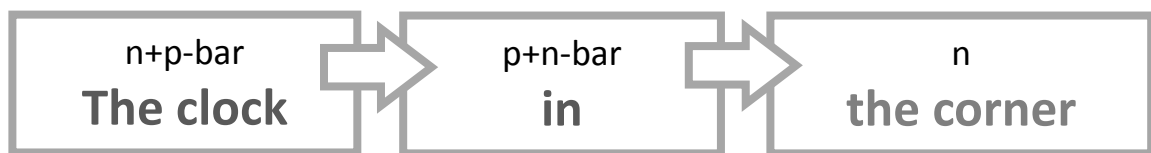


Figure 2. 3: Universal grammar N-bar theory

"Each grammar is simply a description of a certain set of utterances, namely those which it generates". Utterances can be described as observations. Utterances, speech and sequence consist of one or more words.

"Any scientific theory is based on a finite number of observations, and it seeks to relate the observed phenomena and to predict new phenomena by constructing general laws in terms of hypothetical constructs.

Observation is also addressed by Popper, in reasoning about logic. This helps us understand the role of logic in semantics. He states that "we are justified in inferring universal statements from singular ones, no matter how numerous; for any conclusion drawn in this way may always turn out to be false: no matter how many instances of white swans we may have observed, this does not justify the conclusion that all swans are white. The problem with induction may also be formulated as the question of the validity of the truth of universal statements which are based on experience, such as the hypotheses and theoretical systems of the empirical sciences." (Popper 1992)

Chomsky considers utterances as observations, the role of observations are key to identify and infer grammar rules: "A grammar of English is based on a finite corpus of utterances (observations) and it will contain certain grammatical rules (laws) stated in terms of the particular phonemes, phases... These rules express structural relation among the sentences of the corpus and the indefinite number of sentences generated by the grammar beyond the corpus (predictions). Our problem is to clarify and develop the criteria for selecting the correct grammar for each language". Grammar rules may be considered laws, while the corpus may correspond to predictions, and grammar to theory: "A grammar of a language is a complex system with many varied interconnections between its parts. In order to develop one part of a grammar thoroughly it is useful to have some picture of

the character of a completed system.” A typical grammar rule could be represented by the following expression:

$$X \rightarrow NP \rightarrow Y$$

If X and Y are strings through transformation, the transformation is responsible for converting object into subject.

A grammar is closely linked with meaning and therefore closely linked to semantics. “The major goal of a grammatical theory is to replace this obscure reliance on intuition by some rigorous and objective approach. (...) A grammar of a given language must show how these abstract structures are actually realized, while theory of linguistics must clarify these foundations for grammar and the evaluating methods”.

The question of meaning, or its logic, is described by semantics. “In describing a meaning of a word it is often expedient to refer to the syntactic framework in which the word is usually embedded”. Linguistic theory must provide an evaluation procedure for grammars, taking into consideration the fact that the most efficient grammars are studied independently of semantics.

To understand a sentence it is necessary to reconstruct its representation on each level, including the transformational level where the kernel sentences underlying a given sentence can be considered. Chomsky bases his universal grammar on theorised principles of an innate human grammar. He believed that we divide words into different systematic categories. These categories, commonly known as word types such as nouns, verbs, prepositions and articles used in accordance with syntax rules, help us construct sentences. The syntax rules are recursive and can be applied repeatedly to obtain a set of results at each stage. Although diversity is an issue in languages, these syntax rules seem to follow the same simple recursive pattern. By isolating this pattern, an abstract grammar can be obtained, the true essence of a universal grammar. The x-bar theory discussed above is a clear illustration of this, since X and Y are categories which, if combined in a sentence structure, can be separated into the x-bar or y-bar according to the clause. The formulation of a sentence in a given language would involve the addition of these bars in a simple schema that can be represented by $x\text{-bar} = x + y + \text{bar}$. This rule is then applied recursively.

$$\begin{aligned} n\text{-bar} &\rightarrow n + p\text{-bar} \rightarrow p + n\text{-bar} \rightarrow n \\ &\rightarrow \text{The clock} \rightarrow \text{in} \rightarrow \text{the corner} \end{aligned}$$

The difference is established between syntax and semantics. Syntax refers to sentence structure whereas semantics refers to the study of meaning, creating a relationship between words, phrases and signs: ‘Semantics in describing a meaning of a word it is often expedient to refer to syntactic framework in which this word is usually embedded’.

The linguistic theory must provide an evaluation procedure for grammars. The best grammars are studied independently of semantics. In order to understand a sentence it is necessary to reconstruct its representation at each level. Including the transformational level where the kernel sentences

underlying a given sentence can be considered. The formal study of grammar structure is directly connected to the syntactic framework and this supports the semantic analysis.

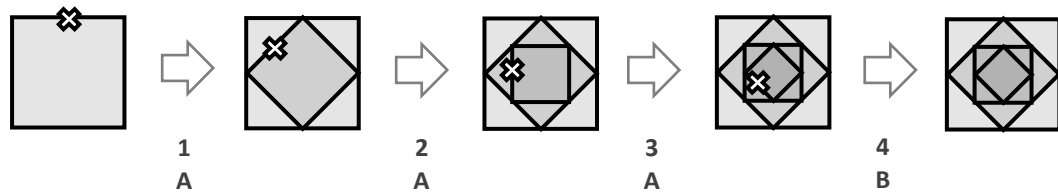
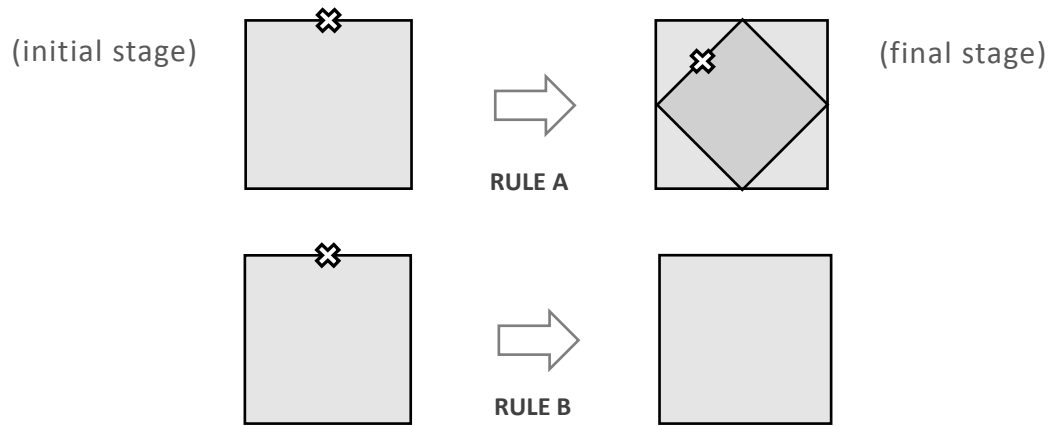
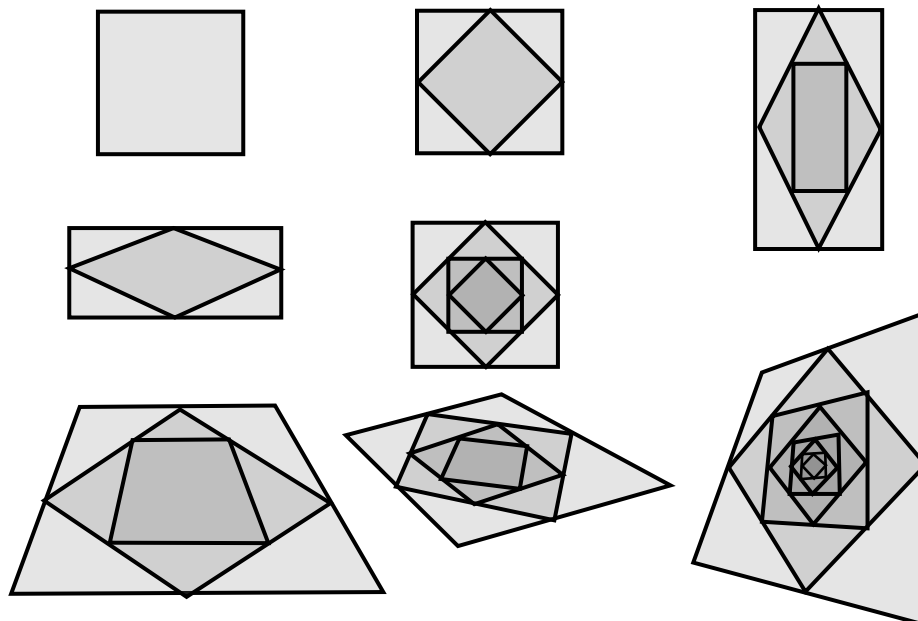


Figure 2. 4: Grammatical shape rules

Figure 2. 5: Derivation

Figure 2. 6: Corpus of shape grammar solutions



2.2 Introduction to shape grammars

The idea of a language to describe the way we speak has been used since early times to describe the way we design. Design language stands as a synonym for design style, and it is not surprising that if a specific design language exists, we can also infer a shape grammar. It is merely a question of representation, and shape is the most common way to represent an object. Typology, function can also represent an object but shape transmits a powerful message.

An object can be also represented by its coordinates, insertion point, material and other properties but in design shape still figures as the most representative way to describe an object. Therefore, if a grammar of a given spoken language is composed of syntax, a lexicon of words and a set of construction rules, a shape grammar would use a lexicon of shapes and a set of shape rules.

This was suggested in the 1960s by George Stiny and was demonstrated by a simple set of shape rules. It applied the same principle as Chomsky's, namely that a finite number of rules can generate an infinite number of solutions. A shape grammar was demonstrated using a geometric transformation (Stiny 2008). A square is transformed into the original shape plus a smaller circumscribed, scaled, centred and rotated second square, as illustrated in Fig. 2.4. The creation of simple and complex shapes can be represented by construction or transformation rules. These rules, grouped as a set, describe a specific language and help create a family of designs, which are a group of shapes that share a similar generative process and therefore share intrinsic design properties and similarities as a final design solution, as illustrated in 2.6. As with linguistics a limited set of shape rules can produce an extensive set of design solutions or corpus.

Shape rules can be established simply by a graphically illustrated initial shape (e.g. a square), a transformation (commonly represented by an arrow which stands for a geometric operation) and a final stage (the complex shape, in this case a circumscribed rotated square). These rules can be applied recursively by a process of embedding. As an illustration of a transformation and rule application, a derivation process is graphically represented in Figure 2.5. Derivation consists of the consecutive, step-by-step application of rules, from the initial stage to the final design solution. It shows how rule A can be used time and time again to generate a solution. Rules A and B together form a 2-rule set shape grammar. Rule B demonstrates another important element, which is a specific condition. This condition is demonstrated by a label. The label describes a special condition that allows the rule to be applied and normally demonstrates that the shape is in the early stages of design and can be transformed.

Shape grammars can be defined as a formulation composed of geometric shape rules or transformations which, if applied recursively and ordered in successive steps, can produce a family of designs that share the same design principles, features or style.

It has proved to be an efficient way to describe architectural styles (Stiny and Mitchell 1978), design (Cagan and Osborne, 2008), (Knight 1980) and even painting styles (Knight 1989). The rules

encompass extended expert knowledge that takes proportions, shapes, and spatial relations into consideration.

The first experiment based on an architectural example was the Palladian grammar (Stiny & Mitchell 1978). This grammar reproduced the bi-dimensional floor plans of the villas and was the first real implementation of the shape grammar theory in a design context. George Stiny and Bill Mitchell re-examined the work of the Italian Renaissance architect Andrea Palladio and, after carrying out extensive research into the different villas designed by the architect, narrowed this down to 72 shape rules that encoded the design principles they had discovered. This grammar was then used to produce a new corpus of designs (G Stiny & W J Mitchell 1978). The new solutions followed the extracted or inferred rules and recreated Palladio's corpus of design in a similar language. The language was recreated by replicating a corpus of existing villas floorplans.

Although highly controversial the shape grammar theory proved to be an operative system that replicates a style or design language, whilst also allowing for exploration of the design and consistent diversity.

The shape grammar rule formulation consists of 3 stages:

1. an initial stage
2. a transformation
3. a final stage.

An illustration of a typical shape grammar rule is provided in figure 2.4. The first stage contains the original shape(s) or simply an empty space. The transformation is no more than a geometric operation, within which copy, rotation, scale, subtraction, addition, subdivision and many more functions are accepted.

"Introduction to Shape and Shape Grammars" (Stiny 1980a), systematised the particular shape grammar formulation which presents and explains many of the basic concepts. The definition of shape is presented as a "limited arrangement of straight lines defined in a Cartesian coordinate system with real axes and an associated Euclidean metric". Complementary shapes or sub-shapes are defined in a similar manner. A line L , in which $L = \{p_1, p_2\}$, "is determined by a set of 2 distinct points". Moreover "a shape is a sub shape (part) of another shape whenever every line of the second shape thus a shape S_1 is a sub shape of S_2 (denoted by $S_1 \leq S_2$, an empty shape is a sub shape of every shape"

Boolean operations are also categorised in this work:

- A shape union (Defined by $(S_1 + S_2)$) "Is the shape consisting of all lines in S_1 or S_2 .
- A maximal line of S_1 + a Max line of S_2 can combine to form a longer maximal line $Q_1 + Q_2 \Rightarrow S_1 + S_2$, $P_1 + P_2$ Shape difference (Defined by $(S_1 - S_2)$) consists of only the lines in S_1 that are not lines in S_2 .

2.2.1 Shape grammar notions

Shape Transformation (Defined by $r(S)$) is the group of geometrical operations responsible for modifying the exact location of the vertexes of a given shape and its sub-shapes relative to its origin. Translation, rotation, reflection and scale are examples of this.

Another important notion directly related to the application of transformations is that of labels or labelled shapes. This involves an initial stage in which the shape is in its original form, the transformation itself and the final stage in which the resulting shape is produced. The label often indicates the initial stage and shows that a transformation may occur. "The labelled shape, consisting of a shape but with no symbols associated with it, is denoted by $\langle S, 0 \rangle$ where 0 is the set of labelled points containing no elements. A label can be a simple graphic representation in the form of a tick, cross, point, bubble or any other expression to signal the transformation and shape. It can be accompanied by a note that provides extra details and information.

In a shape grammar elements derived from the same set of rules constitute solutions that share similar formulations, features and characteristics. The universe of possible solutions, whether finite or infinite, is called the corpus of solutions. A selection or part of that corpus can be called a family of solutions, and can be defined "in terms of a given shape by allowing the component elements to be dimensioned in accordance with certain specified criteria. (Stiny 1980a)

"A family of shapes share a similar language that describes the way shapes are represented in that group. A language of shapes is a means of describing how similarities can be identified. (...) It is defined by a parameterised shape S which is obtained by allowing the coordinates of the endpoints of maximal lines in an assignment of real values to these variables in different shapes that contain similarities. "Shape grammar formalisms allow algorithms to be defined directly in terms of labelled shapes and parameterised labels so that each algorithm defines a language of shapes." The 4 basic elements used to create a shape grammar are therefore shapes, symbols, shape rules and initial shape.

"A shape grammar defines a set of shapes called a language. This language consists of all of the shapes generated by the shape grammar (...) each of these shapes is derived from the original shape by applying the rules, each made up of shapes and sub-shapes of shapes in the set S " (Stiny 1980a)

Shape rules are a group of threefold instructions that progress from an original shape in its initial stage to a transformation (or geometrical operation) and a final stage or solution. In principle they may represent a simple transformation containing just a geometrical operation or a complex variety of transformations occurring concurrently in one step. These shape rules are often represented in a parametric formulation. This means that there is a generic representation as well as a graphic representation of the rule. The parameterisations are expressed by the schema or the algebraic

expression that accompanies the illustration, thus allowing for a more flexible application of the shape rule. For example, the graphic representation might show a square but the expression might include polygons with sides of different dimensions, thus allowing for rectangles, trapezoids and other four-sided shapes. It is often the case that parametric formulations generate rich and unexpected solutions, enabling shape grammars to be used for shape exploration.

A particular type of shape grammars are the parametric examples. These propose formulations based on variables. This allows for flexibility and customization. "Parametric shape grammars are an extension of shape grammars in which shape rules are defined by filling in the open terms in general schemas. A shape rule schema $\alpha \rightarrow \beta$ consists of the parametric labelled shapes α and β ."

"Shape grammars are best used to define languages of shapes with proportional relationships determined by arithmetic or geometric series (...) parametric shape grammars preserve straight lines but their relative dimensions and the angles between them may vary. They can be used to define languages of shapes with proportional relationships determined in anyway." (Stiny 1980a)

In 1980 the first three-dimensional shape grammar was published, inspiring other housing shape grammars such as the Prairie House, Malagueira, Queen Ann and Buffalo Bungalow grammars and entitled "Kindergarten grammars: designing with Froebel's building gifts" (Stiny 1980b). In this study Stiny reflects once more on the importance of design families and languages, stating that five stages must be followed to produce a language:

- 1st - specify a vocabulary of shapes
- 2nd - arrange the spatial relationships between the shapes addressed in the vocabulary
- 3rd - define the space rules according to the shapes selected and the spatial relationships determined
- 4th - establish rules for the shapes in the initial stage, leading to the transformation, which creates the final stage
- 5th - assemble the set of rules to create the grammar.

The complexity and extended number of shape rules adds to the difficulty of applying shape grammars and therefore the possibility of creating computational tools as a strategy has proved useful and time-saving. Often algorithms have been created to encode complex shape grammars. An algorithm is simply a set of precise instructions to accomplish a given task. "Design machines" (Stiny & March 1981), co-authored by Stiny and March, was published in 1981. According to the authors, a design machine is composed of three elements:

- a receptor
- a design language
- an effector

With regard to shape grammars, they state "The theoretical mechanism that would make it possible the construction of a design machine is a shape grammar", a concept initially developed by Stiny

and Gips (Stiny & Gips 1972). A shape grammar is defined as a set of transformation rules applied recursively to an initial shape to obtain a design that may be used for codifying a given house type." The authors overcame these problems by merging the different perspectives into a set of instructions that specify how to generate new instances of the type. There is a semantic equivalence between type and grammar. Shape grammar is the formalism used in the proposed design system for making new designs, solving the generation problem.

According to Stiny "shape grammars provide the foundation for a 'science of design' and for a theory of architectural composition." They are more suited for teaching composition and visual correlates such as proportion and symmetry". "A well crafted grammar (...) may be used to classify designs and to predict unknown or hypothetical ones successfully and it can serve as platform for theories of style that go far beyond compositional issues so far as to explore historical issues" (in Duarte Ex of application in arch practice). Computer applications provide range and power for shape grammars, enabling the non-specialist user to avoid dealing with the technicalities of formulation and offering advanced grammarians the possibility of rapid exploration of rules and design possibilities.

The requirements for a successful shape grammar implementation are the need to clarify the structure and appearance of buildings, to determine whether any building not in the original corpus is an instance of the style adopted and to simplify it, and, finally provide the compositional machinery needed to design new styles of buildings.

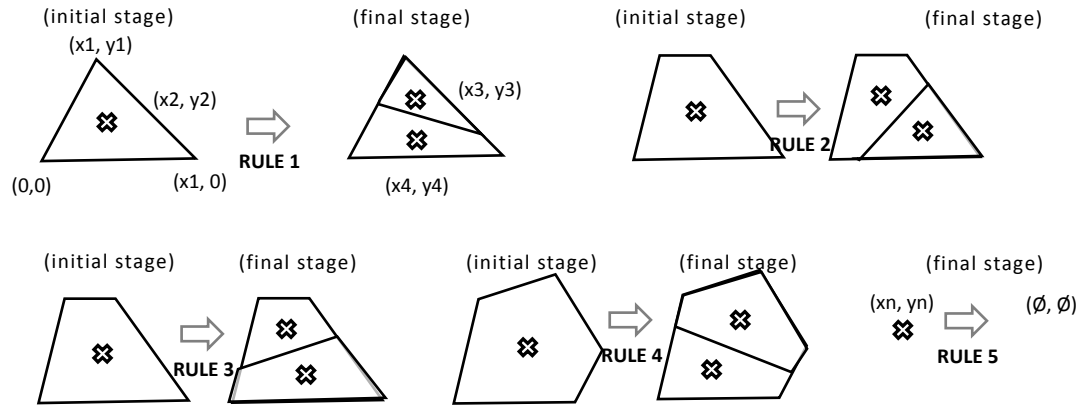


Figure 2. 7: Ice-ray shape rules

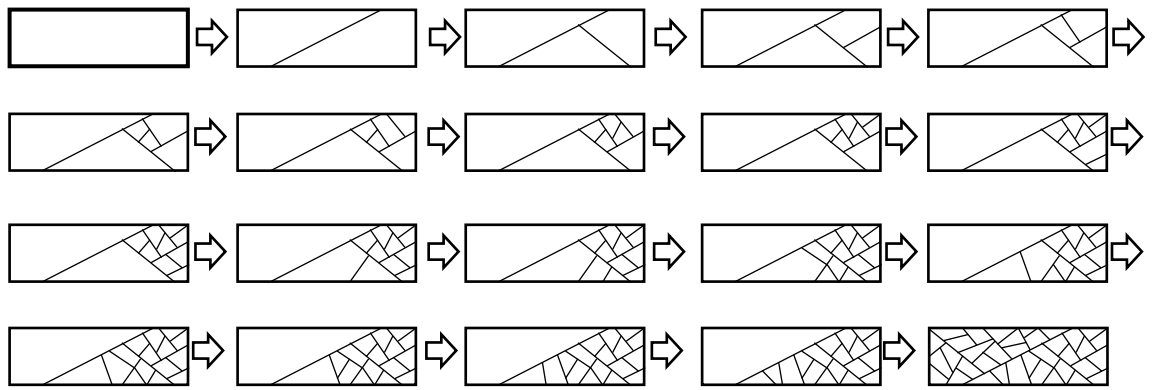


Figure 2. 8: Ice-ray shape derivation

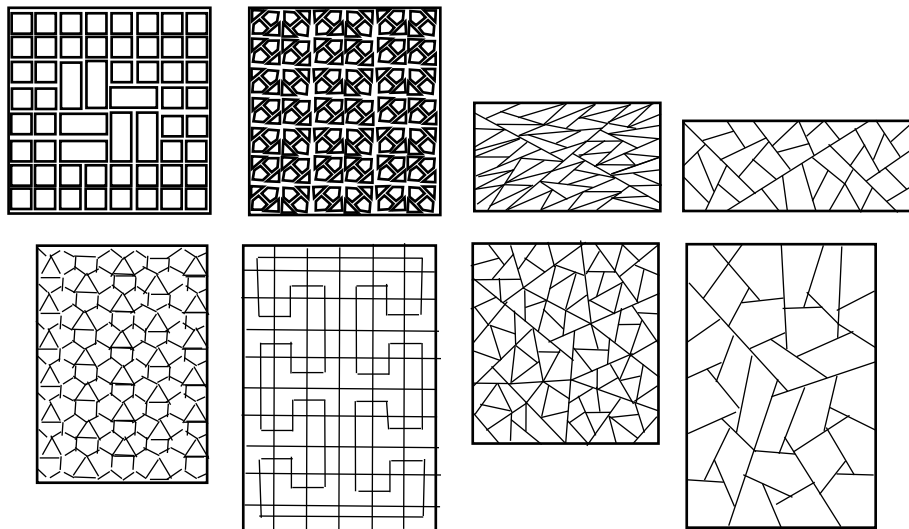


Figure 2. 9: Ice-ray shape corpus of solutions (Stiny 1977)

One grammar inferred from a pre-existing set of designs with defined language but no formal author is the Ice-Ray grammar for generating Chinese lattice designs. (Stiny 1977) The novelty of this proposal was the new type of grammar introduced, the subdivision grammar. The subdivision type was identified according to Knight's grammar classification (Knight 1994), discussed in the next section.

The Ice-Ray grammar was implemented to explain the apparent random design of Chinese frames composed of diagonal mullions and transoms arranged at varying angles. The outcome is a net-like matrix of an irregular triangular grid resembling cracking ice and therefore named after this phenomenon (see Fig. 2.9).

The elegance of this parametric grammar would later inspire other famous housing grammars (such as the Malagueira and Buffalo Bungalow grammars, or even the alternative Palladian grammar presented in a later chapter). It contains a small set of five transformation rules, as shown in Figure 2.7.

1. Rule 1 subdivides a triangle into two shapes, a triangle and a trapezoid. This is a parametric shape rule and generically covers a large amount of possible triangular shapes regardless of their internal angles and lateral dimensions. The outcome is also generalised for any angles or dimensions as long as the main features (shapes) are included. Similarly
2. Rules 2 and 3 divide a four-sided polygon into two smaller shapes. Rule 2 creates a combination of triangle and trapezoid in its final stage whilst
3. Rule 3 generates two trapezoids in the same stage.
4. Rule 4 starts with a pentagon as an initial shape and generates a trapezoid and a pentagon of approximately the same area.

Although there is a very limited set of rules, the outcome of this grammar is quite unpredictable in the sense that it can generate countless solutions, ranging from very regular grids to ice-ray patterns, as illustrated in the sample from the corpus of solutions shown in Figure 2.9.

One of the strengths of this grammar is its tectonic aspect. Although it is quite a straightforward design, it is derived from real window-frame making techniques and is based on the artisan's perspective and the tectonics of window framing. The Chinese artisan, for constructive reasons, would start with the boundary and progress inwards. The overall shape (usually a rectangle and therefore a four-sided polygon) would be then divided into two portions of a similar size by a diagonal element. This is replicated in Rules 2 and 3. Typically, this diagonal element would be structural, spanning opposite sides and creating the two aforementioned shapes. The sequential steps are executed bearing in mind the need to structurally bridge the space between opposite elements until an evenly spaced grid is produced.

There is no limit to the number of recursions that can be performed, although logically the cells cannot be smaller than the width of the frame elements. Derivation consists of the succession of steps and transformational shape rules used in an orderly manner from the initial stage up to the final one. One possible derivation example is shown in Figure 2.8.

The issue of which rule to use and when is frequently debated by the latest generation of grammarians and is a focus of reflection in "What rule(s) should I use?"(Stiny 2011). Generally, shape rules can be used at any time, as many times as desired unless stated by the grammar conditions or labels. Some grammars order specific shape rules to occur in specific stages (i.e. the Palladian Villa grammar proposes a specific stage for spatial concatenation of the nuclear social space and addition and space merging is only applied there). The application of shape rules and its order of application will reflect on the design language itself. The sequence of application can be set by the grammar structure just like the exact rule application sequence.

This grammar showcases the range of possible solutions and diversity when given a limited rule set and exposing the potential of grammar formulation.

The Palladian Villa grammar was developed in collaboration and is the first grammar implementation based on architectural design (Stiny & Mitchell 1978). In a way, it demonstrated the feasibility of the shape grammar theory, showing it is effective as a means of representation and recreation of a design language. From the corpus of existing Palladian villas the grammar proved successful in recreating the floorplans. A derivation for villa la Malcontenta is shown in this work.

The grammar was developed bi-dimensionally, focusing on the main floor of Palladio's villas. The interior layout, which is illustrated using a grid process, is therefore the main focus. A self-contained shape within an external rectangular boundary villa is first sketched from the inside out using a bottom-up methodology:

1. The grid is then filled cell by cell through a process of copying and symmetry until a composition with a 3x3 to 5x3 array of cells is obtained. This constitutes the first of eight consecutive steps discussed in further detail in the next chapter.
2. The second stage defines the exterior wall.
3. The third step rearranges the central space by a process of spatial merging in which adjacent cells are concatenated.
4. The fourth stage completes the layout of the interior using wall realignment to accommodate spaces.
5. The fifth stage defines the entrance point and defining elements.
6. The sixth stage adds decorative features to the exterior, such as classic columns, frontons and cornices.
7. Detailing begins in the seventh stage using a process of subtraction, and walls are also opened to create doorways and windows.

The eighth and final stage is a technical stage which involves deleting labels and terminating the design.

This study created a great deal of controversy amongst architects and art historians and architecture (Hersey 1992). This was only to be expected from a methodology or formulation capable of replacing the designer's creative vision, and is even more controversial if it hints the

future possible computation of rules using an automatic tool (Duarte 2001a), thus touching on ethic and deontological boundaries and raising questions about authorship and creativity.

These topics were addressed in further published studies reflecting on the implications of grammars and specifically the Palladian shape grammar. "Counting Palladian Plans" showed the complete range of solutions made possible by the parametric shape grammar, based on plans with 3x3 and 5x3 cell arrangements (G Stiny & W J Mitchell 1978)(Stiny & Gips 1978). Several of the examples can be found in existing Italian villas and others are pictured in Palladio's treatise "Il Quattro Libri dell Architettura" (Palladio 1997), in addition to numerous new options generated by the grammar. The layout arrangements for plans with a 3x3 matrix include 20 different options. Plans are considered different if they cannot resemble any other when using symmetry, rotation or other geometric operations. Villa Angarano, designed by Palladio is included in this set. With regard to 5x3 villa layouts, 220 different options were found including existing examples such as the Badoer, Seno, Malcontenta, Pisani, Emo, Sarraceno, Ragona and Poiana Villas.

Within the corpus of solutions, several of those generated clearly followed Palladio's intuitive rules. Others were refuted by Palladio scholars as being unlikely Palladian designs. Questions of aesthetics and balance were also raised. "An evaluation of Palladian plans" (Stiny & Gips 1978) aimed to address these issues, its main focus being the extended corpus of solutions generated by the Palladian grammar. Evaluation criteria was established to analyse the results objectively. It was found that the grammar lacked certain aesthetic properties required to assess the results that were carefully generated by the shape rule application. Two types of criteria were therefore proposed. The corpus was divided into two groups, a small one which included the original Palladio designs and another containing the remaining solutions. Since shapes and proportions are not addressed directly by the grammar, this aesthetic evaluation is carried out by analysing the spatial layout of the floor plans.

"The evaluative criteria is based on the arrangements of rooms in terms of an underlying grid". (Stiny & Gips 1978). This aesthetic evaluation is drawn from Palladio's emphasis in the treaty on beauty and symmetry. Two rules were then established by the author: any room that crosses the floor plan from one façade to the opposite one must lie on top of the symmetry axis, as well as the central social space with a north/south orientation. The second rule addresses the rooms not intersected by the symmetry axis, which must be similar in size to the others in the plan, relate directly to the cell size within the grid, and respect the symmetry defined by the north/south axis.

The second and largest group, the corpus of new Palladian designs generated by the grammar, contains examples that do not conform to one or both of these rules. In the 3x3 grids three plans satisfy both provisions, whereas in the group of plans for 5x3 grids fifty-seven do so. In addition to these two rules in the 'selected corpus' another underlying rule was detected and inferred. Four

adjacent cells forming a square must never be either immediately adjacent, forming four similar adjacent independent rooms, or merged within the same space. Even though this 'rule' was empirically inferred, neither it nor the previous two rules were ever used in the grammar. Instead they were used merely for evaluation purposes in order to assess the beauty or spatial quality of the villas. However, in Stiny and Gips (1978) an aesthetic measure was presented for the grammars. "This measure assigns aesthetic values to objects of a certain type or style in terms of their descriptions and the rules used to generate them. (...) The aesthetic value of the object is given by the ratio of the length of the description of the object to the length to the information required to generate it by use of fixed procedure". A method for attributing wall openings to plan layouts according to the array of cells, their layout and the complexity of shapes used was defined. This system of attributing points it is correlated with what the authors called the visual complexity of a plan. The more complex or intricate designs scored higher: not surprisingly the Malcontenta (89), Ragona and Zeno Villas received high scores.

Aesthetic value and the notion of creativity formed the basis of the greatest criticisms levelled against the shape grammar theory, and will be discussed in later chapters.

2.2.2 Rule Schemas

The attraction of shape grammars can be explained mostly by the fact that they can be illustrated figuratively, enabling the formulation of a rule, and consequently its application, to be grasped quickly.

However, shape rules are nothing more than algebraic operations that can be expressed mathematically. These expressions can help provide an accurate description of the rules.

Another method for describing the rules in a more generic manner involves using schemas. Schemas are expressions based on algebraic formulations that describe shape operations whilst maintaining a level of abstraction. They simulate shape rules using a written form. For instance, a shape rule is composed of an original shape, a transformation represented by an arrow symbol and a final shape. Schemas replace shapes and operations with mathematical variables and the operations are symbolised by mathematical symbols. As in shape rules, the transformation makes use of the arrow notation. Additionally, the notion of initial and final stage can be indicated by the position of the expression in relation to the direction of the arrow.

These schemas are simplified expressions that code shapes and transformations, using simple characters or letters to state how the rules work. In Stiny's words 'Shapes may be added or subtracted, and changed by the transformations. The operators involve parts of shapes, transformations and boundaries' (Stiny 2011).

Thus, the following three levels of entities contain:

- 1) $\text{prt}(x)$, which determines a part or portion of a given shape
- 2) $t(x)$, which transforms x , the original shape, into a different shape or shapes
- 3) $b(x)$, representing the boundary of a particular shape

When combined these three elements or schema types provide enough information to calculate with shapes and deliver a given design.

A generic lattice is represented below, with the common schemas grouped in sequence:

$x \rightarrow x'$ parametric variation	$x \rightarrow y$ unrestricted rules	
$x \rightarrow t'(x)$ projective transformation	$x \rightarrow \text{prt}(x)$ part of shape	
$x \rightarrow t(x)$ Euclidean transformation	$x \rightarrow$ erasing rules	$x \rightarrow b(x)$ boundary rules
	$x \rightarrow x$ identity	

Table 2. 1: Lattice schemas, adapted from (Stiny 2011)

The first schema represents the generic transformation, in which x is the original shape, the arrow represents the transformation and x' the resulting object. This schema is representative of any given shape rule without describing the type of transformation which takes place.

The second expression, represented by $x \rightarrow t(x)$, can be described as a Euclidean transformation. Rotations, symmetries, translations or moves, copies and rotations can all be included in these transformations.

It is also possible to identify $x \rightarrow t'(x)$ or projective transformations. These are of particular interest to designers, architects and engineers since they reduce the dimensions of use in the graphical representation of design objects from three to two. The transformations allow the user to project a given perspective or view onto a plan, thus reducing the dimensionality by one.

The schema $x \rightarrow y$ represents the unrestricted rule which allows the original shape x be substituted by y , which terminates the design. In shape grammars it is often a technical rule used in a specific context.

The schema $x \rightarrow prt(x)$ is a rather useful rule in shape grammar formulation. The original shape x is subjected to a transformation that is specific and targeted at the illustrated object. The transformation that occurs applies only to a specific part of the original shape. Therefore, the final step in the rule is represented only by $prt(x)$ or part of the initial shape. This is the first step towards a more complex representation of shape calculation. The schema allows shapes to be divided or subtracted and is particularly powerful when combined with other rules. It is frequently used in Boolean algebras for shape decomposition.

The schemas that follow, as part of a group termed by Stiny the 'empty rules' are often represented by a single arrow ' \rightarrow '. This symbolises a transformation and the transition from an initial shape to the final stage. It is 'merely a technical device'. From this simple schema, two additional schemas can be highlighted. The first is the ' $x \rightarrow$ ' erasing rule. This rule allows for the simple deletion of the original object ' x ' and, as an empty rule, constitutes a very valuable technical process for shape grammars. This schema is often used for the formulation of label deletions in shape grammars and frequently occurs in final stages. The following schema is the identity rule expressed by ' $x \rightarrow x$ '. As the name suggests, this rule converts the initial object into the object itself. At first sight it might appear redundant, but it has proven its usefulness when combined with other operations i.e. ' $x \rightarrow x + y$ '.

Many shape operations performed by designers are based on shape boundary. For this reason the schema $x \rightarrow b(x)$ plays an important role in shape rule schemas. It converts a certain shape ' x ' into its boundary or perimeter and in any transformations that involve the boundary. Once more, this provides scope for interpretation and manipulation when combined with other operators and therefore widens the potential of this schema.

Rule	$x \rightarrow$	$x \rightarrow \text{prt}(x)$	$x \rightarrow t(x)$	$x \rightarrow b(x)$
Inverse	$\rightarrow x$	$\text{prt}(x) \rightarrow x$	$t(x) \rightarrow x$	$b(x) \rightarrow x$
Alternate form	$\rightarrow x$	$x \rightarrow \text{prt}^{-1}(x)$	$x \rightarrow t^{-1}(x)$	$x \rightarrow b^{-1}(x)$

Table 2. 2: Rule, inverse rule and alternate form schemas, adapted from (Stiny 2011)

Since these schemas derive from a simplification of algebraic expressions, the inverse of these operations can also be calculated, thus aiding the representation of inverse shape calculations to a greater extent than previously discussed. Stiny proposes flipping the final stage, with the initial stage thus describing a number of operations that can be foreseen. This is described in detail in the table above.

The first row represents the rule previously described, and the second the inverse operations that result from a simple stage crossover. Interesting operations emerge from these inversions. The erasing rule ' $x \rightarrow$ ' is converted into an addition rule in which an empty stage is transformed into a new object ' x ', with the schema ' $\rightarrow x$ '. Following this, another variation can be performed to represent the expression in a novel way called an alternative form. This alternative form uses the number raised to the power (n^{-1}), the mathematical formula to express the inverse. As Stiny explains, 'These alternate forms are already clear for transformations. Instead of flipping schemas, (...) an inverse notation for parts and boundaries'. In this case, the inverse of 0^{-1} equals 0, therefore the alternate form coincides with the rule ' $\rightarrow x$ '. (Stiny 2011)

In the schema ' $x \rightarrow \text{prt}(x)$ ' which represents part of ' x ', the inverse rule permits the transformation of 'part of x ' into ' x '. This could be interpreted as the enclosure of an open polygon x into a closed shape or merely the addition of an element that complements ' x '. The inverse form of the schema becomes ' $\text{prt}(x) \rightarrow x$ ', whilst the alternate form becomes ' $x \rightarrow \text{prt}^{-1}(x)$ '.

The following schema represents a standard non-specific transformation that could be any Euclidean operation in which ' $x \rightarrow t(x)$ ' or, alternatively, a shape ' x ' is transformed by a rotation, translation, copy or mirror into another object(s). The inverse of this is easy to imagine. A given shape operation is undone and the original object emerges. A good way to exemplify this is with an orthogonal object rotated in relation to the Cartesian axis. To rotate it to a position where its sides are parallel and perpendicular to X and Y , the inverse of ' $x \rightarrow t(x)$ ' can be applied to obtain ' $t(x) \rightarrow x$ ', where the resultant shape is the non-rotated object ' x '. This is an abstraction that can simplify certain operations and their formulations. The consequent alternative form would become $x \rightarrow t^{-1}(x)$. The schema ' $x \rightarrow b(x)$ ' is a particular case of the schema $x \rightarrow \text{prt}(x)$, although it is connected to an important part of the object ' x ', its boundary. Often in the design environment shapes are in fact represented and used taking boundaries into account. $x \rightarrow b(x)$ represents the transformation occurring exclusively in the boundary of x . The inverse of this would be represented by the schema

$b(x) \rightarrow x$ and its alternative form by $x \rightarrow b^{-1}(x)$. One practical example of this kind of operation would be the conversion of a group of lines into a shape resembling an enclosed polygon.

These schemas may seem abstract, but when combined offer various possibilities for designing efficiently, describing not only shapes but also the design process used to produce them. Schemas in the lattice and inverted schemas can also be combined to define compound rules ‘that are widely used in art and design’. Two combination methods may be used: composition and addition.

A simple operation can be represented by: $\text{prt}(x) \rightarrow b(t(x))$

Since it is difficult to describe a transformation that starts with the transformed shape itself, one way to simplify the understanding of this operation is to simplify the first and initial term of the schema from $\text{prt}(x)$ into x . In order to do so, an alternative method in which some of the denominators are substituted by their inverses or alternative forms is proposed, as follows:

$$\begin{aligned}
 &\underline{x \rightarrow \text{prt}^{-1}(x)} \rightarrow t(\text{prt}^{-1}(x)) \rightarrow b(t(\text{prt}^{-1}(x))) \\
 &x \rightarrow \text{prt}^{-1}(x) \\
 &x = x \\
 &\quad x \rightarrow t(x) \\
 &\quad x \rightarrow \text{prt}^{-1}(x) \\
 &\quad\quad x \rightarrow b(x) \\
 &\quad\quad x \rightarrow t(\text{prt}^{-1}(x)) \\
 &x \rightarrow t(\text{prt}^{-1}(x)) \\
 &x \rightarrow b(t(\text{prt}^{-1}(x)))
 \end{aligned}$$

Table 2. 3: Lattice schemas, adapted from (Stiny 2011)

Instead of $\text{prt}(x) \rightarrow b(t(x))$ for a schema that is difficult to grasp from the start, $x \rightarrow b(t(\text{prt}^{-1}(x)))$ can be used, which transforms the original shape into the simplified linear square.

Apart from alternate forms, schemas can also be used as addition processes, combining more than one schema, as in a mathematical expression. For instance, geometric symmetry can easily be represented by the schema $x \rightarrow x + t(x)$, in which a transformed shape, ‘ x ’ or $t(x)$, is added to the original object ‘ x ’. This can also be represented using a different notation $x \rightarrow x + x'$, where x' represents the mirrored version of x . This schema is used in grammars such as the Kindergarten grammars, a typical addition grammar (Stiny 1980b), and Knight’s colour grammars (Knight 1994).

As a result of addition, these schemas can also be used in subdivision, as tested by Stiny in the Ice-Ray grammar. The Ice-Ray grammar is based on a subdivision grammar in which most shapes are divided more than once.

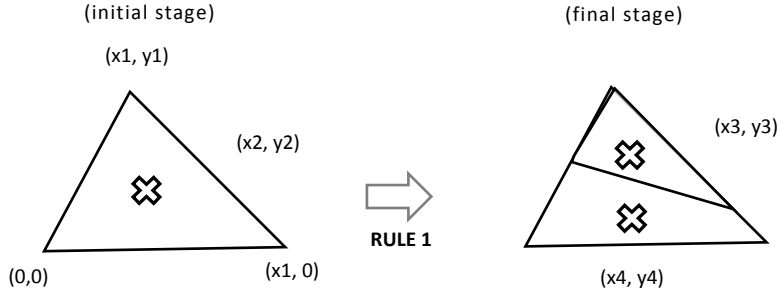


Figure 2. 10: Subdivision rules for the Ice-Ray grammar

‘The rules for ice rays divide polygons into polygons – triangles, quadrilaterals or pentagons into different pairs of these figures. A polygon x goes to a sum of its transformations x' and x'' in a fractal like fashion’ (Stiny 1977). If we focus on the first shape rule, the triangle constituting the original shape is divided into 2 objects, a smaller rectangle and a trapezoid. The schema that would demonstrate or explain this transformation can be easily expressed as:

$$X \rightarrow \text{prt}'(x) + \text{prt}''(x)$$

In which the triangle x is transformed into 2 independent objects, each derived from the original x and, in fact, a constituent part of x . Both the new triangle and the resulting trapezoid can be represented by $\text{prt}'(x)$ and $\text{prt}''(x)$ respectively.

In order to further simplify this expression, Stiny proposes another schema that produces it:

$$X \rightarrow \text{div}(x)$$

According to Stiny, ‘The schemas $X \rightarrow x' + x''$ and $X \rightarrow \text{div}(x)$ produce the same result when the operator $X \rightarrow \text{div}(x)$ divides X into x' and x'' . Addition and division look alike although the same – notably Christopher Alexander – try to make a big deal out of this timeless distinction’ (Stiny 2011). Although this schema was not included in the lattice schemas, its importance should still be acknowledged. It is the base schema for grammars like the Ice-Ray, Knight’s De Stijl paintings grammar (Knight 1989) for the work of George Vantongerloo and Fritz Glarner, Duarte’s Malagueira housing grammar for Siza (Duarte 2001a) and, as Stiny notes, the medieval building plans in Venice or even Frank Gehry’s roofs and façade cladding. However, due to the fact that this schema is merely an abbreviation or synthesis of other schemas such as $X \rightarrow X$ or $X \rightarrow \text{prt}'(x)$, it was not included in the lattice schemas.

Addition can be taken even further. The original Palladian grammar, for example, used a shape rule that simultaneously allowed the mirror of a single cell to be moved to a neighbouring position and the symmetrical group of cells to be replicated in the opposite wing of the villa.

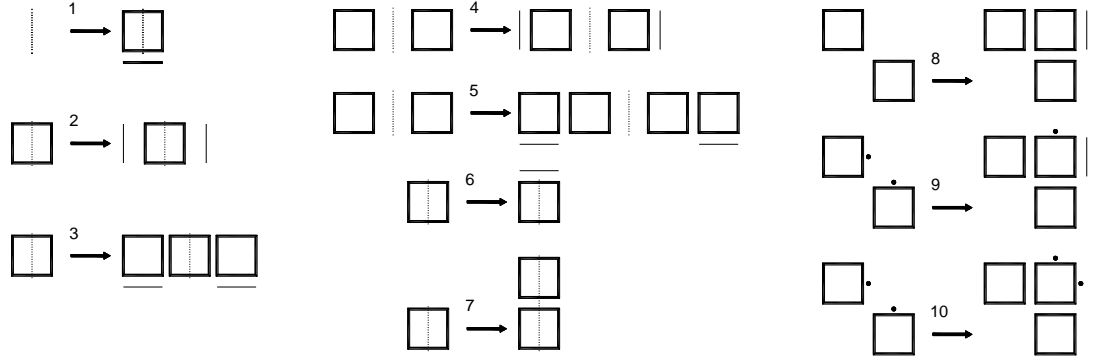


Figure 2. 11: Grid cell rules for the Palladian grammar (Stiny & Mitchell 1978) (adapted)

This rule can be interpreted as the addition of two rules: the symmetry of one cell $X \rightarrow x + x'$ and the same replicated by the symmetrical transformation of the opposite side $t(x) \rightarrow t(x + x')$. As these two rules are represented as one in Stiny and Mitchell's grammar, if we add them together:

$$X \rightarrow x + x'$$

+

$$\underline{t(x) \rightarrow t(x + x')}$$

$$\text{we obtain: } x + t(x) \rightarrow (x + x') + t(x + x')$$

Generically, taking into account that $(x + x')$ equals to $\text{div}(x)$, we obtain:

$$x + t(x) \rightarrow \text{div}(x) + t(\text{div}(x))$$

Technically the latter schema is not performing the same job as the former, even though it can represent the same operation. Interestingly, this later schema is also useful for representing another shape rule that uses bilateral symmetry in the same grammar, namely the concatenation rule for central rooms. If we use the inverse or even the alternative form for this schema it is not difficult to see that this can represent the concatenation procedure, in which $\text{div}(x)$, added to its transformed version, $t(\text{div}(x))$ will generate x and its transformed version.

$$\text{div}(x) + t(\text{div}(x)) \rightarrow x + t(x)$$

Moreover, another schema lattice can be shown in which simple schemas are implemented using additions:

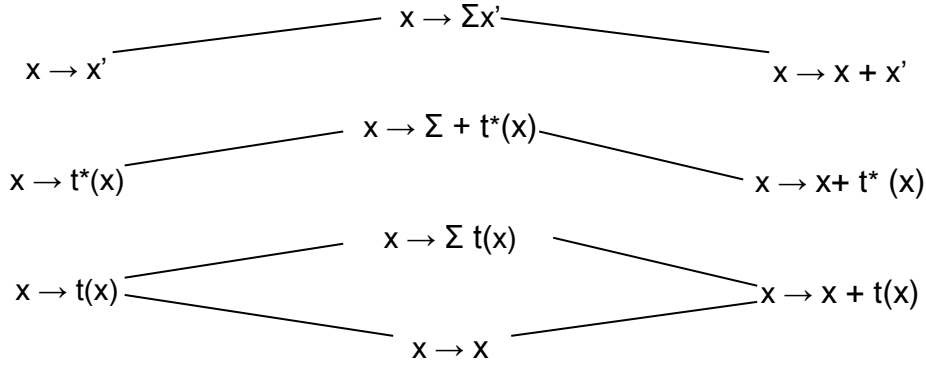


Table 2. 4: Lattice schemas , adapted from (Stiny 2011)

The left-hand column of the lattice focuses on elemental transformations with one operator at each end. X transforms into x or a given transformation of x . In the right-hand column the addition processes are considered and other items symbolising shape rules with an added value of complexity are added to the elemental rules, for example the symmetry rule already discussed, represented in the lattice by the schema $x \rightarrow x + x'$.

On the other hand, the central column represents more intricate rules with many applications. These rules are often preceded by sigma which, in algebra, symbolise a sum. These rules are linked to division rules of the type $X \rightarrow x' + x''$ or $X \rightarrow \text{div}(x)$ and, as described, are often connected to subdivision grammars such as the Ice-Ray or Malagueira grammars. However, their full potential also allows for the description of fractals or any fractal-like pattern design. A schema such as $x \rightarrow \Sigma x'$ can be decomposed into $x \rightarrow x + x' + x'' + \dots + x^n$ (or, alternatively, $X \rightarrow \text{div}(x)$), essentially representing the small or large sum of parts of x . In accordance with previous explanations, this could also take the form of the sum of parts of the shape x , resulting in $X \rightarrow \Sigma t(\text{prt}(x))$ or its inverse.

2.2.3 Embedding

The notion of embedding is relevant to studying and understanding schemas. In algebra, embedding is an instance of a certain mathematical structure contained within another instance, i.e. a subgroup contained within a group. In shape theory or design, embedding can be explained as the ability to identify a shape within another or within intersections of several shapes.

Shape grammars manipulate the embedding notion its benefit. When calculating with shape the grammarian looks for familiar shapes in different proportions or sizes in the overall picture. These can appear in intersections of two or more shapes.

'Shape grammars make art and design mathematics – well, at least calculating – by showing that rules in schemas match what is tried in practice. Whether you are aware of it or not, you are using schemas and rules – there is calculating anytime and anywhere you look. Design equals calculating. (...) 'Art and design enlarge calculating with embedding, while calculating goes on with schemas and rules. Embedding is the key.' (Stiny 1980a)

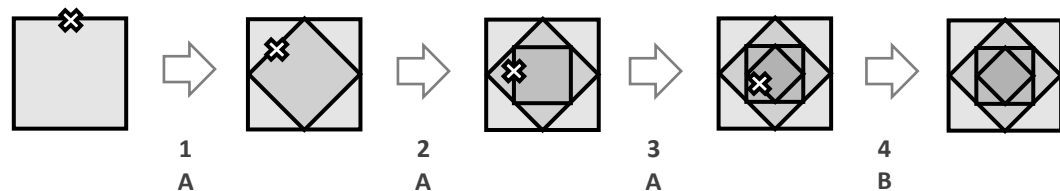


Figure 2. 12: Embedded square

In the picture above, for example, several shapes are embedded within others creating not one, but n shapes. The original square is followed by two squares, one of which is rotated and scaled down. The next derivation step allows the rule to be applied twice. A smaller square can be designed in the centre of the composition or a larger rotated square as a bounding shape. This is a common example of embedding. It is a particularly useful notion in design and its full potential can be explored by using the schemas. The shape rules of a given grammar can be applied in many ways to the overall shape, to one of the internal or sub-shapes or to all of them, producing several different types of design solutions.

'Without embedding neither schemas nor rules allow me to see what I choose' (Stiny 2011)

'Shapes and rules may also have a relationship between schema and correction. Schemas are outlines in tree-trunks or clouds of earth, and rules are for correcting and refining lines (boundaries) and between schemas as archetypal shapes, and rules to change them to meet the ongoing demands of experience and circumstance seems to be a common idea in art and design. This works with shapes and the rules defined in my schemas.' (Stiny 2011)

‘In architecture, and sometimes in philosophy, a distinction is made between schemas and rules to contrast a synoptic view of possibilities. Maybe schemas and rules are parallel ways of doing the same kinds of things with the emphasis placed either on similarities in origin and classification (schemas) or on differences in process and elaboration (rules). Using schemas or rules is simply a matter of taste, or perhaps it is just a matter of words.’

‘Schemas to define rules, and rules (inverse compositions and addition) to define schemas may imply another kind of equivalence with further interconnections. Changing a schema and combining shapes leads to the same things, the differences between schemas and rules are trivial.’ (Stiny 2011)

The notion of schema is a flexible notion that holds great generative power. Shape rules have the benefit of using a graphical representation to convey a message. The schemas can replicate that message but make use of a level of abstraction. A shape rule is often used in its literal sense without much negotiation. The schema allows for its application over a wider range of situations without tying itself to a restrict representation. In addition, allows for ‘calculating with shapes’ in an optimized manner, where complex operations can be performed algebraically first and then replicated to the final stage to shapes.

These benefits of rule schemas associated to shape rules will be furthermore explored and used by the work described in this dissertation.

2.3 Shape grammar typology and analysis

Two types of grammar classification can be made, one referring to the grammar range and to the grammar rule set type.

Knight defines six types of grammar which refer to its range and type of restrictions applied. (Knight 1999) (Knight 1983a; Knight 1983b; Knight 1983c) Basically there are two types of restrictions, linked to two factors: rule ordering and rule format.

These six types are the basic grammar, non-deterministic basic grammar, sequential grammar, additive grammar, deterministic grammar and unrestricted grammar. The first three types all have in common the fact that they present closed restrictions with effects on rule ordering and rule format. Additive grammars restrict the rule format but place no restrictions on rule ordering, which means that any shape rule can be applied at any stage without endangering the final design. The deterministic grammar does place restrictions on rule format but is relatively free with regard to rule ordering. These grammars present a stratified sequence of events and are divided into stages. In each stage, a group of rules can be applied but these are exclusive to a particular step.

The three more restrictive grammars - basic, ND basic and sequential - represent grammars that guide the user step-by-step in order to control the final product. They all contain a defined set of rules to be used in a set order.

The basic grammar is a standard shape grammar with all the conventional signs: stepped stages for rule applications, the original, transformation and final stages, labels to describe the original shape and a final shape. Most of the rules in the set of rules are additive and there are no subtractive rules in the set.

The ND basic grammar is a variation on the basic grammar. The rules are also essentially additive, although they allow for one particularity previously described by Stiny as embedding. Embedding, in a grammar context, means finding a specific shape within a set of shapes or, more precisely, applying a transformation rule that is only applicable to a specific shape but, once completed, produces a shape with a similar geometry but different proportions, position or even rotation, resulting in a more complex element. This grammar is therefore termed 'non-deterministic' because, unlike the previous example in which it is easy to envisage the final result, its results are less obvious or even unintelligible at first, therefore creating a higher level of unpredictability. The results are often a conglomerate of simple geometries resulting in a complex array of shapes. According to Knight, 'Clearly ND grammars are more powerful than basic grammars. In particular the class of languages generated by ND basic grammars are useful for after initial explorations of design possibilities with simpler basic grammars.'(Knight 1999)

Sequential grammars are almost self-explanatory and in terms of level of restriction are less restrictive than the basic and non-deterministic types. Unlike the basic grammar, the set of rules are

not limited to additive rules, since labels can be erased and replaced by other shapes. The labels themselves can be deleted by specific shape rules which erase them, a new feature that is not evident in the basic grammar rule set. They are also deterministic, as the illustration shows. The rules are applied recursively in an orderly sequence and always refer to the restrictions defined by the labels.

Additive grammars constitute some of the most popular grammars and are also the easiest to describe. Their construction is less restrictive than the previous types, since they do not restrain the order in which the rules are applied. Schematically, these rules can be explained by the expression $A \rightarrow B$ in which A is different from an empty shape. It is also a non-deterministic process whose result might not be grasped from the earlier design stages. Most of the shape rules included in the rule set are additive and impose the addition of a new element in the design. In this sense, most of these grammars use a bottom-up approach in which the design begins by adding a small element which is generated by a recursive process of adding parts to the whole until it is complete. They differ from basic grammars since there are no restrictions on order and at any moment any of the rules can be applied. One real-life example of an addition grammar is the Prairie House grammar (Koning & Eizenberg 1981).

Deterministic grammars constitute a type of grammar that does not restrict the rule format. In simpler terms, the rules are defined in a way that is predictable or easily envisaged even if the grammar is infinite. In fact, this type of grammar only allows either finite or infinite results depending on the rule set, although in both cases the result is intelligible: 'there are different external and internal control mechanisms such as rule indexing or labelling, that can be used to impose determinism in a grammar.' (Knight 1999) Deterministic grammars may be confused with the idea of a simple, neutral grammar, although 'deterministic' refers to the language used and the predictability of the set of solutions described. As stated by Knight 'In general, deterministic grammars are more powerful than any of the previously defined shape grammars.' (Knight 1999) One functional example of a deterministic grammar is the Malagueira House grammar. In this grammar, a well-defined set of parametric shape rules is carefully ordered to reproduce a family of related designs that respond to the style requirements designed by Siza.

The last type identified by Knight in this classification is the unrestrictive grammar.

As can be easily understood, this type does not require any onerous conditions and does not impose restrictions, either on the shape of the rule format or the order. However, this does not mean that it is a type of grammar that requires informed, expert use. They exclude rule-based systems such as parametric shape grammars or colour grammars which impose conditions and define a particular rule system, and they can be applied in any format or order, making them the most unpredictable and simultaneously the most powerful mechanisms for computer implementation. Knight compares them to Turing machines, hence justifying their potential. From a grammar-based point of view they are difficult mechanisms to develop due to their unpredictability,

complexity and extensive corpus of solutions. Nevertheless, they combine great generative power with design exploration potential.

The second type of classification reflects on the nature of the shape rules required to develop the basic design solution. There are three types as pointed by Prats (Prats 2007): grid, addition and subdivision grammars.

Grid grammars use a subterfuge based on a grid to prompt the design. This grid assists the derivation process providing a foundation for the generation process. A good example of a grid grammar is the Wren's city churches grammar as shown in figure 2.15 (Buelinckx 1993). The grammar allows for the creation of the churches floorplans that are originally based on an orthogonal structural grid. The design of the building is then derived from this key generative principle.

Notably grid grammars use a sequence of addition rules that promote repetition.

Addition grammars are common formalisms that use addition rules as basis for generation. These are the most common grammars due to its easy formulation and absence of complexity. The major difficulty faced is the usual higher number of shape rules involved. One of the first examples of addition grammars is the kindergarten Foebel building blocks grammar (Stiny 1980b). This grammar was based on the children's building blocks game and its lexicon corresponds to the number of pieces provided by the game. The grammar then explores the spatial relations between pieces and the rules reflect the combinations amongst pieces. Other grammars follow these principles such as the Prairie houses grammar (Koning & Eizenberg 1981) and the Queen Anne houses illustrated in 2.15 (Flemming 1987).

The latest type is the subdivision grammar. Mostly using top-down approaches, these base the generation of solutions on subdivision rules. Usually a bounding shape is introduced in early stages of design and is further detailed by consecutive subdivisions. This was the principle used in the ice-ray grammar (Stiny 1977) previously discussed, in the Buffalo bungalows (Downing & Flemming 1981) and in the Malagueira houses (Duarte 2001b).

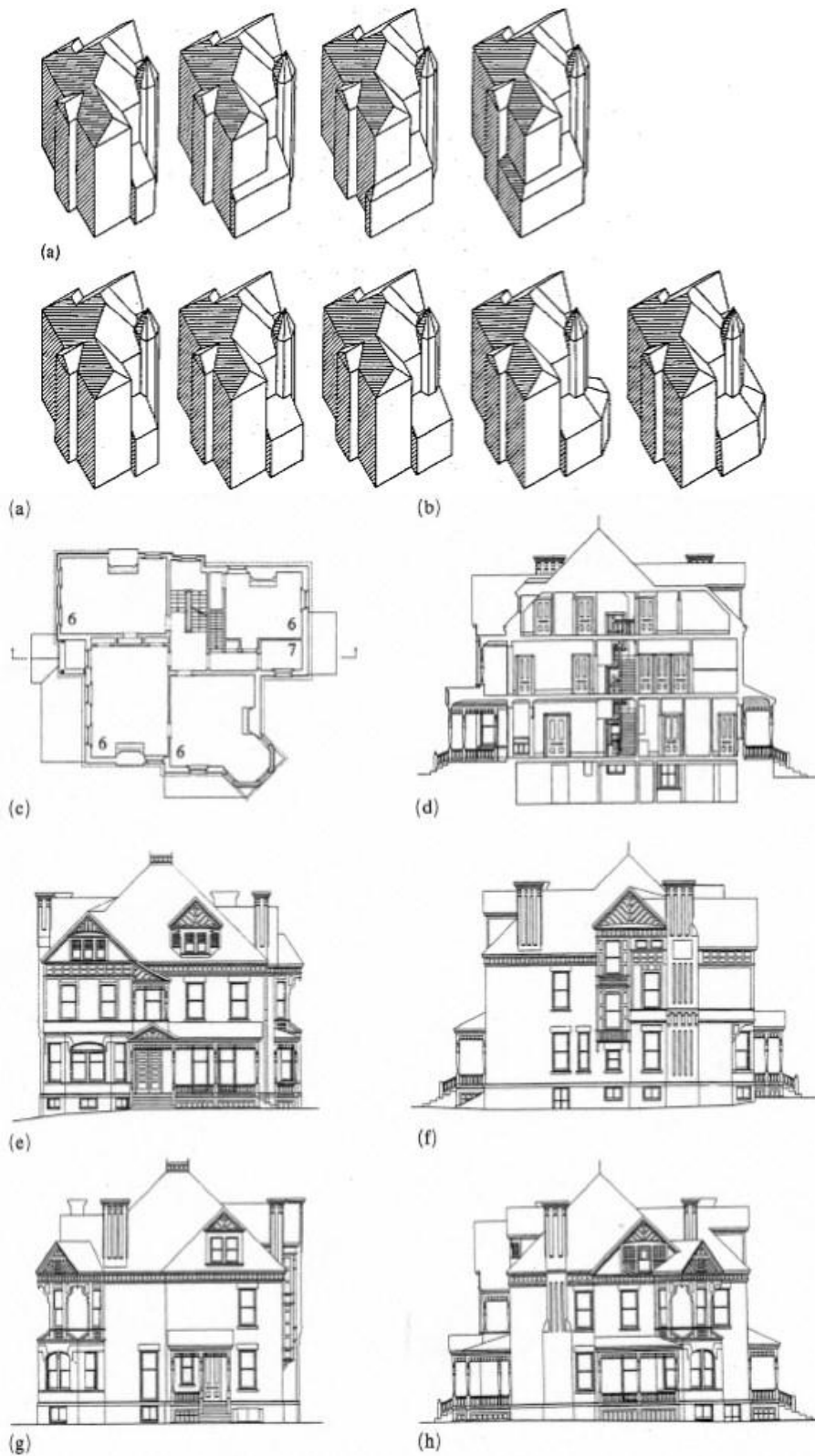


Figure 2. 13: The Queen Anne House grammar: an example of an additive grammar (Flemming 1987)

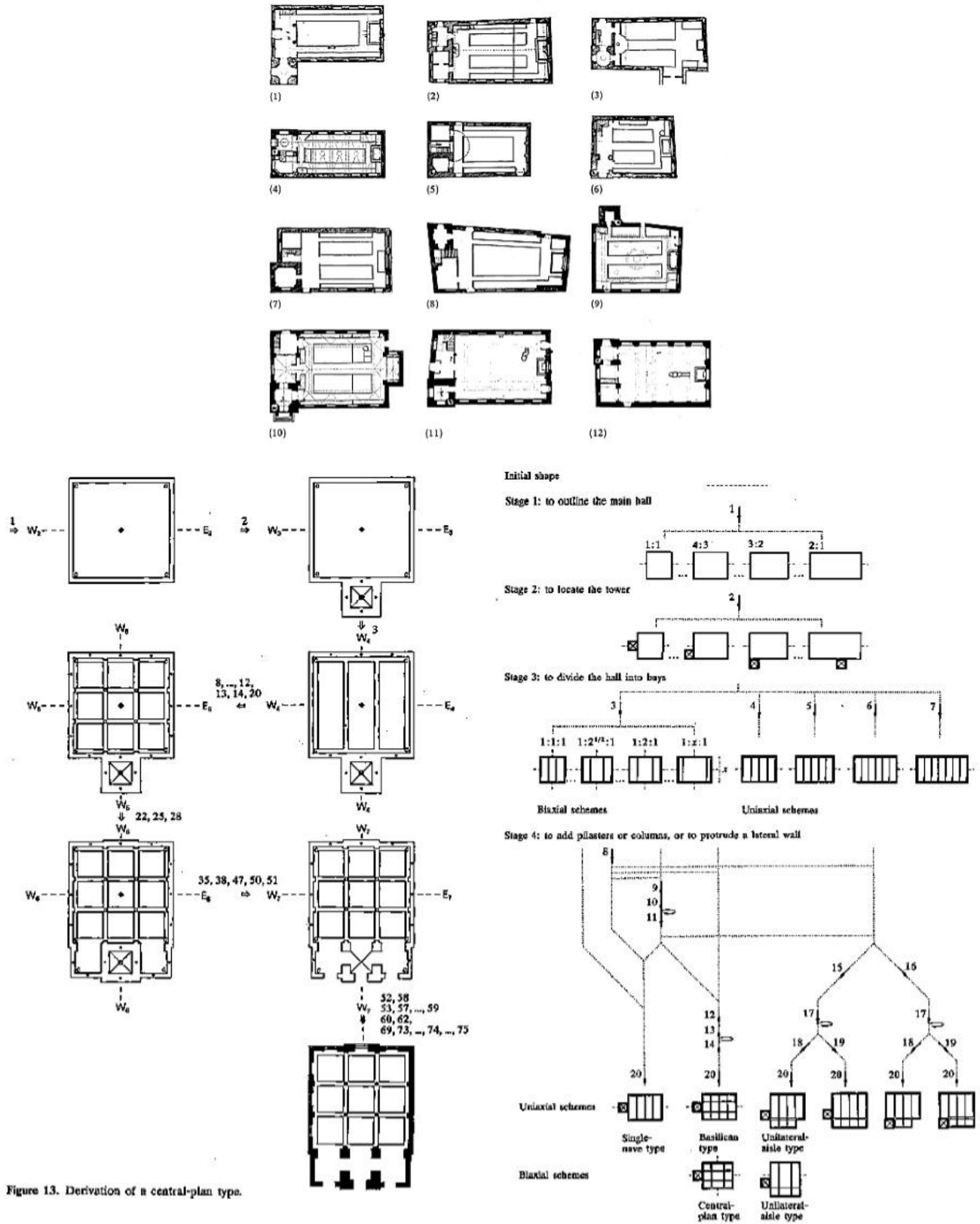


Figure 2. 14: Wren's City Church grammar: an example of a grid grammar (Buelinckx 1993)

This classification for the grammar process is attributed to Knight and cited by Prats (Prats 2007). This focuses mainly on the design construction system and the process used to apply rules. The major three categories of shape grammars identified describe their construction process. Knight distinguishes between 1) Grid, 2) Subdivision and 3) Additive Process Grammars. All three types, regardless of whether a top-down or bottom-up approach is used, work initially at a lower level of abstraction, as noted by Miguel Prats (Prats 2007).

1) A Grid Process grammar proposes a grid matrix system as the starting point for the design. Just as designers start designing a floor plan by establishing a matrix grid system, often combining structural requirements, this can be seen in grammars such as the Palladian Villa and Mughul Garden grammars (Stiny & Mitchell 1980) and the Japanese Tea House grammar. As an organisational system it proves quite effective in guiding the designer/grammar user through a top-down approach due to its empirical and intuitive perspective. Typically the design starts with the outline or boundary in floor plan view, evolving through further detailing in each grid cell. Alternatively, as shown in the original Palladian villas, a process of consecutive cell addition is followed in order to generate the grid. It is important to note that the grid process can include designs that do not necessarily present a rational or immediate grid form. It represents a good principle for design organisation, enabling complex forms to be designed simply by adding design rules for merging or deleting cells.

Another example of grid grammars has resulted in a very different case study and application. “A study of emergence in the generation of Islamic geometric patterns.”(Jowers & Prats 2010) explores the wealth of patterns provided by geometric Islamic patterns and proposes a generative design methodology that is powerful enough to recreate a large number of these patterns.

Like the Palladian Villa grammar, this grammar does not attempt to replicate the designer’s role by reproducing the original design process. Instead it aims to recreate the patterns by shape emergence, using different Islamic patterns as a case study and analysing their inherent shapes.

Sixteen different shapes were identified as recurrent and frequent in different compositions. All of these polygonal shapes have more than 4 sides, with the exception of a squared triangle. Most of them are regular polygons based on bi-dimensional designs, although there are exceptions with only one level of symmetry. Among the shapes, stars, wedges, squares, pentagons, arrows and hexagons can be identified, as well as other less recognisable shapes.

A straightforward grammar may be derived from the tile pattern generation based on addition methods, in which one shape is associated with another. This is responsible for the generation of a significant amount of shape rules and create quite a large shape rule grammar set that would be

difficult to handle and manipulate in order to obtain all the possible patterns. It would also generate a rather restrictive grammar lacking in generative power. Generative power would only be controlled by the number of rules, possible arrangements or spatial articulations, which would limit the extent of the corpus of solutions.

Jowers and Prats therefore came up with a methodology that could boost the desired range of designs. In order to achieve this, the different patterns and basic shapes were observed, leading to the conclusion that most of the boundary lines of the designs are orientated either vertically, horizontally or rotated to 45 degrees. Another breakthrough was the concept of square tiles: since tiles are usually manufactured in series or batches the question of repetition is relevant. It is not by accident that most of the basic shapes identified reveal symmetry in their genesis and this issue of symmetry is closely related to the question of repetition.

Taking this into account, the grammarians used the square tile shape as a starting point and selected only a half portion. This half portion was selected through the main tile diagonal, which could be easily mirrored. The shape obtained is a square triangle with a vertical, a horizontal and a main diagonal. These boundaries became the guideline for the entire pattern design. The grid was then set as an auxiliary design method. The grid lines were traced parallel to the border of the tile triangle forming a diagrid consisting of construction lines represented graphically by dotted lines. Labels are placed between the dotted lines, mainly at their endpoints. These labels, in the shape of dots, help identify the endpoints of the designs created. By filling in between the dots a continuous line is generated, allowing for changes of direction and configuration.

Once the line drawing is acceptable, Rule 7 is responsible for label deletion and Rule 8 for the deletion of the construction lines.

The full tile is then generated by mirroring the diagonal as the axis with Rule 9, and Rule 10 exemplifies the mirroring process to obtain adjacent tiles. The design is then replicated by the recurring application of Rule 10.

Although this grammar is very different in theme, object and corpus from the Palladian grammar, it illustrates a similar formulation based on a grid. It also shows a different type of grid which does not need to be an orthogonal matrix, as previously shown. It successfully sums up the potential of shape grammar formulation and its generative power even when using a limited set of ten rules which allows for an unrestricted set of examples and a significant number of possible design solutions.

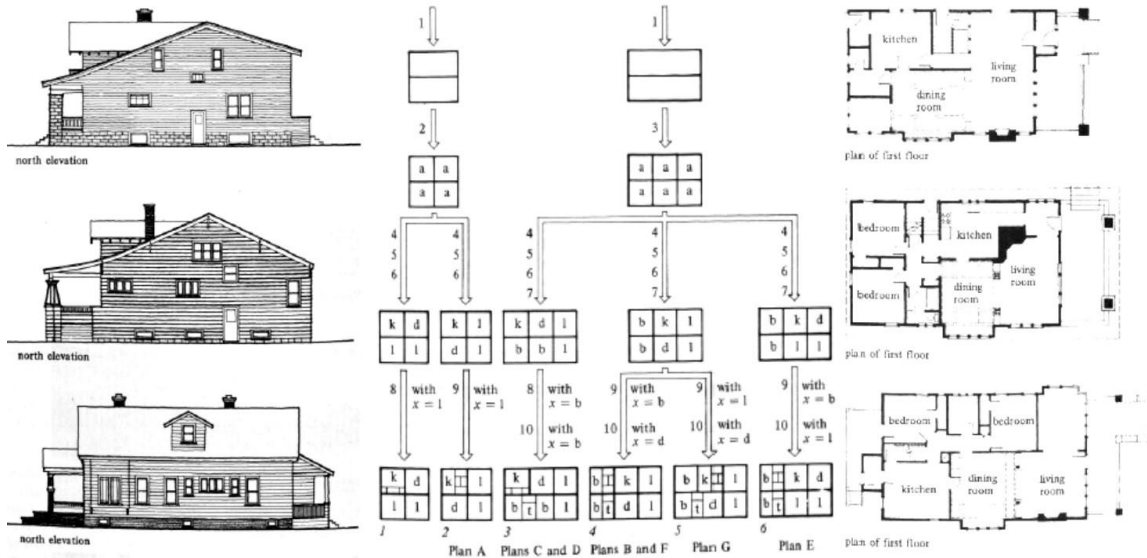


Figure 2. 15: The Buffalo Bungalow grammar: an example of a subdivision grammar (Downing & Flemming 1981)

2) The Subdivision Process grammar is a popular system for operating with grammars that illustrate architectural designs, since it is closely related to the way in which architects conceive of design and therefore feels more natural for the general grammar user. Most of the scenarios covered by this process use top-down approaches to design, starting with the boundary and working their way into further detailing. The subdivision process was first attempted in the Ice-Ray Grammar, in which all four design rules are typical subdivision rules showing shape transformations when the starting point is a closed polygon and the final stage of transformation is two closed polygons. Other examples are the Malagueira, Buffalo Bungalow and Hepplewhite Chair grammars (Knight 1980) and the Alternative Palladian Grammar, presented in a later chapter (Benros 2012). The design sequence starts with the outline shape which is divided recursively until the design requirements are satisfied, very much like a typical architectural project in which the designer is faced with pre-existing conditions such as a plot or surrounding buildings and a brief, then creates the design from sketches based on a floor plan, drawing, sculpting and cutting through the available space and creating detail.

3) The Addition Process Grammar is a type of construction process that proposes a bottom-up approach, developing the design step by step from an initial shape and adding successive elements until it is complete. It can be illustrated by the Prairie House grammar, since this is a typical example of the addition process. The creation initiates with the addition of the main focal point of the house, as described by Frank Lloyd Wright himself, namely the fireplace. This is both metaphysically and architecturally the heart of family life, combining living, dining and food preparation, as well as family and social life. The whole house and its array of spaces are arranged

round it. In other words, the process consists of a series of additions that take immediate pre-existing additions into account in order to continue the detailing. It is an intuitive process, although the outcomes are not immediately predictable, and creates a wide range of design results and potential for design exploration. It also allows for complex designs that are not self-contained, with a rational polygonal line. Other examples of grammars that use addition processes are the Queen Anne House grammar (Flemming 1987) and the Kindergarten Building Block Gifts (Stiny 1980b). In the case of the former, a hint is given by its title 'More than a sum of parts'. Often these types of grammars are restrictive in essence and combine a set of additive rules only. The design arises from a recursive application of the shape rules until it is completed.

Other important concepts and notions relating to grammar types are 1) parametric shape grammars, 2) parallel grammars, 3) colour grammars and 4) graph grammars. Although shape grammars, they have a degree of independence on the grounds that they show versatility and address issues that the shape grammars previously described struggle with.

1. Parametric shape grammars are a specific type of shape grammar. However they refer not only to shape, but also to algebra to illustrate and describe their applications. They constitute a restrictive type of grammar *per se* that indicates and instructs the way is applied, the circumstances and the results. The concept of parametric really refers to the use of parameters and variables to describe shape. Like parallel grammars they allow for a higher level of description that extends beyond the geometry itself. To accompany the shape transformation an expression is added which mathematically describes how the rule is applied and how the shape transforms. As in an equation, the variables and operations are carefully indicated, together with their universe of application and permitted intervals. This allows for the controlled design of family-related objects, using algebra as a control mechanism instead of shapes alone. In this way, if the grammarian intends to generate geometrically similar designs, this can be better managed. Over the years parametric design has become a useful tool in many ways, especially in industrial design, product design and architecture. Blanko Kolarevic has written extensively on the subject of parametric design in works such as 'Architecture in the digital age' (Kolarevic 2005) exploring how real life applications of parametric design allow for the fast and effective construction of complex geometries using parametric shape rules in the design process. Another useful application of parametric design can be found in the mass customisation of automatically manufactured products. In this case parametric design is used to standardise the basic features of a product, whilst others can be manipulated or adapted. One good example of this is automobile production lines such as those at Toyota, where cars are custom-made to user requirements. The user can input different features such as inside out colours, finishes, and accessories and the production line can be programmed to deal with these changes with no additional costs or time restrictions. This is also the case with

production lines for prefab systems, where beams, columns, infill panels and other modular or bespoke building components can be specified in particular sizes, quantities or proportions using parametric design principles to script the computer tools that manage the lines. One example of a parametric shape grammar is the Malagueira grammar. Every single shape rule is described graphically and by algebraic expressions that describe each component and shape. This also controls the range of variations and restricts the design to fit the house plot, which is one of the most important and noticeable restrictions on the design. The fixed area plot for all house types ($A = 12 \times 8 \text{ m} = 96 \text{ sqm}$) has to include all the elements of the construction and therefore every rule from the Malagueira set is derived directly from this restriction and is a variable of these measurements.

2. Similarly Malagueira is a good example of a parallel grammar. This name derives from the way in which the grammar is graphically represented. Shape grammars like the Palladian grammar, the Ice-Ray grammar and others are represented bi-dimensionally, either by plan, section or elevation. This seems to be an efficient way to describe these grammars and, in a way, defines a shape grammar. The Froebel Building Block and Prairie House grammars were illustrated three-dimensionally communicating the design focus better, using representation through axonometries. Nevertheless, they could have used a parallel grammar to showcase their rule system. Malagueira does this, using two simultaneous graphic systems to explain the transformation taking place, both in a plan and in section. However, a parallel grammar is not limited to dual representation through different views. It can also add another level of information to the shape rule, such as adding function to a space. In this case apart from the shape rule illustrated and the potential inclusion of labels, written information can add another level of design description.
3. Colour grammars were mentioned by Knight to show numerous significances. Two relevant studies are 'Shape grammars and colour grammars in design' (1994), in which these grammars are explored, and 'Transformations of De Stijl art: the paintings of George Vantongerloo and Fritz Glarner' (1990), demonstrating an operative colour grammar, although studies using colour had appeared much earlier (Knight 1994), (Knight 1989). Effectively Krishnamurti and Chris Earl used a colour grammar to enumerate and count the Palladian grammar, work that was later published by Stiny and Mitchell as part of their reflections on the Palladian Villa corpus of results (G Stiny & W J Mitchell 1978). Colour grammars are variations on shape grammars, in the sense that instead of using shapes and shape transformation they use groups of colours as their lexicon. Each colour has a symbolic value and the transformation is then described by colour value. This is illustrated in the Vantongerloo and Glarner grammars, although they also present an underlying subdivision grammar. This means that the two grammars are parallel grammars in which a subdivision process takes place while a colour grammar is also described. The geometrical

De Stijl paintings are described by both sets of rules in parallel and intrinsically connected step by step.

4. Graph grammars are another variation on shape grammars. They are in essence graphical, as the name indicates, but are enriched by a level of knowledge that ordinary shape grammars do not possess. Using a series of symbols such as circles, rectangles, lines, vectors or arrows, they represent flows, connections or adjacencies. Their added value allows for the description of adjacency graphs, functional diagrams or organisation charts. When combined with conventional shape grammars they can add an incremental level of information.

2.4 Automation in shape grammars: the car industry, façade detection

In the last decade certain studies were developed in order to find methods to automate shape grammars and infer rules. The following paragraphs discuss representative state-of-the-art developments in the automation of shape grammars.

Cagan and Osborn published work on the automation of shape grammars applied in the car design industry (Orsborn et al. 2008) and a group from ETH Zurich led by Pascal Müller has developed work on how to automate and model façades using a formalism close to the shape grammar universe (Muller et al. 2007; Parish & Müller 2001).

The work of the ETH Zurich group proposes an algorithm to automatically reconstruct a three-dimensional model from a high-resolution façade image, using shape grammars to analyse the image and generate the output. Their approach mimics the procedure modelling methodology of computer graphics to analyse a façade image using a top-down approach. A subdivision is applied to the image, taking into account windows, floors, doors and other architectural elements that may help analyse the overall façade structure. The algorithm starts by identifying repetitions and is followed by a method to derive a top-down subdivision scheme. This subdivision is influenced both by computer vision techniques and procedural modelling knowledge, which in this instance is based on architectural know-how. Shape grammars are automatically inferred from the information available and analysed, and shape rules are derived from the complex images.

The work is prompted by the use of imagery from ground or even aerial photographs. The urban façade reconstruction is carried out by what the authors call a 'semi-automatic method' based on Debevec and using a self-designed tool for imagery rectification which is believed to be more trustworthy than the free public domain tools available.

The methodology is based on four stages:

1. Detection of façade structure
2. Tile refinement
2. Element recognition
4. Editing and extraction of shape grammar rules.

The first step encompasses detection of the façade structure, in which the input is a single façade. Once these elements are detected, a tiling procedure takes place in which a tartan grid is overlaid in order to simulate or replicate the floor and vertical structure location.

The second step involves tile refinement. The grid established in the previous step is progressively detailed. In order to do so, the algorithm draws inspiration from split grammars, using translational symmetry as a means of achieving a feasible subdivision. Some clustering is performed within each tile, grouping together a set of architectural elements and separating sets of similar information in repetitive tiles.

The third step involves element recognition. Each clustering tile detected in the previous step is matched and cross-checked with a 3D library of architectural elements in order to identify it. Each element corresponds to a three-dimensional entity that is delivered as the output of a complete textured model, accompanied by a semantic structure represented by a shape tree diagram.

The last step allows for the editing and extraction of shape grammar rules. The shape tree allows for the rule extraction that supports the façade to be inferred and derived. The semantic façade can be manipulated and used for the rule extraction or even to refine a model that has already been produced.

2.5 Previously inferred shape grammars: Palladian, Prairie House and Malagueira

ARCHITECTURE	DESIGN AND FINE ARTS	PRODUCT DESIGN PRODUCT
PALLADIAN GRAMMAR	ICE RAY CHINESE LATTICES GRAMMAR	HEPPLEWHITE CHAIRS GRAMMAR
PRAIRIE HOUSES GRAMMAR	VANTONGERLOO PAINTINGS GRAMMAR	BUICK GRILL AND FRONT VIEW GRAMMAR
MALAGUEIRA GRAMMAR	GLARNER PAINTINGS GRAMMAR	COFFEE MAKERS GRAMMAR
WREN'S CITY CHURCHES GRAMMAR		KINDERGARTEN BUILDING GIFTS GRAMMAR
BUFFALO BUNGALOWS GRAMMAR		VICTORIAN WINDOWS GRAMMAR
QUEEN ANNE HOUSES GRAMMAR		
MUGHUL GARDENS GRAMMAR		
TAIWANESE HOUSES GRAMMAR		
JAPANESE TEA HOUSES GRAMMAR		
TURKISH HOUSES GRAMMAR		
MARRAKESH URBAN DESIGN GRAMMAR		

Table 2. 5: Previously inferred shape grammars, organised by field

Based on a number of architectural shape grammars inferred in the past, including the Wren City Church, Buffalo Bungalow, Taiwanese House and Japanese Tea House grammars, many people have tried to encode original compositions to recreate languages or styles and analyse known or existing design languages.

The table above shows some of the most relevant inferred grammars, organised into three major fields: architecture, design/fine arts and product design.

However, the Palladian Villa, Prairie House and Malagueira House grammars stand out within this group, due to their innovative demonstration of classical style (Palladian), relevance to categorising a style with numerous existing solutions of acclaimed design quality, and recreation of a living master's view, using the grammar to produce novel solutions accepted by the original architect and linked to a computerised tool (Malagueira).

These are the main reasons why these grammars were selected as a case study for this research project.

In addition, these grammars represent three of the different types of grammars typified by Knight, namely the Grid (Palladian), Addition (Prairie) and Subdivision (Malagueira) grammars.

Other selection criteria were also considered. First all the grammars have a common ground, since they all deal with housing, and more specifically individual dwellings. Secondly, they present different views of architecture, reflecting three very different architectural styles. They also produce

different architectural languages by different authors from different countries and time frames. Last, but not least, they represent different grammar construction processes.

Therefore, the grammars reflect stylistic design differences and functional similarities that benefit shape rule variety and allows future comparison.

The Palladian grammar (Stiny & Mitchell 1978) explores the universe of Palladio built solutions and proposes a set of rules to recreate the design vision of Andrea Palladio. The authenticity of the grammar is proven by the possibility of recreating the original designs by applying the proposed set of 99 rules in 12 sequential design stages. The first design stage in this grid grammar consists of grid assignment. All the rules included refer to the definition of the tartan grid, considering important aspects of Palladio's language, orthogonality and symmetry to produce grids of 3 to 7 cells. The second stage places the boundary around the predesigned grid. The third stage focuses on.

The Frank Lloyd Wright Prairie House grammar (Koning & Eizenberg 1981) represents a typical addition process. Two thirds of the 23 proposed stages constitute addition processes with transformational shape rules that add extra elements to the original design, culminating in the final solution.

The Malagueira House grammar is an example of a sub-division process. The significant steps in the extensive 162 shape rules consist of dissecting an original shape (the house plot) and splitting it in two. Spaces then result from this recursive process.

The methodology used in this research required a comparison of the different grammars and a consistent expression of their sets of rules. The Prairie House grammar diverged in its representation by expressing its rules three-dimensionally, while the Malagueira grammar, although addressing three dimensionality, opted for a parallel grammar in which the rules are represented by both a plan and a section view. Taking this into account, an effort was made to represent all the rules in a bi-dimensional manner, following the example of the Palladian grammar, since it did not provide three-dimensional information. Basically a dimensionality reduction was performed, as previous work in style archetypes suggests (Hanna 2007).

This enabled the different rule systems and steps to be compared accurately. Furthermore, it allowed the three stages and tree diagrams to be juxtaposed to compare differences and similarities.

The Palladian and Prairie House examples reveal several similarities in the stage sequence and shape rules applied, which can be explained by the process used to create the grammars. Both grid and addition grammars can be described as pure addition processes with different construction rules and slightly different stage sequences.

The Malagueira grammar, although resorting at times to addition rules, starts by focusing on a fixed predesigned element, the plot, and sub-dividing this element recursively.

Regardless of the similarities and differences, from the 300 rules observed (90+99+162) within the set of design rules, 4 standard or generic rules can be extracted that aim to encompass all the particularities of these grammars, as will be explained in the next section.

Another way of explaining the differences and similarities between the grammars is through their structure. The Malagueira and Palladian grammars are closer, since both use a top-down approach and start by exploring the overall design or spatial outline and detail sequentially.

Conversely the Prairie House grammar starts by positioning a specific part of the design and evolves step by step, adding elements of design. It is only in the final stage that the outline and final composition is defined, using a clear bottom-up approach. Nevertheless, the characteristics of this grammar also reveal close affinities to the Palladian Villas grammar.

The grid structure (used in the Palladian grammar) and the addition use methods which favour addition processes. The grid in the villa design generation is produced by assembling different cells. These are arranged one-by-one bi-dimensionally. Other elements of the design, from porticos to staircases, columns and frontans, are also assembled sequentially. In the pure addition process used in the Prairie House grammar it is not difficult to grasp the true nature of the addition, in which each room is added in each step to support a specific function and spatial area of the dwelling.

Fundamental differences in the level of detail of information provided in each grammars, were encountered of. All the grammars are parametric and display parameterised shape rules. However, chronologically it seems that each one has tried to overcome the difficulties and imperfections of the previous attempts.

The Palladian grammar proposes a bi-dimensional structure. The shape rules focus mainly on the villa's main floor plan, exploring the spatial and geometric generation process tested by Palladio. It does not address or label spaces and it does not assign functions to them.

The Prairie House grammar, described over a decade later, explores the volumetric nature of Wright's creation. Therefore, it proposes three-dimensional rules that are embedded and labelled with functions. This assignment of volumetric function is prescribed in an evolutionary manner. The next function to be included always reviews whatever has already been designed and which restricts the subsequent degree of freedom, thus conditioning the next step. Another innovation was the representation media for the grammar, since it was displayed three-dimensionally using axonometric views. However, the rule parameterisations were not available and the embedded expert knowledge of the proportional factors of the elements was not discussed in the paper.

The Malagueira grammar also used a different approach. Focusing on the three-dimensionality of the subject, it used a parallel grammar to represent its shape rules. A parallel grammar is a grammar that simultaneously provides a plan and a section view, or a plan and elevation view for a more accurate representation. This seems to be particularly useful in architectural grammars, allowing a proper distinction to be made between the building envelope and the construction elements. This grammar also demonstrated the parametric expressions for each rule, enabling

proportional and geometrically consistent solutions to be recreated. In addition, the grammar was implemented using a computerised tool to aid design. This allowed for a guided step-by-step design process that always observed the basics of the design. This computer tool was supplemented by an interface that allowed the user to explore design solutions and generate a complete house by introducing user input and needs. The interpreter would then use this data either to generate a house in real time, showing the 3D model visually in each step, or to optimise the best solution and randomly produce it.

2.6 Shape exploration

In addition to the ability to analyse and describe design, one of the other of features of shape grammars is shape exploration.

Many studies have been made on shape exploration and how to ascertain the large number of design solutions which grammars generate. In fact, by extracting shape rules from expert designs, grammars later allow these rules to be applied to a wider range of options which over time become intangible or invisible to designers who integrate series of preconceived ideas, stereotypes and self-imposed conditions into their work in a way that cannot be achieved by objective formulations.

Immediately after inferring the Palladian grammar, Stiny and Mitchell reflected on the extent and quality of the solutions provided by the grammar in their paper 'Counting Palladian Plans'(G Stiny & W J Mitchell 1978). This offered proof of the large number of villas supported by the grammar and the range of possibilities not designed by the author himself which, nevertheless, seemed to meet the criteria and comply with the set of rules. Likewise, the Malagueira grammar also revealed a large corpus of possibilities.

This can be explained by two factors; firstly, the fact that the case study was based on an existing set of nearly 300 houses built between the 1970s and 1990s, and secondly because the shape rules inferred were large, detailed and assessed by the architect himself. This resulted in a set of thousands of possible solutions.

Other grammars have investigated the concept of design generation, the corpus of solutions and their possibilities in terms of design exploration.

The concept of ruled-based design systems for architecture has always been considered throughout time. From Vitruvius to Durand, Alberti to Serlio and even Ruskin to Neufert, architects have tried to systematise, catalogue and plan the design of several types of buildings. Vitruvius coded the classical style rules into what is considered to be the 'bible' of architecture, in which the three orders, Corinthian, Doric and Ionic are described and their shape rules discussed informally. It is impossible to determine the extent of the influence of this book on classical and neo-classical architecture. Countless buildings were erected following the rules included in this treatise and although most of them conform to classical dogmas, the results are as diverse as possible (Vitruvius 2001).

Like Palladio, Alberti, Bramante and Serlio all left treatises and writings which, rather than simply describing the genre and style they pursued over a lifetime of work, aimed to leave a legacy of know-how and expertise embedded in the set of rules they described, even if at times this might be disguised in formal approaches. Often these writings are accompanied by drawings and references to their work, but they are no more than role books for the architecture they follow. On another level, Neufert provides a catalogue of solutions based on the optimum options for building types. This

catalogue focuses on real functional problems and specific building/room types and tries to provide solutions for each one, illustrated with prototypes and often with real built examples. There are few similarities with shape grammars but this information gathering method is what grammarians often use as a starting point for their shape rule inference processes. It also broadens ideas and, consequently, design solutions, or even the horizons of the corpus of new solutions. As with shape grammars, the goal is design exploration (Neufert et al. 2002).

In a similar vein, Miguel Prats published his doctoral dissertation on shape exploration in design, focusing particularly on shape grammars. In his opinion 'Shape grammars have a significant potential to bridge the gap between traditional sketching techniques and modern computational methods of design'. Shape grammars provide two methods of exploration, first through shape decomposition and second through shape transformation (Prats 2007).

The shape exploration is also threefold:

- it provides the guiding principles of composition (according to Stiny)
- it allows for reflection and perception via cognitive processes (Oxman 2006)
- it creates personal activity, based on context and the design conditions.

Shape grammars have a fixed set of requirements. They are a generative design system which acts as a complement to the mental process. The concept is often underlined and materialised in design by a series of sketches or mental visualisations, whereas shape grammars 'can capture and externalize partly formed thoughts and ideas'. (Prats 2007)

It is not surprising that architects and designers use support systems and their set of guidelines for design purposes. Corbusier argues that the use of regulating lines provided him with sources of inspiration. 'As a matter of fact 'regulating systems can be used for analysis and synthesis of designs'. Guiding principles offer the composition process a means of interrelating form and a method for achieving visual balance, according to Elam (2000).

There is no doubt that shape grammars are useful in constructing a mode for formally generating and exploring designs. They are defined by a set of rules (shape rules) which convey design requirements. They also encode important information at different levels. By coding shape, intrinsically we are also coding function, which is somehow interconnected. As in Louis Sullivan's famous quote 'form follows function' and Frank Lloyd Wright's 'form and function are one', in shape rules form and function appear side by side.

On the other hand, as Ching explains, shape also encodes important information on style, which is by far one of the strengths of the shape grammar theory. 'The shape determined by the contour of an object holds high information content about the object, such as aspects of style'.

Shape can stand alone as the representation of an object. This is one of the fundamentals of the shape grammar theory, given that by studying shape we are unravelling other levels of knowledge embedded within it, quoted by (Prats 2007)

The study of shape, often addressed in the inference process (one of the first steps in setting up the grammar), is based on shape decomposition. Research on cognition states that there is sufficient

evidence to demonstrate that human vision uses part-based representation for shape recognition. This means that in order to perceive and recognise a shape the first mental process to take place involves decomposing the shape into its parts. This can also be transferred to design. Often designers start with partial representations or even abstractions of the shape until they reach the final product. Shape grammar uses shape decomposition to analyse shape and define shape rules into an operative design generation system.

Two properties can be pin-pointed for shapes: structure and outline. These two factors are used both to describe and decompose shape.

The design generation process uses these two properties. Often designers construct their design in a cyclical manner, transforming it recursively. Each progressive sketch is a transformation and development of the preceding sketch, involving additions, deletions, modifications and replacements of certain elements. The range of shape grammars allows for a process that is in some ways similar. It is based on decomposed parts progressively generated by each rule up to the completion of the design.

The emerging shapes may appear by means of two processes, an interpretive process or a transformational process. The interpretative process is based on an analysis of the original shape and the application of shape rules that will result in a final design. In order to achieve this, the grammar user has to interpret the original shape and apply the rule accordingly by analysing the different elements or parts of the design. The transformative process implies a transformation. The original shape has to allow for a straightforward geometrical operation that will, in fact, change the existing geometry and generate a different design.

Stiny stated that 'Designers calculate with shapes when exploring design' (Stiny 2008) and, in fact, design exploration amounts to no more than playing with shapes and performing geometric operations. Shape grammars provide a platform and a structure for shape calculation and therefore for design exploration. The shape rules are set up in such a way that the user will be guided through a sequential process of applying simple and targeted rules whilst retaining the integrity and coherence of the design. This integrity follows the design standards and style requirements whilst offering some scope for creativity. Rules can be applied in a creative way as long as they are guided, and exploration is made easier and more efficient. They also provide a tool for explaining design spaces and exploring new designs, even when they are difficult to infer and develop.

The design exploration aspect of shape grammars has already been put into practice in the product industry. The car manufacturer brand Audi and its design team led by designer Maria da Silva have been applying the design principles of shape grammars to their design production. Shape grammars have proved to be efficient in maintaining brand identity and brand signature style whilst allowing for novel designs and design exploration and development. In order to create these automobile grammars, the brand has had to define their guidelines and over the years interpret which key elements and lines are fundamental to the brand identity. These are then coded into shape rules and new designs are generated by the grammar. In this way a true language of designs

is created, which defines a set of designs based on common generative principles, as described by Stiny and Gips in their work (Gips 1974).

As a specific type of ruled-based system or even a particular type of expert system, shape grammars have been compared and contrasted with these. They are all production systems that can be applied as generative design systems. The latest systems either imitate or reproduce the human cognition system and simulate the process of 'if' implies 'then' or action/reaction. Even in his work on language grammars Chomsky describes generative systems for phrase grammars and, as we know, many languages allow for the possibility of recreating or creating new formulations (i.e. the German language allows new words to be composed by a process of conglomeration). Similarly 'shape grammars use an alphabet of shapes' (Stiny 2008). They also provide a framework to explain architectural styles and to generate them.

As previously stated, the first step in shape inference is often design decomposition and this is really a way of addressing and analysing shape.

'It's hard to imagine a better way to describe shapes than to resolve their parts and to show how the parts are related. This is what decompositions are for, but if parts are fixed errantly then this is a poor way to understand how shapes work when I calculate' (Stiny 1987).

Design decomposition is a basis for describing design requirements, thus designers can maintain design requirements whilst exploring designs by transforming the outlines and structures of shapes through generative rules.

2.7 Chapter reflection

This Chapter focused on the literature review and state of the art of shape grammars. The shape grammars discussed proposed specific formulations to describe one singular design language. From architecture, to design all grammars presented targeted novel designs with recognisable styles. The assumption 'to one grammar corresponds to one corpus' comes to mind, if not by the range of examples showcased, but also to the bibliographic record presented.

To designers and grammarians seems obvious that any given design language corresponds to a specific parametric space where all family related designs can be found. A successful shape grammar can be identified by how well circumscribes its design range. This design space should be well expressed by the corpora if not even quantifiable.

Shape grammars also provide a pragmatic tool that replicates the design process. The design process is replicated by the results it produces rather than the method used. Often shape grammar make use of alternative processes to attain a specific result. The Palladian grammar is an example of this. They also provide a clear algorithm with the process used which can be applied recursively and consistently. A designer does not design with such precision or method. A designer will develop their language as a result of creativity, exploration, experience and culture and often this language develops and transforms. This constitutes a developing process not a static method such as grammars describe.

Nevertheless, the pragmatic and descriptive nature of the shape grammars showcased in this literature review is essential to the understanding of the so called 'parametric space' of a design language. This important concept allows us to formulate an important hypothesis for this thesis:

Design solutions within a corpus parametric generated use a restricted range of parameters that can be specified within a shape grammar.

Each of the shape grammars revisited are style specific and illustrate specific parametric rules to prompt a design. They do so by placing conditions and establishing particular and limited shape rules. This method allows for the consistency in design often named as design language or style. They provide an algorithm or routine to replicate the design process and recreate the language consistently.

3.	Case study: three architectural housing grammars	77
3.1	The Palladian Villa grammar by Stiny and Mitchell	78
3.1.1	Grammar type and classification	78
3.1.2	Grammatical Structure	81
3.1.3	Rule System	84
3.2	The Prairie House grammar	89
3.2.1	Grammar type and classification	89
3.2.2	Grammatical structure	89
3.2.3	Rule System	92
3.3	The Malagueira House Grammar	98
3.3.1	Grammar type and classification	98
3.3.2	Grammatical Structure	101
3.3.3	Rule System	103
3.4	Comparison of the case study shape grammars	108
3.4.1	A shape grammar comparative study	108
3.4.2	Comparison and results	110
3.5	Chapter Reflection	114

3. Case study: three architectural housing grammars

The present chapter focuses on a case study of representative shape grammars. All three grammars represent examples of singularity and univocal relationship between grammar and corpus. To each one of the grammars is linked a specific corpus. Each corpus is linked to a recognisable design language, previously identified by architectural and art history experts as well as perceived by laymen. All of the case study grammars focus on a clear and easily identifiable language. The designs within each corpus are easily recognisable and perceived. Moreover, within each corpus the designs share similarities and differences but are always family related to one another. All these parametric grammars are responsible for a parametric space where each design solution occupy a particular position. This seems to relate to the perception that one has from a specific solution from a corpus and how we identify familiar relationships.

In addition the grammars describe a specific design process. The design process, sequence of design stages and rule system of successful grammars will assist identifying correlations between the parametric space of a grammar and our intuition of similarities in design as we will see.

Since the development of the shape grammar theory, many grammars have been inferred with the aim of implementing a shape grammar strategy. Shape grammars describe a family of shapes.

“A family of shapes can be defined in terms of a given shape by allowing its component elements to be dimensioned in accordance with certain specified criteria. More precisely a family of shapes is defined by a parameterized shape ‘s’ which is obtained by allowing the coordinates of its end points of the maximal lines in a given shape to be variables. A particular member of this family is determined by an assignment ‘g’ of real values to these variables” (Stiny 1980a).

To this point specific grammars were developed to recreate specific languages. The idea of developing a generic housing shape grammar using generic rules and applying them to design houses of more than one specific style is put in practice in this work. This will make a contribution to knowledge by helping to explain how different styles and languages can be derived from a similar branch set of rules. It is believed that successful housing languages share design principles and other common features and this generic grammar would therefore help in understanding these notions. A generic grammar for housing would allow:

- evidence that grammars for different languages of design share similarities and processes.
- facilitate the development of other particular grammars
- help explain design formulations and optimize them
- allow for a better understanding of the shape grammar formalism and its application

Three grammars were selected for a case study.

The grammars selected reflect the three types of shape grammars identified and described in Knight's grammar classification (Knight 1999). The three grammars include the Palladian Villas by Palladio (Stiny & Mitchell 1978), the Prairie House by Wright (Koning & Eizenberg 1981) and the Malagueira Houses by Siza (Duarte 2001), which represent grid, addition and subdivision types respectively.

All grammars shared the following aspects:

- Formulation applied to single-family housing
- Featured distinctive differences in style, layout, and history,
- Proposed grammars for languages composed by architect's with different background and signature,
- Represented different geo-graphical locations, contexts and periods of construction

Despite these differences, the common features allowed comparison. The differences enriched the corpus of the study and helped highlight the specific characteristics of each grammar.

The selection of the case study was followed by observation and comparison of their different processes and shape rules and, finally, a proposal for a new set of rules combined in a new shape grammar.

3.1 The Palladian Villa grammar by Stiny and Mitchell

3.1.1 Grammar type and classification

The Palladian grammar was the first architectural grammar to be inferred and published with a complete set of shape rules for generating the floor plans of Palladio's sixteenth-century villas. The large number of surviving villas that can be seen today in the Veneto region of Italy and Palladio's extensive writings have provided art historians and researchers with a range of material. The Four Books of Architecture or 'Il Quattro libri' provide detailed reflections on Renaissance and Mannerist architecture, constituting a treatise for architects, stonemasons and builders (Palladio 1997). Most of the villas described were, in fact, built and many of them still exist today.

This inspired Stiny and Mitchell to apply and test the shape grammar theory. The Four Books provided a set of preconceived and precise design rules and the additional the material allowed the remaining rules to be inferred. The original research proposed a new way of replicating a design vision by recreating design rules extracted from an original architectural corpus of buildings and drawings. It claimed that it was possible to replicate the design language by using the embedded rule system from previous designs.

Stiny and Mitchell proposed a system of rules containing up to 72 shape rules separated into 12 sequential stages.

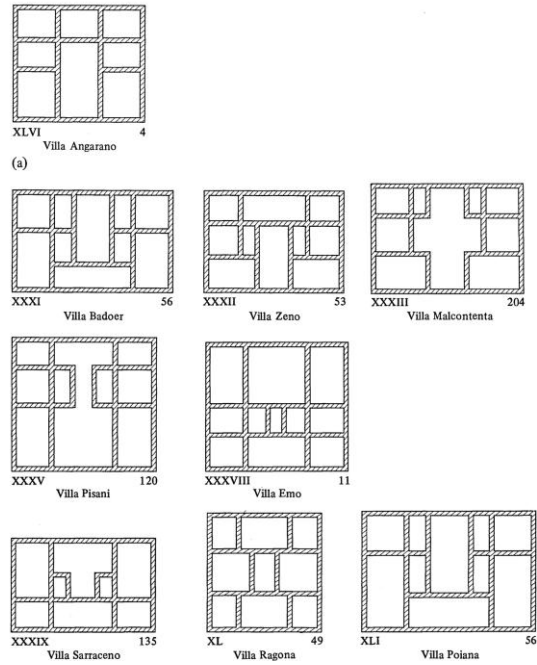


Figure 3. 1: Nine existing Palladian villas with 3x3 and 5x3 floor plan grids (G Stiny & W J Mitchell 1978)

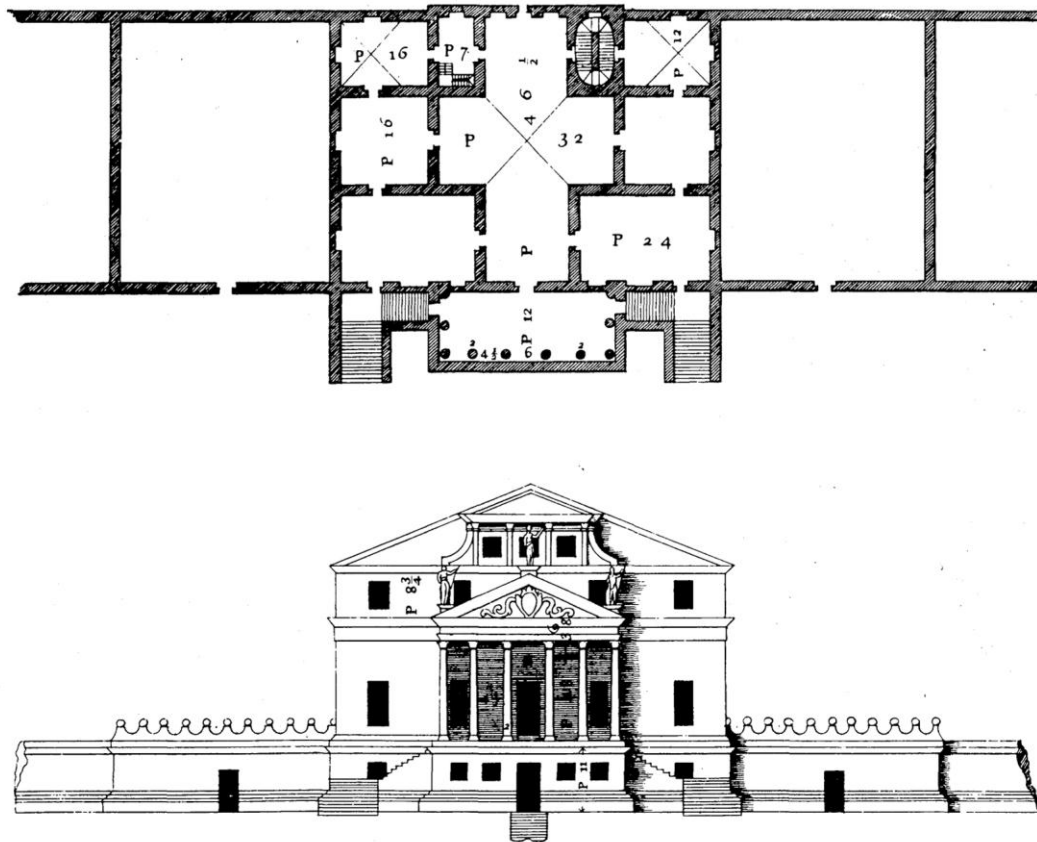


Figure 3. 2: The 'Malcontenta' Villa 5x3 grid floor plan and elevation (G Stiny & W J Mitchell 1978)

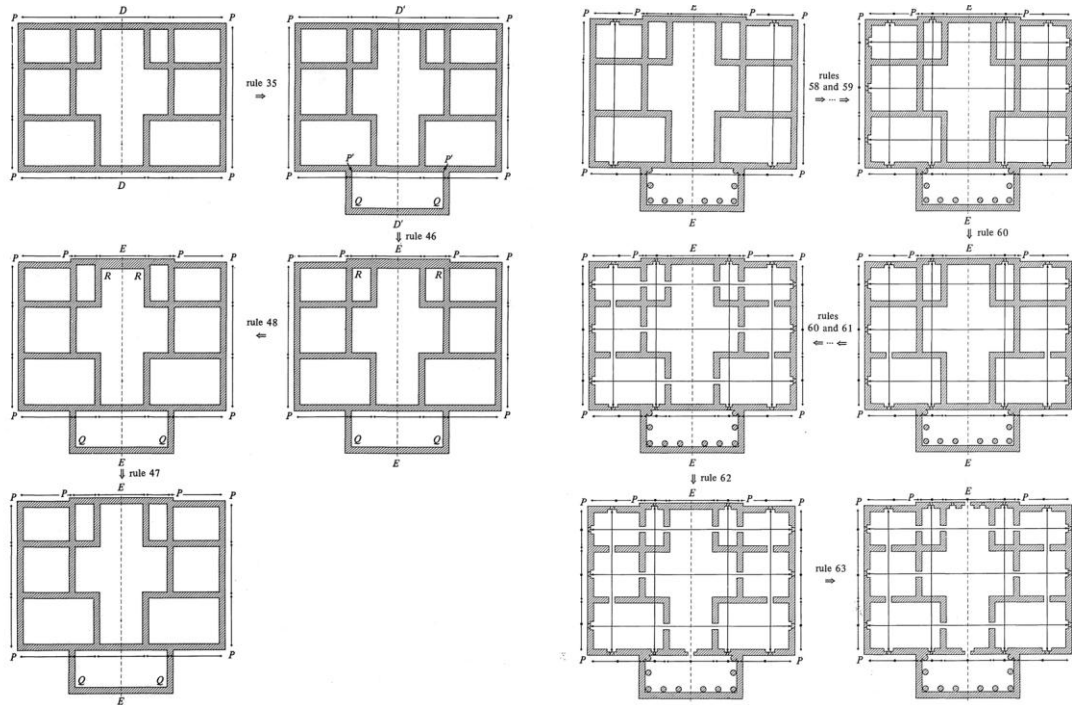


Figure 3. 3: Partial derivation of the 'Malcontenta' villa using the Palladian Grammar (G Stiny & W J Mitchell 1978)

Their observations revealed that the Palladian villas have a floor plan usually built within a 3x3 grid (in villas such as Angarano), or 3x5 cells (as pictured in the Baldoer, Emo and Pisani villas and others shown above in Figure 3.1). The design is inspired by Roman classicism. One characteristic described by Palladio himself in his extensive writings, was symmetry, an important feature of this style and evident in the villa floor plans and elevations. The graphical representation illustrated in the west wing of each layout is mirrored in the east and the centre of the villa stands on the axis of symmetry. It can be identified as the main social area which occupies a focal point and stands out in the spatial hierarchy due to its size and the often intricate geometry resulting from the merging of adjacent cells. Doors and windows are arranged symmetrically and aligned with the axis of symmetry in each room. Palladio established a parallel between the façade of the building and the symmetry of the human face, thus creating the rule for the arrangement of doorways and windows. Anthropomorphic design is used in the creation of the façade. Doors, like the mouth or nose, must be centred and uneven in number, whereas windows, resembling the eyes of the dwelling, should frame the door openings and appear in even numbers.

Stiny and Mitchell provide a complete generation of the ground plan of the Malcontenta Villa, using a process known as derivation. Derivation can be defined as the graphical illustration of the rule application and the shape transformation from the genesis of the design to its conclusion. The Malcontenta derivation, its illustration and schematisation of each grammatical stage is shown in Fig. 3.3.

The inferred rules are either direct translations of Palladio's explicit design canons or, in other cases, are based on the illustrated examples of villa plans in the *Quattro Libri*.

Most of the original villas have a specific floor plan geometry. Palladio proposed an actual architectural system rather than merely reinterpreting the classical style. He was the author of the translation and re-edition of Vitruvius' treatise on classical architecture which became popular in the sixteenth century. This was published with the help and support of one of his wealthy clients, the owner of the Foscari, or 'Malcontenta', villa. Wittkower (1952) sees the distinguishing feature of Palladio's villas as "the systematization of the ground plan". In fact the main floor represents the centre stage of the house, serving as a social hub. This floor plan translates into a succession of rectangular rooms, often arranged orthogonally in a sequence of evenly numbered bays to create a perfect symmetry based on the central space. The central space is the largest room, which was designed for entertaining and is the only room that extends from the front to the back façade. Bilateral symmetry is evident in both the floor plan and the elevation and is a key feature in Palladianism and in classical and Mannerist architecture. In Italian classical architecture this floor was commonly known as 'piano nobile' and due to its position in the social hierarchy of spaces was the most represented plan by Palladio. In the '*il Quattro libri*' only this level is illustrated. This is the main reason why the Palladian grammar focused its rule system on this alone.

3.1.2 Grammatical Structure

The grid grammar bases its structure on additive processes and is built up from a simple shape to a detailed group of shapes which, in this case, represent the floor plan of a house. Housing grammars simultaneously address form and function and which are closely connected. The floor plan design is based on a grid matrix and constitutes an abstraction or generalisation of the ideal floor plan. This is a useful method for achieving regular and symmetrical outcomes which conform to classical design criteria.

Eight of the twelve stages highlighted and described by Stiny and Mitchell are responsible for the housing design layout. The other stages, although equally important, contribute to the design aspirations and maintain language coherence. These eight design stages are illustrated in Figure 3.4:

1. First stage – Define grid
2. Second stage – Define external wall
3. Third stage – Room layout
4. Fourth stage – Interior wall realignment

5. Fifth stage – Define entrance
6. Sixth stage – Exterior ornaments
7. Seventh stage – Windows and openings
8. Eight stage - Terminate

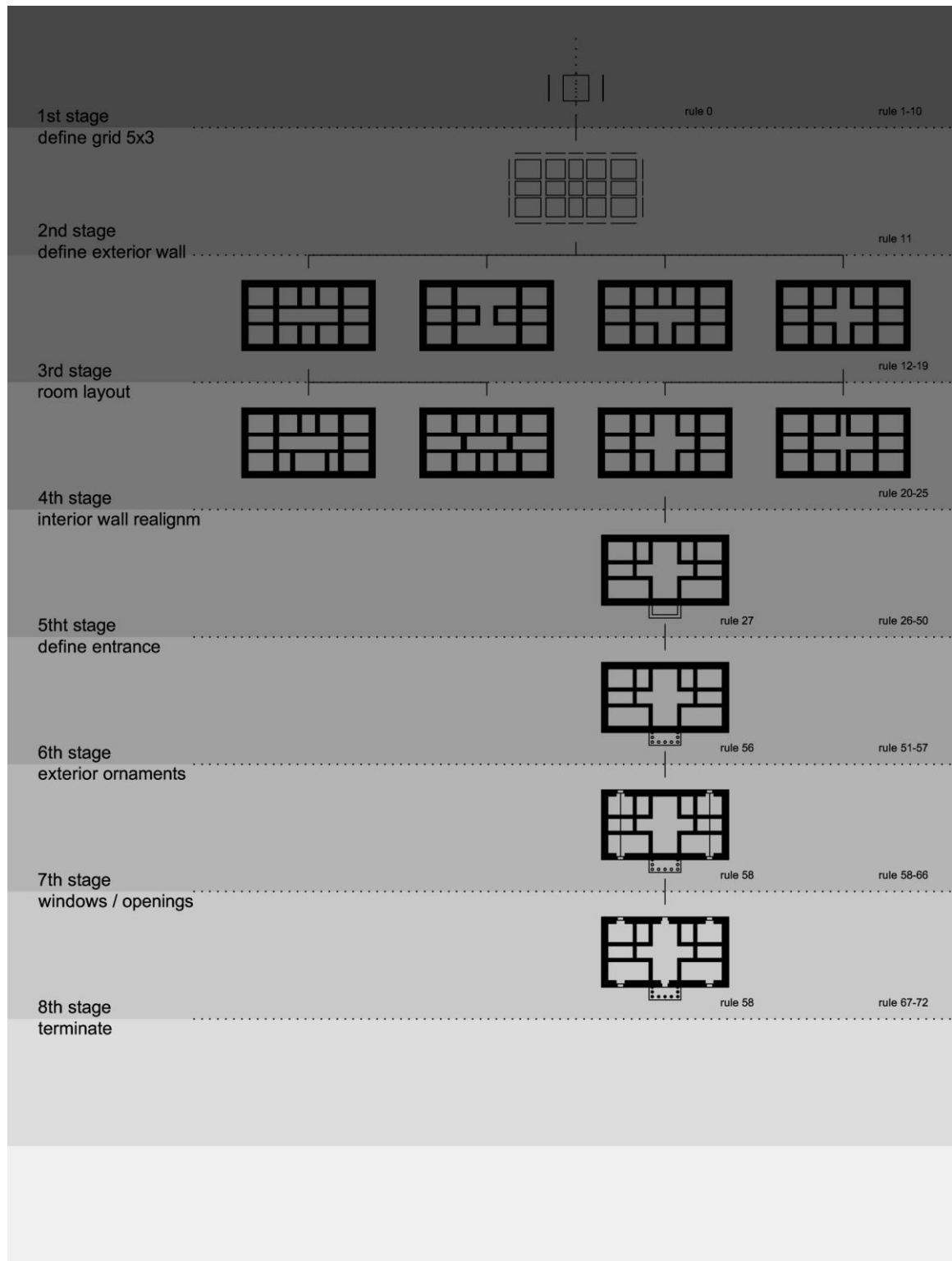


Figure 3. 4: Stiny and Mitchell 1978 structure of the Palladian Villa grammar

3.1.3 Rule System

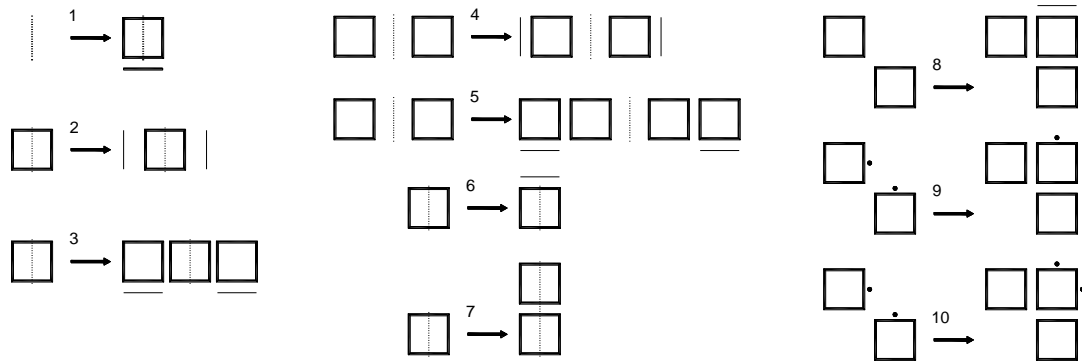


Figure 3. 5: Cell rules 1-10 for the Palladian grammar, adapted from (Stiny & Mitchell 1978)

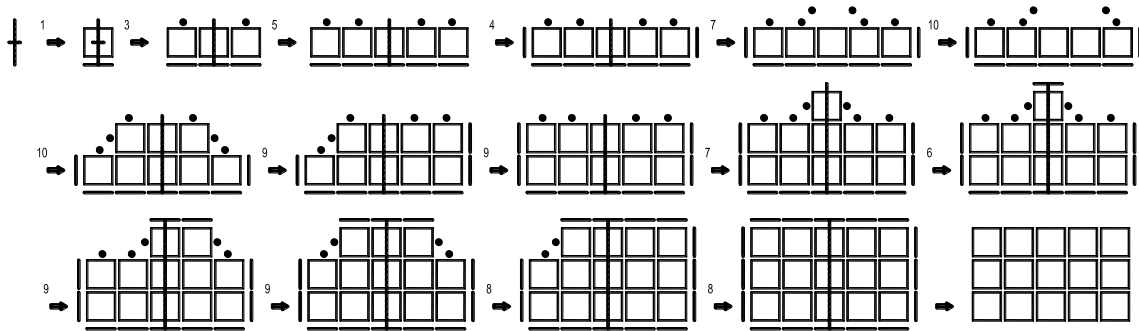


Figure 3. 6: Application of the Palladian grammar rules to generate a 5x3 grid floor plan

The grammar was established using a top-down approach, developed from general to particular. The first stage consists of creating the grid by adding the first room cell, followed by a number of copies and mirroring operations to fill the entire grid. It comprises Rule 1 (adding a rectangle) to Rule 10, as illustrated in Figure 3.5. As shown, the rules are mostly addition rules. Rule 1 initiates the design by adding the first shape. This square shape is accompanied by a straight-line label that describes the boundary of the design. It also displays a central axial label that indicates the centre of the design. Other labels are then introduced, as shown in Rule 3, as two full circles which describe the potential for future transformation, tagging the shape as a preliminary design.

This grid grammar uses an addition mechanism to generate its structure. It is used firstly to maintain the desired proportions and secondly to incorporate the build-up of wall thickness in the grid generation process. The grid generation process is illustrated in Figure 3.6 above. As shown, the recursive application of the ten initial rules generates the design, illustrated here as a 5x3 grid. The first rule to be used is Rule 1 which introduces the first shape/room, followed by Rule 3, responsible for the addition of another room and the immediate symmetry, and Rule 5, which sets the outer bays. Rule 4 creates the boundary so that Rule 7 can start generating the design for the middle

bay. The rest is self-explanatory up to the last step where the grid is now recognisable and the labels erased. In this 5x3 grid, fifteen steps were required to complete the grid design.

The rules did not provide a parametric formulation, which makes it harder to fully apply the grammar to design other examples. There is an underlying suggestion that the shapes to be transformed are square in origin and this is further confirmed by Stage 4 which is responsible for the wall realignment (Figure 3.7). In this stage, shape rules 20 to 25 are applied and the original squares are altered to create rectangles with new ratios. The new ratios are not stated but certain assumptions can be made from other research work and from Palladio himself. In his writing, he refers to the classical proportions and the benefits of harmony. The square or 1:1 rectangular proportion is not only favoured by him but was also well-known in classical architecture. Another preferred proportion is the golden ratio, or rectangle with the square root $\sqrt{2}$:1 proportion. This rectangle is generated by using the square diagonal as the longer side and the square side as the shorter one. Other square root rectangles can be generated, including the 2:1 which is the sum of two squares, or simply $\sqrt{4}$:1 = 2:1.

According to (Hersey 1992), Palladio's intention was to design housing with mostly square rooms or, alternatively, in 1:1, 4:3, $\sqrt{2}$:1, 3:2, 5:3 or 2:1 proportions. The only exception to this rule is the main reception area of the house which varies from a square to a rectangle or more complex shapes such as 'T', 'I' or 'X' shapes. However, these are often obtained from space merging operations and therefore their dimensions are derived from the 'canon'.

Other important geometrical features are linked to the bilateral symmetry of every design step. This is achieved by establishing an axis of symmetry which divides the plan into east and west, linking the main entrance, secondary entrance and core space. This axis is centred within the central bay and also dictates the positioning of internal doors along this line, aligned with the two external doors. It also dictates that the largest room is the one which lies on the axis or, as previously mentioned, the core space. No other rooms can be larger than this or larger than two squares. The central bay hosts the main room with perfectly aligned openings. No walls should be aligned with the central axis. The additional internal doors should lie on a parallel or perpendicular axis. In terms of dimensions, a room should not be shorter than 7 Vicentine feet (a Vicentine feet equals roughly to 245 mm).

The rules above mentioned are integrant part of the first stage of design. The following stages are comprised of:

1. The first stage is responsible for the outline. Uses one shape rule, the insertion of a polygonal shape.
2. The second stage contains only one rule, which is responsible for the insertion of the exterior wall. It is a version of an offset or parallel copy of the perimeter cells edge.
3. The third stage is responsible for the creation of the central core of the villa containing the largest social area. The rules proposed in this stage are four different concatenation or

shape merging rules for adjacent cells which recreate rectangular, T', 'I' or 'X' shaped rooms, as shown in Rules 12-19.

4. In the fourth stage the grid is modified by a series of rules (20-25) that allow for the shifting, realigning and stretching of room walls. As previously explained, these rules allow for some spatial flexibility, diverging from the preliminary fully square grid.
5. The fifth stage positions the entrance, which is aligned and centred within the building layout. It proposes a number of solutions ranging from porticos to loggias and porches, and contains a total of 25 different shape rules.
6. Some exterior classical ornamentation such as cornices, classic columns and other elements are introduced in the sixth stage, comprising Rules 51 to 57. This also marks the last additive step in the grammar. Thereafter the grammar structure changes direction and starts detailing what has been designed so far. Therefore, instead of adding new elements it starts the detailing and modification operations.
7. Stage seven is a typical subtraction stage in which windows and doorways are opened in the walls positioned in earlier stages. These obey the aforementioned Palladio design rules and principles of symmetry and alignment and are encoded in 10 shape rules.
8. The eighth and final stage terminates the villa design and initiates the deletion of any remaining labels. It comprises Rules 67 to 72.

The complete set of design rules proposed by the grammarians is shown in Figure 3.7 in which all the formal representations of the parametric rules, although the precise parameterisations are not available. All of the basic shapes illustrated are squares which is a good foundation *per se* for a Palladian villa (and one of the most widely-used proportions), although the other rectangular ratios values are not available.

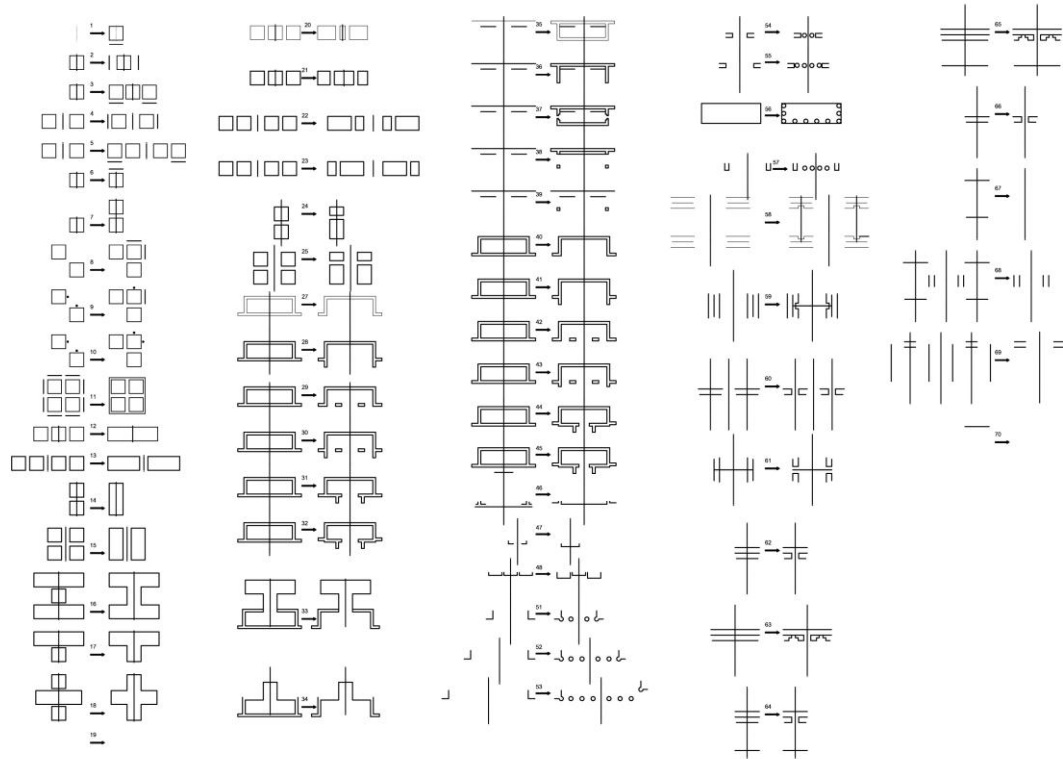


Figure 3. 7: Complete Palladian Villa grammar rule set, adapted from (Stiny & Mitchell 1978)

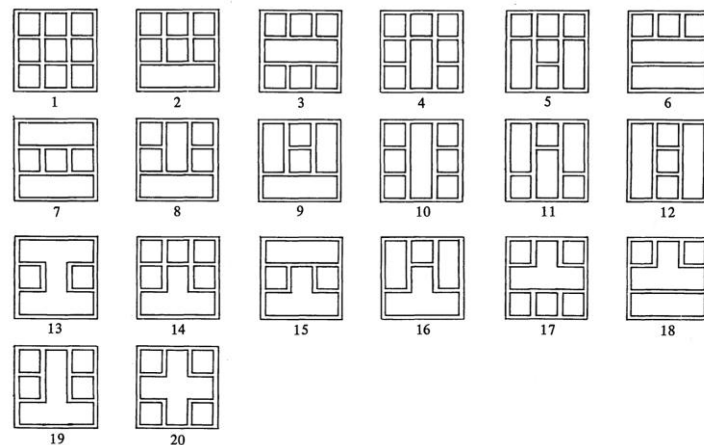


Figure 3. 8: Corpus of 3x3 grids for Palladian villas generated by the shape grammar (G Stiny & W J Mitchell 1978)

Derivations of existing and new examples constitute a good robustness test of the efficiency of a given grammar. As shown in Figure 3.3, the proposed Palladian Villa grammar was tested using these rules in the derivation of the Foscari 'Malcontenta' villa built on the outskirts of Venice in the 1500s (Giacconi & Williams 2003). The shape grammar design is a valid example of the efficiency of the grammar and its ability to design family-related houses that fit the language criteria. However, another test was needed to determine whether the shape grammar would enable new designs to be produced that fitted the family criteria. The 'Counting Palladian plans' (G Stiny & W J Mitchell 1978) presents a full set of simplified designs with all the possible room distributions, as shown in the

illustration above. Apart from the 20 possible designs for plans with 3x3 grids, an additional 210 were generated for larger 5x3 grids. This proves not only the efficiency of the grammar, but also its design potential.

Some abstractions were used; for instance, only rooms with a square footprint ratio were illustrated, although this clearly allows for further modification and grid manipulation as predicted in subsequent design stages.

Not surprisingly, this corpus of designs includes a variety of floor plans either built by Palladio or illustrated in his writings. With a parametric variation Type 1 could easily illustrate the Emo villa (as shown in 3.1) or, with the right grid manipulation, the Ragona villa, and, with different parameterisations, the Type 4 Caldogno or Angarano villas and Type 20 Ragona villa. The rest of the villas presented are new examples. Although some, such as Types 2, 6, 7, 12, 18 and 19, are quite unlikely Palladian designs (possibly because of a few missing rules/conditions in the grammar), others, such as 8, 10, 11, 14, 15, 16 and 17, would make good candidates

The conditions that could further refine the grammar were later identified by the authors as:

'1. Any part of a room that extends from one exterior wall to its opposite must lie along the north-south axis of symmetry in the plan' and '2. the exterior rooms intersected by the north-south axis of symmetry must be as large as any other room in the plan.' (Stiny & Gips 1978).

However, this grammar does not produce the full range of existing design solutions and struggles to accommodate villas such as the Rotonda and other types with 3x4, 5x2 and 5x4 grids.

Similar generative systems, such as the system tested by the 'Possible Palladian villas' (Hersey 1992), also fail to reproduce these designs when faced with variations. This study proposes a tool named Planmaker with generative power. This tool uses an expert system computer software to design Palladian floorplans based on musical harmonies. However, this tool faces some limitations as described by their authors: 'Planmaker cannot produce circular rooms as in the Rotonda.' 'Similarly, Planmaker is at present unable to design T, I and Greek cross shaped rooms and these are not rarities in Palladio' (Hersey 1992).

3.2 The Prairie House grammar

3.2.1 Grammar type and classification

The Frank Lloyd Wright Prairie House grammar was inferred by Koning and Eizenberg in the 1980s (Koning & Eizenberg 1981). One of the novelties of this grammar was the proposed three-dimensional approach to Wright's design language. It was based on the extensive number of built examples of prairie houses and scanned the work of one of twentieth century's greatest architects. The grammar aims to recreate Wright's vision by replicating its design process. The starting point is the chimney and fireplace, referenced in Wright's writings as the focal point of family life. Spaces are then arranged around this focal point, where the more public areas, such as living and dining spaces, converge, followed by service areas and more private spaces such as bedrooms.

Unlike the Palladian Villas approach, this grammar followed a bottom-up process. Basic elements and what may be considered a detail, such as a fireplace, become the design process trigger and the grammar's first stage. This grammar proposes 99 shape rules divided into 23 different design stages, progressing from ground level to the upper storey and roof. The grammar structure is represented in the tree diagram shown in Figure 3.9.

Another significant difference is the generation process and grammar type. The Prairie House grammar is an additive grammar resulting from the combination of several additive steps. As a pure bottom-up approach, it starts with one element and is built up by assembling various elements until the dwelling is complete.

3.2.2 Grammatical structure

This grammar can be defined as a typical example of an addition grammar. This type of additive grammar has particular restrictions to its structure.

'Additive grammars have restrictions on rule format only. Like the rules of basic grammars and sequential grammars, each rule of an additive grammar is based on a spatial relation $S + T$ between two unlabelled shapes S and T (neither equal to the empty shape). Each rule has the form $s \rightarrow s \setminus 1$, where s and $/$ are the shapes S and T labelled in any way whatsoever, or not labelled at all, provided that the rule is not indeterminate (does not apply in infinitely many ways).' (Knight 1999)

It is also formulated as a restricted sequential grammar.

A sequential grammar is a linearly ordered and indexed set of rules. Each rule 'rt' is defined on the basis of a spatial relation $S, -h\ 1]$ between two unlabelled shapes S , and Tt .' (Knight 1999).

This grammar is classified as a restricted grammar since it only allows the use of one specific rule in each specific stage: at no point can the addition process allow the inclusion of a previously used rule since this would distort the functional diagram in place. In this case, sequence and restriction are interlinked. This results into a grammar that shows some apparent repetitions. Shape rules with similar formulation are often introduced with a parametric difference in the values allowed. For this reason, this is a very precise grammar, but not a very elegant rule set. A rule to design a floor slab can be repeated for ground and upper floor for example.

The grammar structure can be seen in Figure 3.9, which exemplifies the real tree diagram shown by the authors, and the simplified bi-dimensional tree diagram of the system in Figure 3.10. As illustrated, the design starts with a small element, following the authentically additive nature of the grammar grows in size and complexity as the chronology progresses. In the first stage only the fireplace is created, progressing to the first, second and third rooms in stages two, three and four. The bottom-up approach is also evident. Whereas in other grammars the outline of the solution is visible in the early design stages, this approach starts with one element of the solution and gradually adds to it.

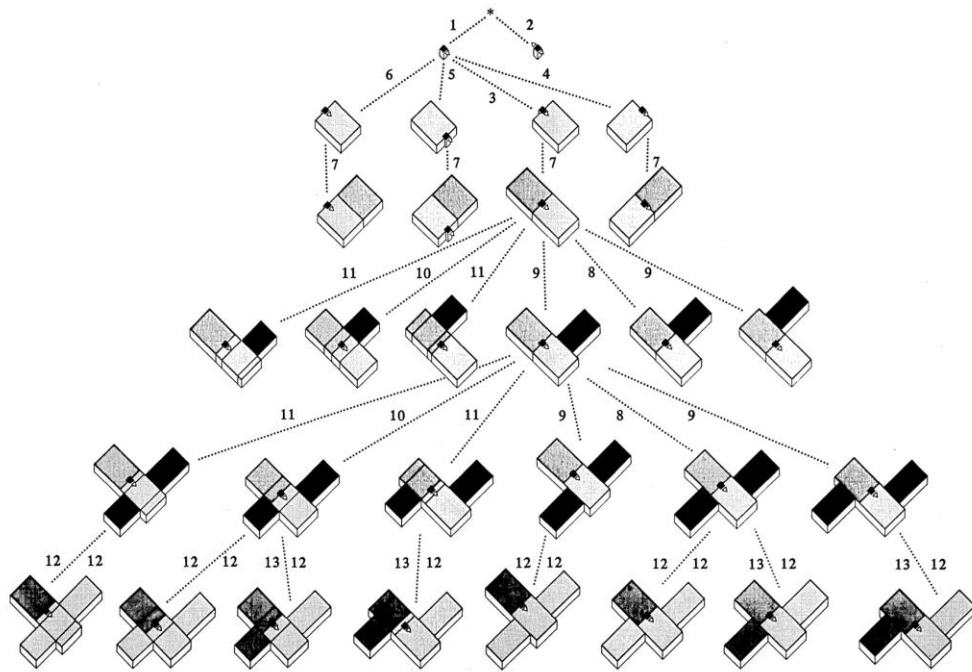


Figure 3. 9: Prairie House grammar rule structure (Koning & Eizenberg, 1981)

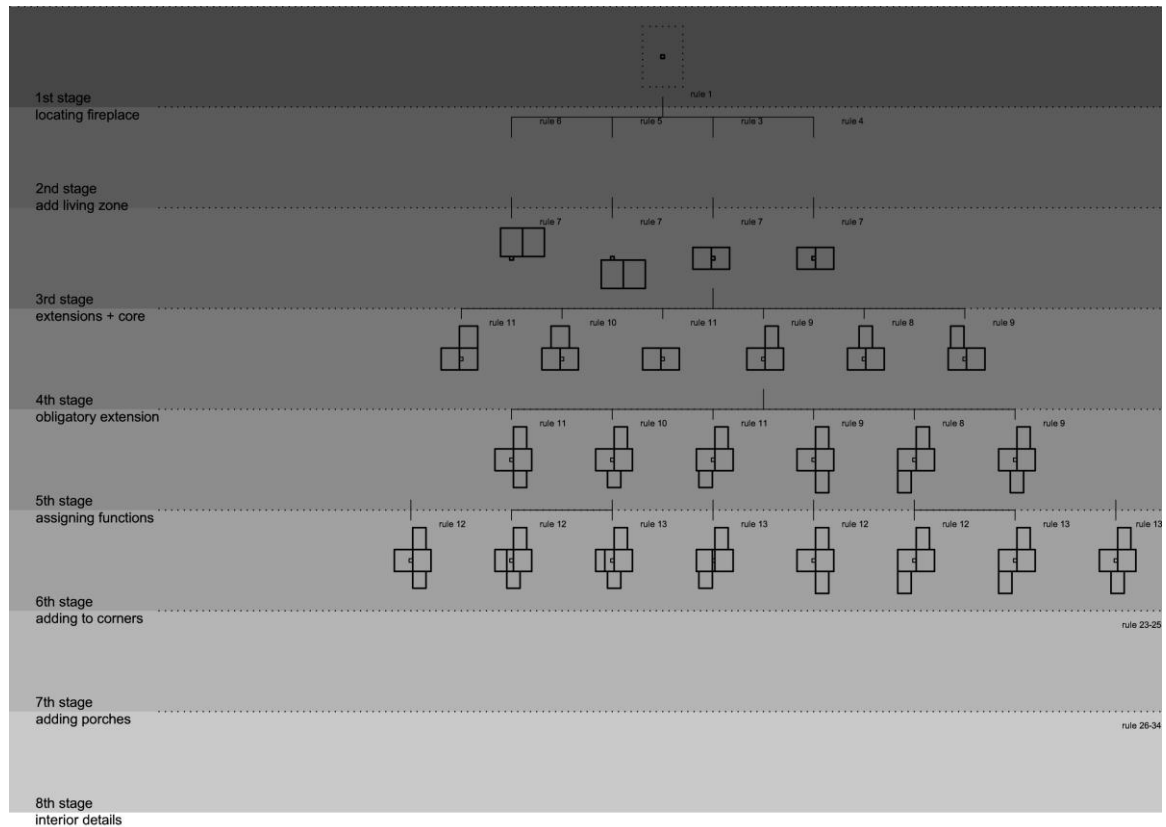


Figure 3. 10: Original Prairie House grammar rule structure

3.2.3 Rule System

The Prairie house grammar rule system comprises the following steps:

1. The starting point is the positioning of the fireplace. This stage combines two shape rules with two positioning options that allow for different arrangements.
2. The next stage is directly related to the fireplace and is responsible for the organisation of the living spaces adjacent to the focal point. Rules 3-6 suggest different room layout options.
3. The third stage continues designing the social area, creating an adjacent living area in the form of an obligatory extension and a core, using Rules 7 to 10.
4. The addition of living areas continues in stages four (Rules 9-11)
5. Inclusion of internal spaces takes place in stage five, in which functions are assigned (Rules 12-18)
6. Stage six proposes the addition of corners (Rules 19-22)
7. Stage seven is responsible for the incorporation of porches (Rules 23-25).
8. The eighth stage proposes interior detailing, using Rules 26 to 34. The details may range from wall shifting, inclusion of columns, merging of adjacent areas or further subdivision of space. At the end of this step the design of the main level, the ground floor, is complete. The following stages almost repeat the previous ones, continuing the design in a similar manner and the process is consequently very similar to what has already been described.
9. In the ninth stage, the basement level is designed, using Rules 35 to 40 which terminates the design.



Figure 3. 11: Rule system for the Prairie House grammar, adapted from (Koning & Eizenberg 1981)

10. The tenth stage designs the terraces and constitutes another addition process, incorporating an exterior space adjacent to the one previously created as an extension of the interior spaces. Rules 41 to 47 present different alternatives.
11. Rules 48 to 53 propose different elements as exterior ornamentation or detail. They constitute the 11th stage and represent either an additive process (e.g. creating a corner element) or a subtractive process (detailing by intersection processes, such as chamfering or pierced corners).
12. Stage 12 proposes two types of solution that enable a porch to be included using a subtractive process.
13. The next stage, comprising Rules 56 to 60, is a technical step which deletes the remaining labels from the house design.
14. The design of the upper floor begins in stage 14, which encompasses the design of the bedroom floor using two different shape rules that govern the placement of two rectangular compartments (Rules 61 and 62).
15. The next stage, comprising Rules 63 and 64, adds bedroom extensions to the previous design
16. Stage 16 adds further detailing to the spaces already created
17. An important feature of the Prairie House grammar is introduced in the 17th stage, which introduces double height spaces in the living areas, incorporating a characteristic feature of Wright's designs for social areas. This may be analysed as union by merging spaces or concatenation, and is encoded in Rules 67 to 72.
18. Stage 18 is another technical stage required to erase the remaining bedroom labels.
19. The next stage establishes the roof eave lines, a process graphically described in eight shape rules, namely Rules 75 to 82. Stage 19 is responsible for identifying the outlines of the design volume and offsetting this to create an overhang. As this is a pure additive process (placing a roof over the construction), it is a geometrically and bi-dimensionally scaled copy that maintains the original geometric centre.
20. In stage 20 balconies are proposed. As in the case of stage 7 (adding porches) or stage 10 (adding terraces), this 20th step is not new but is another additive process, placing a rectangular shape adjacent to other designed elements. It is summarised in two shape rules, Rules 83 and 84.
21. Much of what has been described so far appears to be repeated for the upper level, with stage 21 resembling stage 13 as a deletion process for erasing labels.
22. Stage 22 combines two Boolean extrusion and intersection operations. It specifies the roof formation using an initial typical additive process, then continues by dividing different sloped planes to create a pitched roof. Ten rules for different shaped intersections are described, concluding with Rule 97.

23. Rules 98 and 99 complete the design of the house by adding extra details, such as the round roof or the projection of the fireplace chimney over the roof. This constitutes stage 23 and marks the end of the house design.

The rules derived from this grammar enable the user to design a corpus of nearly 90 different solutions.

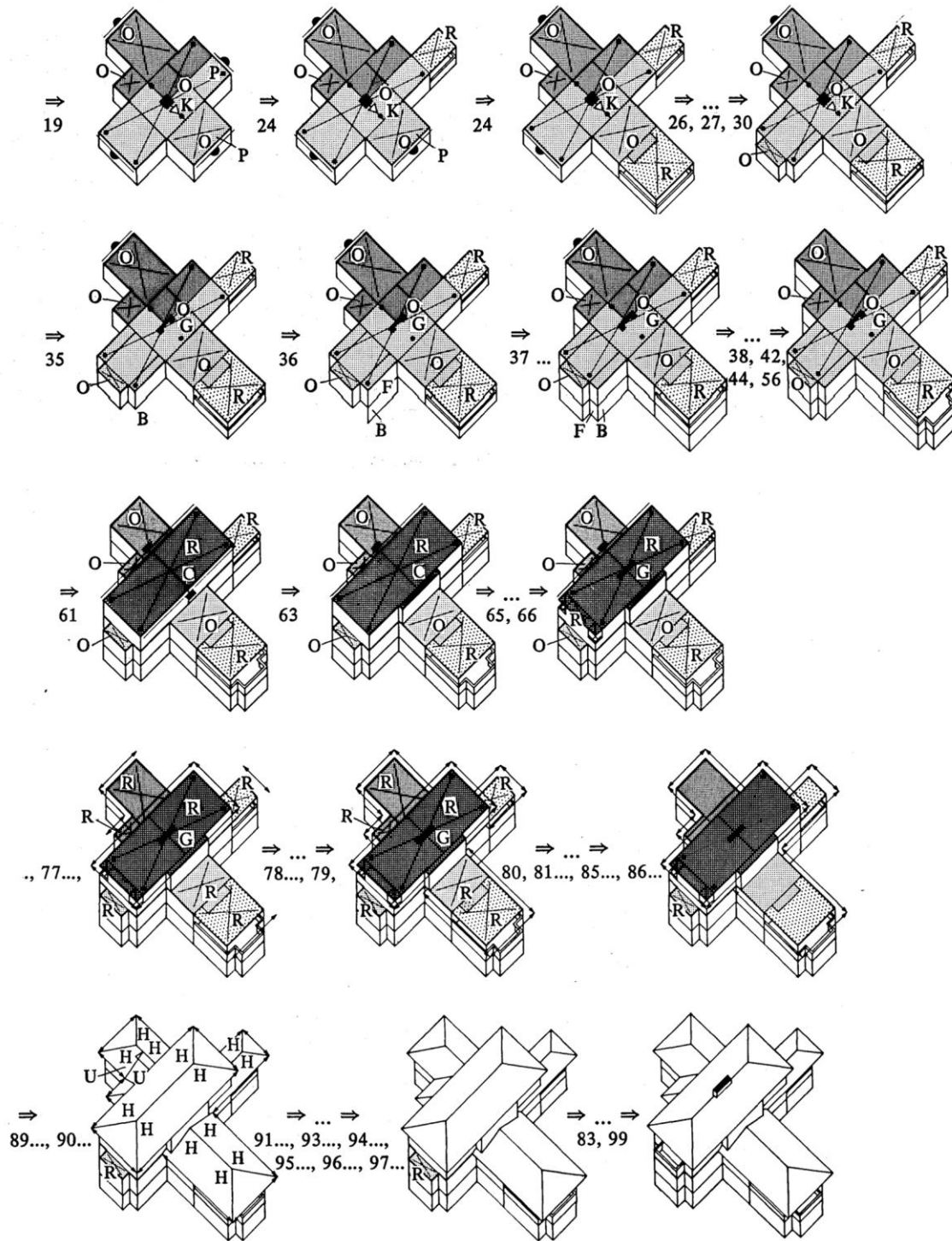


Figure 3. 12: New Prairie grammar houses: Stiny house derivation (Koning & Eizenberg, 1981)

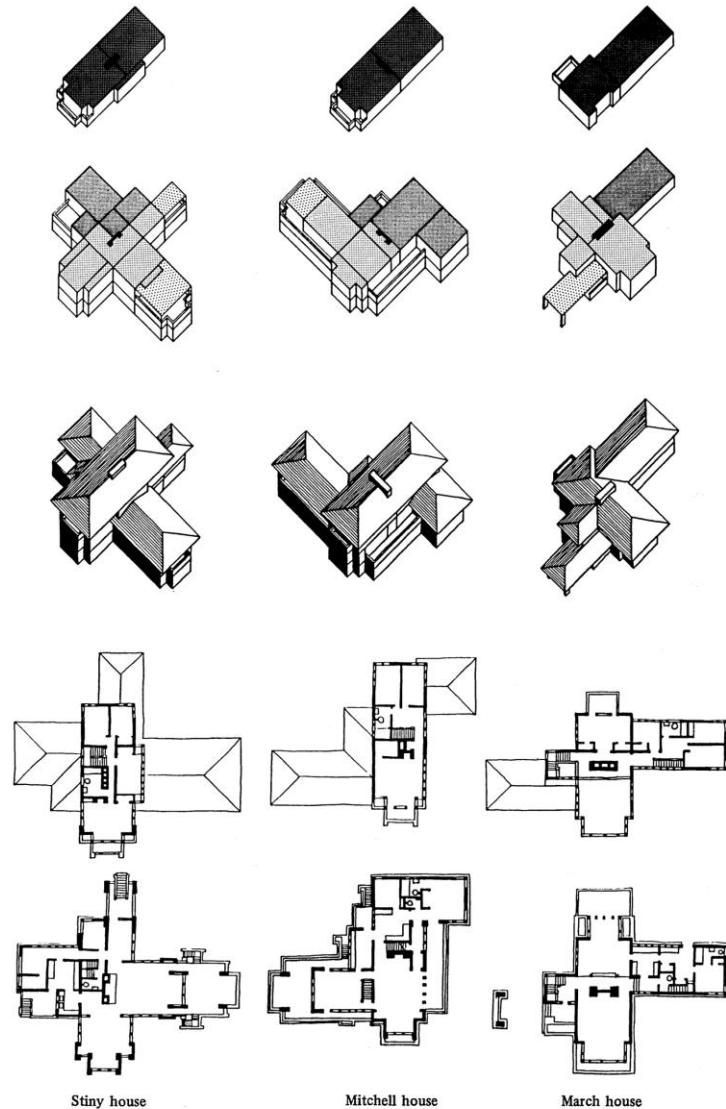


Figure 3. 13: New Prairie grammar houses: Stiny, Mitchell and March (Koning & Eizenberg, 1981)

Figures 3.12 and 3.13 show three new designs generated by the Prairie House shape grammar, all of which demonstrate the efficiency of the grammar. Figure 3.12 demonstrates the derivation process and rule application system. The so-called Stiny house is generated from a cruciform plan and resembles Robie house. The fireplace occupies the centre and the living spaces are arranged around it, whilst the service areas occupy a separate wing. The top floor is smaller, allowing for external terrace space. The volumes are juxtaposed, as in the Froebel box games, a known influence on Wright's work, and the roofs cantilever over the external spaces and extend beyond the perimeter, creating separate entities for each box shape.

A visual analysis of the results confirms the accuracy and efficiency of the grammar. Although the authors' results do not provide a derivation of a known example, the use of the grammar processes and rules would allow accurate and detailed examples to be designed.

3.3 The Malagueira House Grammar

3.3.1 Grammar type and classification



Figure 3. 14: Malagueira house, type Ab: a three-bedroom, semi-detached house with courtyard, plans and elevation from (Duarte, 2001)

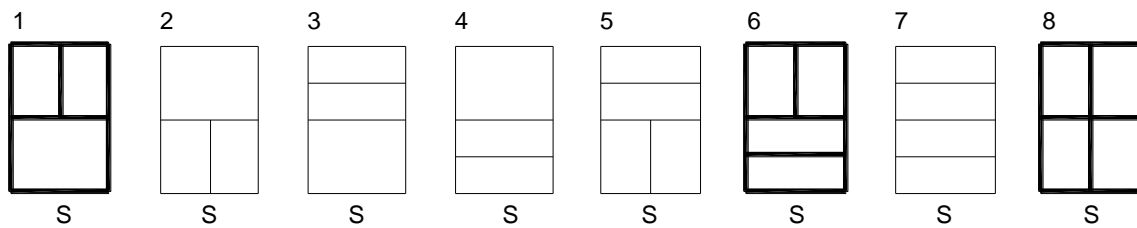


Figure 3. 15: Malagueira subdivision patterns from (Duarte, 2001)

The third grammar in the case study is Alvaro Siza's Malagueira House grammar. This grammar was created and its shape rules inferred by José Pinto Duarte and describes the work of a contemporary living architect (Duarte 2001) (Duarte 2005).

To the best of our knowledge it was the first time the actual architect has been involved in the rule extraction process, grammar Inference and vetted a shape grammar.

Siza was able to question his own design choices and point out his architectural concept, design aims and the reasons behind certain choices. This may be the reason why the grammar stands out as being complete, combining up to 160 different shape rules separated into 20 different stages. Another advance was the use of a computerised tool in the implementation of the grammar. The set of design rules were encoded in AutoLISP and run on an AutoCAD platform to allow for real-time modelling of the customised solutions.

Unlike the Palladian, but similar to the Prairie House grammar, this grammar addresses three-dimensionality, although it uses a parallel grammar representation instead of a conventional 3D rule illustration. It contains both a plan and a section or elevation view showing the impact of the shape transformation rule applied in both cases.

The grammar is able to design not only the 200 solutions built in the Malagueira neighbourhood, but also a vast corpus of new solutions, some acknowledged by Siza as feasible and fitting within his design vision.

The proposals were designed as affordable, single-family housing in the south of Portugal and the contemporary; purist design draws its influence from vernacular southern Iberian housing, which has its roots in Islamic architecture. One example is shown in Figure 3.14. It bears a vague resemblance to courtyard houses and even the envelope and finishes are reminiscent of this influence. Similar grammatical studies have been carried out into the fabric of Islamic cities, namely the Marrakesh urban grammar (Duarte et al. 2006).

Unlike the grammars previously described, the Malagueira House grammar uses the third grammar typology, thus constituting a subdivision process. This means that the generation process for each house starts with an overall generic plan and detail is provided by series of spatial subdivision procedures directly related to the architect's design process and local requirements. The subdivisions take place by consecutive polygonal division.

Each house was assigned a similar 12x8m rectangular plot, resulting in a plot area of 96 sqm. The house was then designed by defining a reasonable portion for the courtyard (either $\frac{1}{2}$ or $\frac{1}{3}$), followed by the selection of the interior area and the division of the living, circulation and service areas. The subdivisions follow specific rules, from which certain common patterns can be seen to emerge (Figure 3.15). These allow for the development of the interior layout, whilst maintaining the external space destined for the yard. They can be observed in the existing built examples and although varied, some patterns, such as 1, 6 and 8, have more in common than others, as highlighted in the image. It is also worth mentioning the relationship between these patterns and the access and adjacent street. This helps to establish some of the interior distribution by typically positioning living spaces and yards close to the access, and the more private zones such as service, kitchen and sleeping areas in the interior of the plot.

Despite its typical subdivision nature, the Malagueira grammar can be considered a deterministic grammar. Deterministic grammars place a restriction on rule ordering only. This restriction is a generalisation of the rule-ordering restriction for basic grammars. (Knight 1999). The results are in any way predictable, the number of rules is finite and ordered and determines when and where they can be applied. The corpus of results is, in many cases, infinite, and in this case, it provides thousands of solutions.

It is also a discursive grammar, since it generates a house which follows the original house brief, establishing an extensive set of rules that describe the transformation in detail. This description is a parallel description, both in plan and section mode, or alternatively in plan and elevation when

required. The resulting solution has a three-dimensional output but is illustrated bi-dimensionally in each rule.

The grammar, unlike the other grammars (such as the Palladian and Prairie houses grammars), follows a top-down approach. The designer formulates an overview and gains an insight into the overall system, followed by the detailed refinement of other subsystems. Although different in nature and in typology, it is related to the Palladian grammar due to this top-down approach. The general plan is described and detailed progressively, rather than starting with detailing.

3.3.2 Grammatical Structure

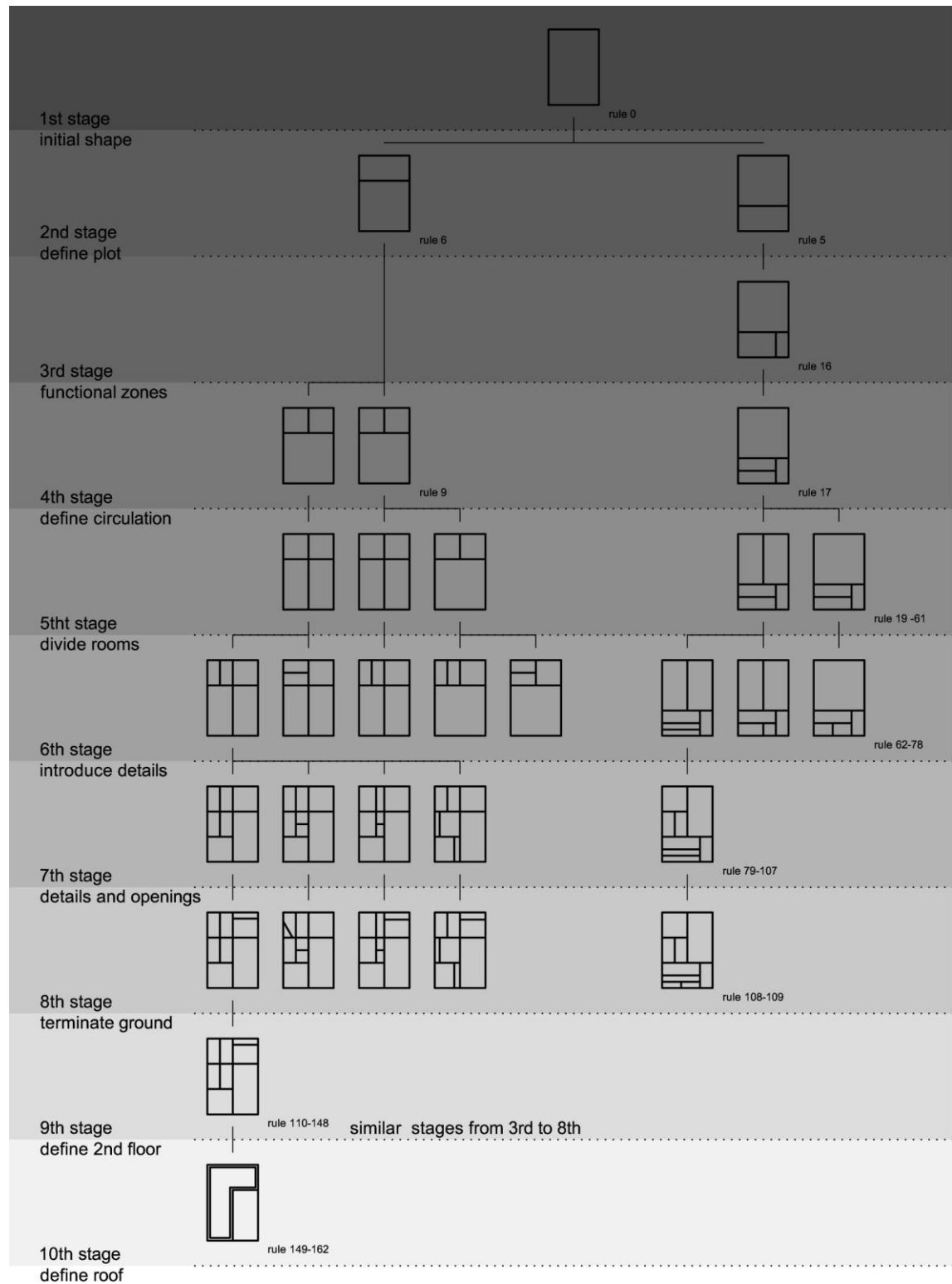


Figure 3. 16: Original Malagueira House grammar structure

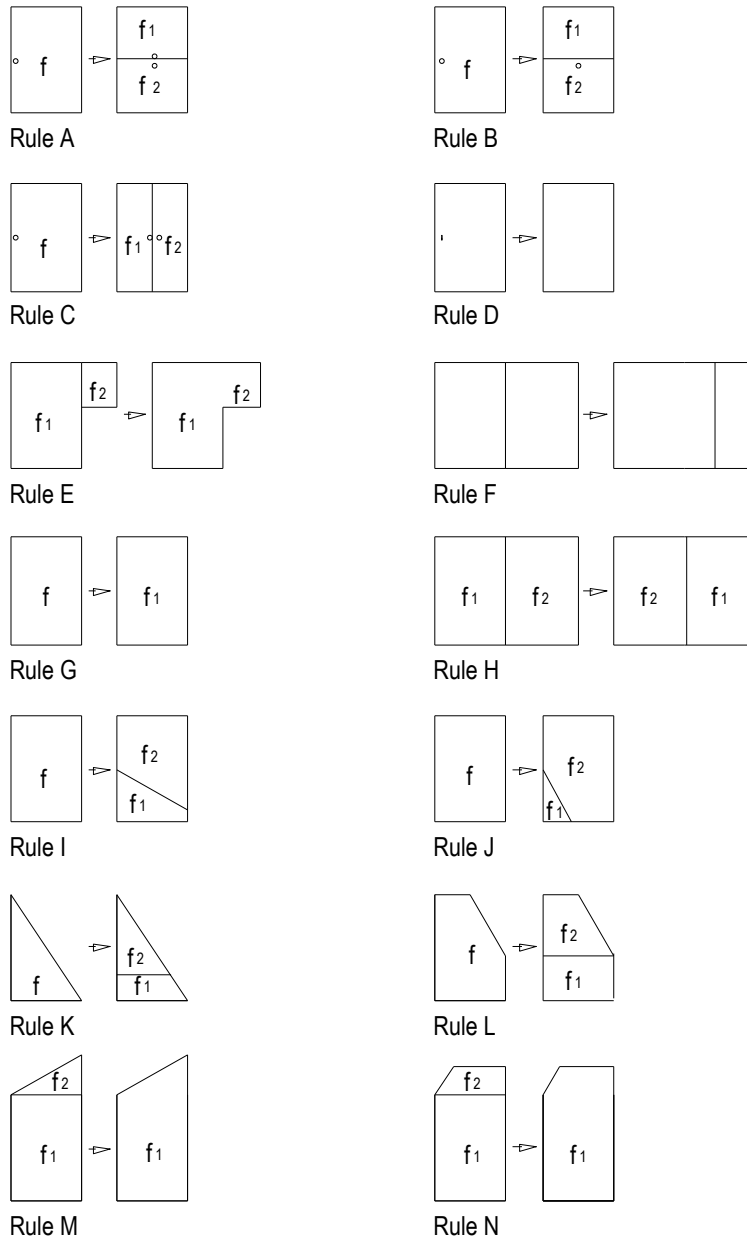


Figure 3. 17: Summary of the Malagueira subdivision rules (Duarte 2005)

3.3.3 Rule System

Like the other grammars, the Malagueira grammar starts with an additive process despite being a subdivision grammar. The tree diagram in Figure 3.16. describes the house generation process.

It also proposes a common additive rule, this is illustrated in the placement of a rectangular shape representing the fixed plot size.

1. This first stage is mandatory and is represented by Rule 0, which establishes the plot. Rules 1 to 4 describe a series of rules to define the plot and create the boundary wall. Special mention should be made of Rule 2, which represents an offset, another commonly featured rule. This is responsible for the boundary wall placement and, in geometrical terms, duplicates or executes a scaled copy of the plot.
2. In stage 2 the plot is defined by another (rare) additive process.
3. In the 3rd stage the grammar type emerges, proposing a typical subdivision process. It is the first of a series of recursive subdivisions that occur in the house generation and can be seen clearly in the grammar tree diagram. Similar grammar arrangement techniques are evident in the Ice-ray (Stiny 1977) and Hepplewhite grammars (Knight 1980). The most typical subdivision rules are summarised in Figure 3.17. This set of 14 parametric subdivision shape rules constitutes the range of options allowed by the grammar. Simple rules such as A, B, C, E, F, and G represent straightforward divisions of a rectangle into two smaller rectangles. Rules G and H do not represent shape subdivision but spatial functional assignments. Rules I and J are special rules that are paradigmatic of the Malagueira style. They create diagonal subdivisions with a diagonal wall placement in the remaining space. Siza created these subdivisions using triangles with ratios of 2:3 in order to incorporate pentagonal living spaces, a variation to the highly orthogonal design of the house. Rule K allows for one level of subdivision in the triangular shape previously created by rule J. Finally, rules L, M and N propose subdivisions to the pentagon previously generated in rule J, namely a pentagon and a square space (L), a rectangle and a triangle (M) and a small pentagon and a rectangle (N). All of the rules described can be applied in stage 3. This stage encompasses Rules 5 to 12 to subdivide the plot into functional zones. The parametric rules convert simple rectangles into two separate rectangular elements segregating the courtyard and built area. The proportion of the resulting spaces is the key to maintaining the character of the design language and was inspired by observation of authentic built examples and their blueprints.
4. Circulation is defined in the 4th stage by a set of six rules extending to Rule 18. Using a subdivision process once more, the interior space is divided into two areas. The narrower one is reserved either for the entrance (Rules 13 and 14) or the positioning of the staircase

(Rules 16 to 18). Moving from the general to the particular, functional zones slowly converge into proper divisions.

5. In the 5th stage, zones are divided into rooms taking the subdivision process to a more detailed level. This is one of the lengthiest stages, comprising more than forty different rules (Rules 19 to 61). A complete set of room shapes is explored at this stage, with an emphasis on Siza's preferred rectangular ratio of 2/3 and on angled rooms with 2/3 angular walls (explored up to Rule 58). In Rule 59 an exception is included: using a mainly subdivision process a shift rule proposes the union, merging or concatenation of adjacent spaces. The rest of the rules in this stage propose detailing.
6. The subdivision is then interrupted in stage 6 by the inclusion of details such as chimneys or risers (Rules 62-67), the adjustment of wall thicknesses (Rules 68-73), the inclusion of steps or further detailing of staircases (Rules 74-76), and adjustments to the patio wall, which concludes this stage (Rules 77-78).
7. Stage 7 is responsible for creating doorways and window openings. The parallel grammar allows for the accurate description and parameterisation of these obtrusions, showing the correct positioning of windows and doors in elevation view. It contains Rules 79 to 107, which constitute a group of Boolean subtraction operations.
8. Stage 8 resembles stages from other grammars and is responsible for label deletion. In this case it erases the remaining ground/main floor labels (Rules 108 and 109) and prepares for the design to proceed to the next floor.
9. In stage 9 the upper level is defined, using both additive and subdivision processes. The first shape rule (Rule 110) is very similar to Rule 0, assigning a rectangular floor slab above the previously designed ground floor. What happens next is a repetition of the process described so far. The successive subdivision steps are executed to create the interior layout of the upper floor and Rules 114 to 119 describe division rules similar to Rules 5 to 12 in stage 3.
10. As in stage 4, the 10th stage describes the derivation of the circulation areas. Rules 120 to 125 present staircase layouts that are related to Rules 13 to 18.
11. In the 11th stage zones are converted into rooms by a subdivision process similar to the one described in the 5th stage. Rules 125 to 132 explore the different possible geometries, while Rules 133 to 136 assign functions to the spaces that have been designed. Not surprisingly, this is also a subdivision process.
12. The 12th stage parallels the 6th stage, introducing details into the divisions. The rules extending to Rule 146 show how the chimneys and risers are related to the upper level. This process can be described as both additive and subtractive, very much like any other detailing operation.
13. Stage 13, not surprisingly, draws inspiration from the precedent set in stage 7 and creates the top floor (window and door) openings. It proposes a set of 27 rules in which Rules 91 to

93 create exterior openings and Rules 94 to 99 interior openings, whilst the remainder up to Rule 107 generate window frames and mullions.

14. As in stage 8, the 14th stage terminates the design of the upper floor by deleting the leftover labels.
15. A new loop starts with stage 15, in which the terrace or flat roof is designed. At this point an additive process is introduced by including the roof slab as a rectangular element, as described in Rule 149. The final roof slab must duplicate the design of the upper floor and a new stage is therefore implemented to transform the simple shape into the desired one. This takes place in the 16th stage with the application of a series of subdivision rules, namely Rules 150 to 153. Once again, a detailing stage is recalled (similar to the 6th or 12th stages) in order to design the chimneys that protrude from the roof slab. Rules 149 to 160 graphically describe this process.
16. The process ends with the final deletion of the remaining labels expressed in Rules 161 and 162, which combine to create the final stage.

Subtype Ab

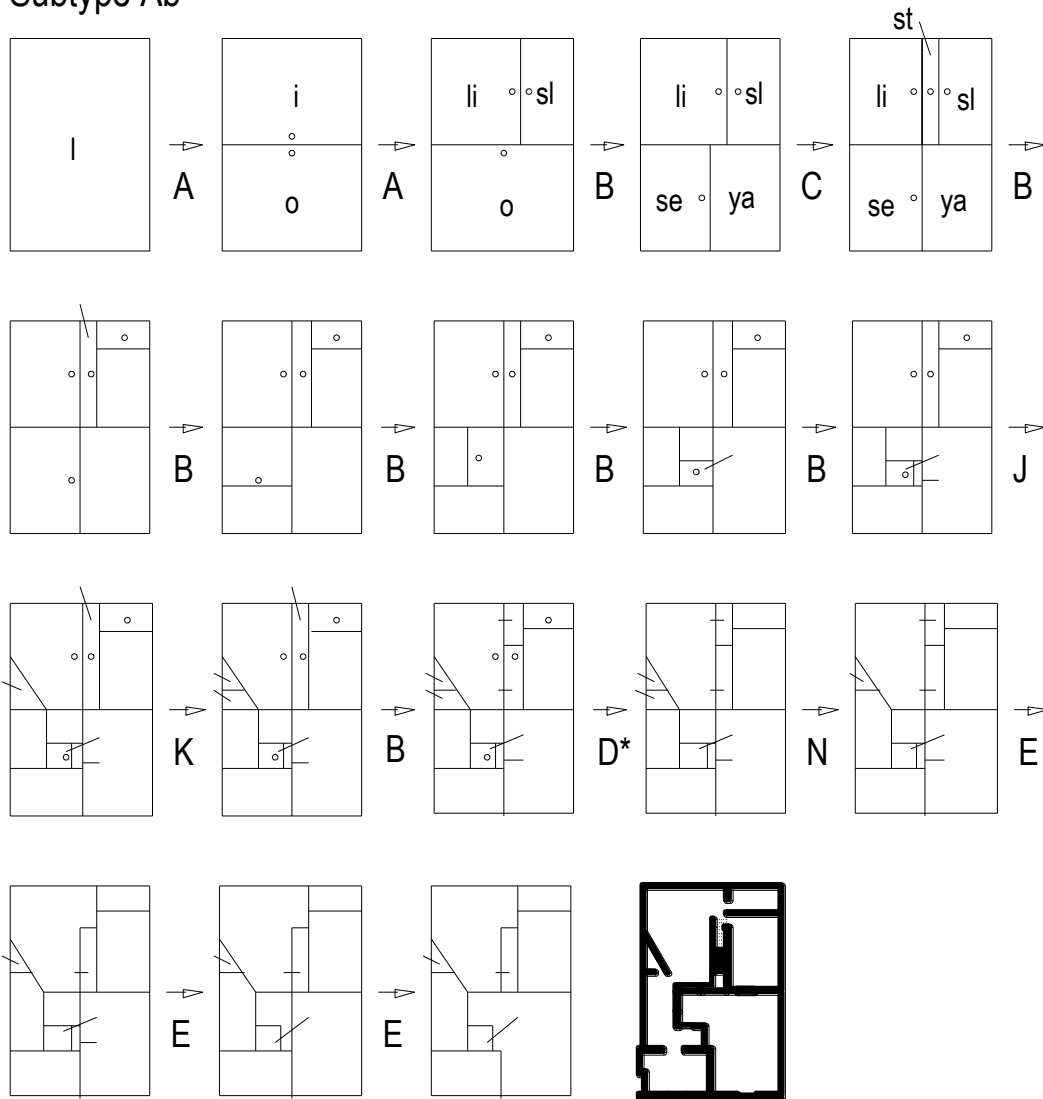


Figure 3. 18: Malagueira grammar derivation for house type Ab (Duarte 2001)

The grammar efficiency was verified using two methods:

- the derivation of an existing example (as shown in Figure 3.18 above)
- the generation of a corpus of new designs (Figure 3.16).

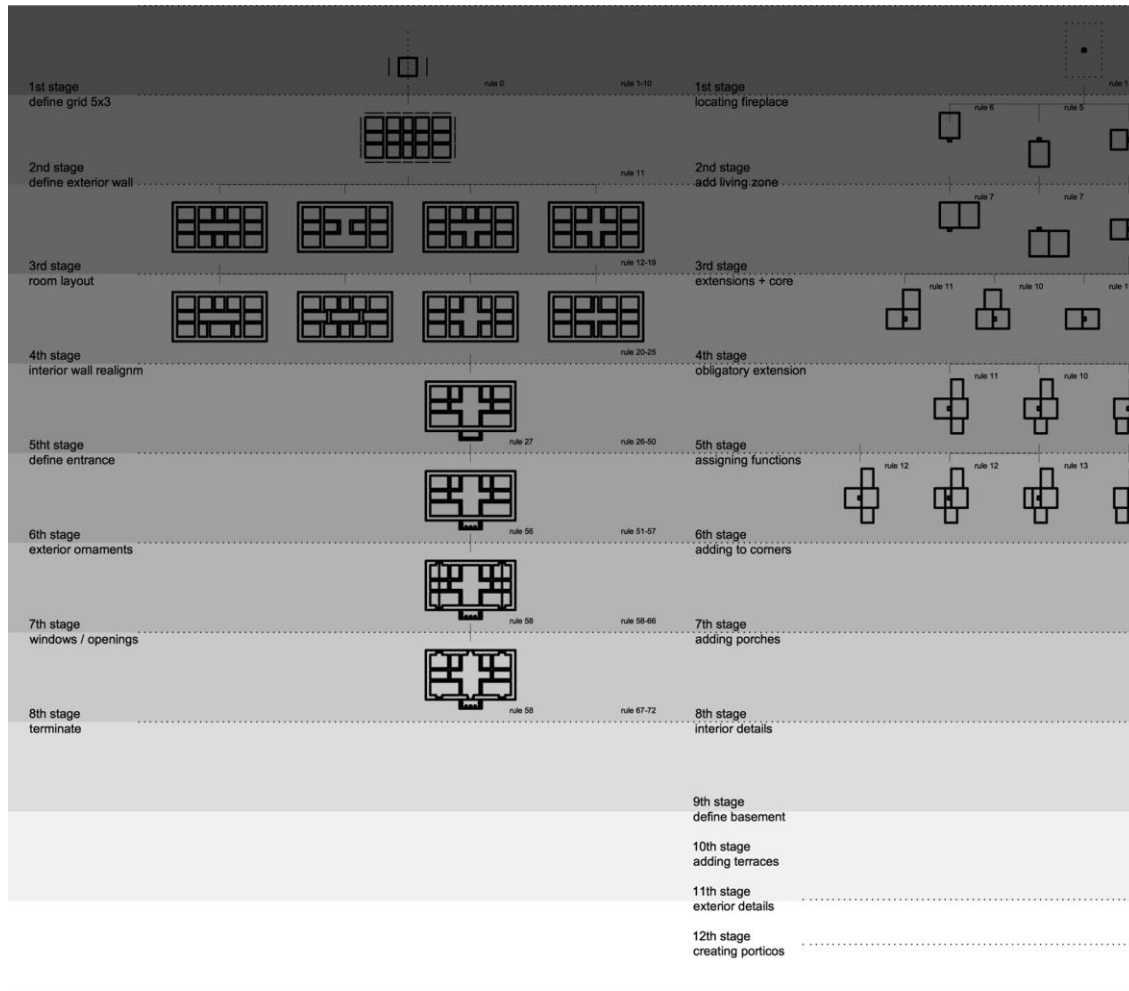
The Malagueira house example of a typical three bedroom two-storey semi-detached house with a backyard occupying 1/4 of the plot, named type Ab is shown to illustrate the grammar generated examples. The existing house derivation is shown as a typical subdivision process that begins with a rectangular shape, subsequently subdivided into the interior/exterior, and the interior further divided into service and social areas. The grammar rules proved to generate the desired house, in

addition to a corpus of new houses. Four of these houses, ranging from two to five-bedroom types, are illustrated above, generated by the same set of rules provided by the computer software. Some of these examples were overseen and approved by the architect as potential candidates for the architectural system, baring the same design criteria and language features as the built corpus. The success of the Malagueira grammar can be attributed to the wide range of consistent new solutions that potentiate and to its unprecedented computer implementation. The well-crafted grammar is supported by an extensive rule system that describes in detail the full range of existing examples. That allowed for a comprehensive system that describes the full extent of all known solutions. By doing so it describes the design language.

Since the system is so complex and extensively described by hundreds of rules and parametric conditions, the computer implementation takes the onerous task of putting the system in place and recreating new solutions. This constituted an advance in the state of the art of grammar research.

3.4 Comparison of the case study shape grammars

3.4.1 A shape grammar comparative study



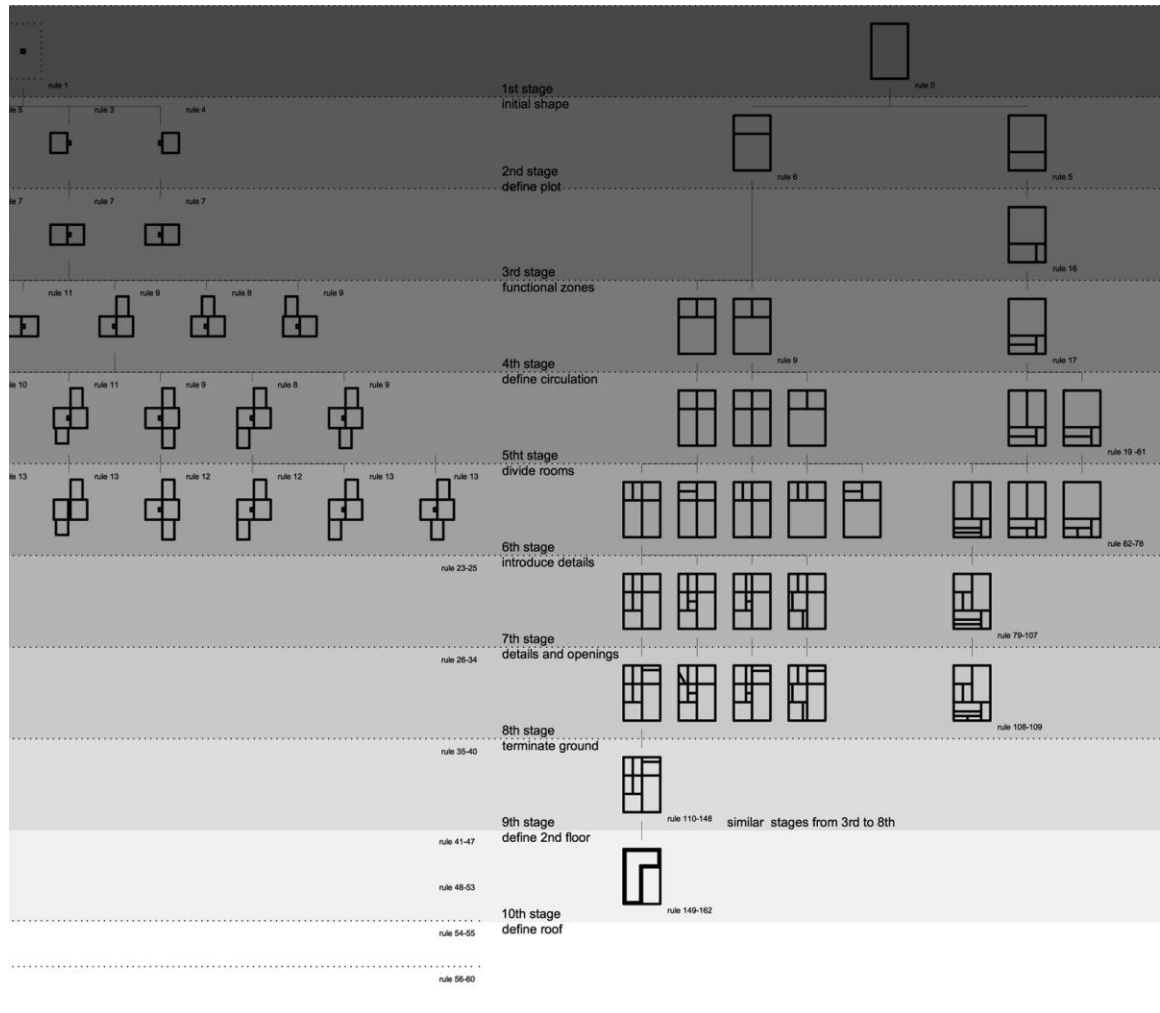


Figure 3. 19: Comparison of all case study grammars

The three tree diagrams for the grammars part of the case study were placed side by side for further comparison and analysis.

This enabled each grammar and the processes used to be analysed stage by stage.

The method used enabled each grammar to be decomposed in the same way on the basis of the following criteria:

- Bi-dimensional representation
- Segregation of sets of shape rules by function
- Chronological decomposition of rule structure in a tree diagram, regardless of the grammar type (grid, addition or subdivision) or process used (bottom-up or top-down)

3.4.2 Comparison and results

The comparison of these 3 grammars highlights certain important aspects which are worth analysing.

There are clear differences in the grammars, such as:

- Different design approaches followed by the original designers
- Distinct design languages, reflecting very different geographical environments, social situations and even different periods of construction
- Independent grammarians with different backgrounds, experiences and approaches
- A corpus of housing solutions from different periods, namely Palladian villas from the 1500s, Prairie houses from the early 1900s and Malagueira houses from the last quarter of the twentieth century, all reflecting different dwelling experiences and aspirations
- Top-down and bottom-up approaches in the three grammars. A top-down approach was followed for the Palladian Villa and Malagueira House grammars, whereas the Prairie House grammar used a bottom-up approach, starting with one detail and working towards the overall plan
- Different ways of operating with design processes, ranging from additive to subdivision processes
- Different representations used in each of the grammars, namely bi-dimensional (Palladian), three-dimensional (Prairie House) and a bi-dimensional parallel grammar that addresses three-dimensionality by expressing it in a dual orthogonal projection (Malagueira)
- Different grammar typologies, ranging from Grid (Palladian) and Additive (Prairie House) and Subdivision (Malagueira)

On the other hand, despite the apparent divergences, there are also similarities:

- All the grammars refer to the same architectural typology - single family housing
- All the grammars focus on various built examples of a common and well-known design language

- The grammars benefited from the amount of written documents provided by the original designers, expressing clear design intentions and original design concepts
- All the grammars allow for the generation of the existing corpus and a corpus new of solutions which follow the conceptual design rules and criteria for the design family
- The three grammars have a similar stage-by-stage formulation and construction structure
- The number of design stages can be narrowed down to 8 operative stages
- Although the grammars present different amounts of rules within the design set, this can be narrowed down to a similar number of different types of shape rules
- Some shape rules are more frequent and appear in every grammar with similar formulations, such as the addition of a rectangle, the offset, the subdivision of a rectangle and the concatenation of adjacent spaces.

The analysis and cross comparison of the grammar selected for the case study required adapting the shape grammar representations to a common format, which had to be either two- or three-dimensional. Two of the grammars could be represented fully in three dimensions but the Palladian grammar addressed plan view only and therefore did not convey enough data for further analysis. The first step was therefore to revert all grammars to two-dimensional representation and convert all the illustration to plan view. A tree diagram was then elaborated for each grammar, illustrating the main stages and eliminating recursive or repetitive stages.

Other comparisons were also possible, namely the number and type of rules used in each stage. By placing the three grammar tree diagrams side by side it was possible to have a clearer overview of their similarities and common features. This comparative tree diagram is illustrated in Figure 3.14.

The processes that had been used were observed stage by stage. Not surprisingly, the Palladian Villa and Prairie House grammars reveal eight similar shape operations or processes. Both use additive processes most of the time, following a stage sequence. Nevertheless, one grammar (Palladian Villas) adopts a typical top-down approach and the other (Prairie House) a bottom-up approach- The Malagueira House grammar diverges slightly, reflecting similar processes in 3 of its 8 stages. This could be explained by the actual nature of the house design generation. Whereas the two other examples only use addition, Malagueira starts with the boundary shape and divides recursively until it reaches the smallest compartment. Malagueira resembles the practice used in the Ice-Ray Chinese lattice grammar (Stiny 1977) where generic and sequential diagonal window frame divisions create random mullions and transoms in a spider-web pattern from a rectangular boundary shape.

Despite the differences, three common stages occur in these grammars at the same point in the design process. The two first stages and the final stage are similar in concept.

The three grammars studied seem to be prompted by the introduction of a simple shape, a rectangle, and addition is always the first generative process.

From the second to the sixth stage the Palladian Villa and Prairie House grammars display several similarities and are different to the Malagueira House grammar. Whereas in the Palladian and Prairie House grammars the generative processes continue by adding new elements to the object being designed (the 2nd stage adds the exterior wall by offset or designs a living room, the 3rd stage rearranges the interior layout, the 4th stage carries out interior modifications, the 5th stage adds another element, either an entrance or an extra room, and the 6th stage deals with further detailing), in the Malagueira subdivision grammar the 2nd to 6th stages present a succession of hierarchical subdivision processes using a top-down approach. After the addition of the plot and courtyard wall, the design steps progress from a large rectangular plot to a small compartment by repeating the series of divisions.

In the 7th stage, the grammars seem to converge and use similar processes. Once the overall design and functional house layout is finished, detailing follows regardless of the approach taken. At this stage details are conceived of as either creating window or door openings, as in the Palladian or Malagueira grammars, or creating exterior details such as porches or concave corners, as in the Prairie House grammar. Although they are different processes, they share a common formulation. Both start with a subtraction process in which a solid intersects a set of other solids. The first of these and its intersection is absorbed or eliminated, followed by the addition of smaller elements such as window frame elements, doors or columns.

The 8th stage represents a technical stage in the shape grammars, consisting of the deletion of labels. Labels are annotations or notes left during the design process to show important clues or restrictions which should be taken into account while designing. Once the design is finished these notes must be erased and, although they may be periodically erased during the course of the grammar derivation, it is necessary to remove any that remain at the end. This is a process common to all the grammars observed.

The tree diagram allows us to observe that the design of Palladian villas requires 8 stages, while the Prairie House grammar 23 stages and the Malagueira grammar 16. This structure was defined according to the authors. However, the grammars that illustrate multiple storey houses (Prairie House and Malagueira) a recursive loop is established to design the upper floors. The same processes and similar stages/shape rules are recursively applied to design the upper storeys. A stage repetition occurs in these cases. In the Palladian Villa grammar, only the main floor, which Palladio called the 'piano nobile' is designed, which was also the only floor illustrated in his treatise.

Despite the already mentioned differences, there is an overall consistency in the evolution of the design process. Regardless of the approach there is an initial level of abstraction that allows for future concretization and detailing. This is observed even in bottom-up approaches. Whether they start with a generic shape or a specific room, they progress from room shape to wall confinement, followed by detailing, such as doors and windows. The design process is embedded in the rule system mimicking the decision-making process.

The Palladian Villa grammar displays particular features that relate directly to Palladio's design vision. The notion of symmetry is embedded in most of the rules. The same is true of proportion: 1:1, 4:3, $\sqrt{2}$:1, 3:2, 5:3 and 2:1 rectangular room ratios seem to be recurrent in his work, orthogonally transcribed by the grid system. The shape rules used translate frequently into addition, symmetry, union and stretch operations.

The Frank Lloyd Wright Prairie House grammar reflects the main concept of the architect. The notions of orthogonality and horizontality are patent, translated into the assembly of stacked boxes, each representing a different adjacent room. The grammar was designed on the basis of the main idea expressed in writing by Wright that the focal point of the dwelling was the fireplace and all the other spaces extended from this. Therefore, taking the Froebel Building Gifts kindergarten grammar as a reference, a similar process takes place (Stiny 1980b). In this grammar the most frequent rules are addition rules and addition, extrusion and offset are quite frequent and recursive.

The Siza Malagueira House grammar introduces the notion of subdivision, resembling the Ice-Ray grammar. The perimeter and maximum rectangular shape is defined in the early stages and a series of divisions and subdivisions subsequently take place. Addition and subdivision are the most recurrent processes, although union and subtraction also occur at key points.

A comparison of the grammars revealed a similar formulation for the three grammars used.

Apart from the fact that they contain 8 main stages with a similar structure, within these 8 stages the design of the beginning and end stages (stage 1, 2 and 8) is very similar.

Despite the already mentioned differences of stages, there is an overall consistency in the evolution of the design process. Regardless of whether they start with a generic shape or a specific room, they progress from room shape to wall confinement, then detailing for doors and windows. To a certain extent the design process is embedded in the rule system replicates the design choices idealized by the original designer.

3.5 Chapter Reflection

The selected case study provided useful insight into the shape grammar theory and application. An important correlation can be formed between the parametric space described by a shape grammar language and our intuitions regarding the similarities amongst the resulting design solutions. When observing a corpus of designs from a specific design non-expert users can easily identify similarities and differences. These are intuitively perceived by an observer which can establish relationships amongst geometries, proportions and spaces generated.

Grammars have been successful recreating design related solutions therefore they should contribute to a better understanding of the relationship between design intuition and design perception. The understanding of its process as explained in this chapter 3 allows for the recreation of similar processes. Likewise, its process might assist in determining how commonalities are intuitively perceived by observers. This relationship between observation, perception and intuition is key whilst understanding design languages. There is more than consistency within a design language. There are embedded similarities and intimate geometric relationships derived from a common genetic formulation.

The case study of the three grammars showcased that grammars can not only replicate the generative process but can assist explaining how design perception and intuition can be linked as will be discussed in the following chapters.

4. Generic grammar rules

4. Generic grammar rules	117
4.1 Introductory Notions	118
4.2 Rules Schemas	120
4.3. Four generic shape rules:	120
4.3.1 Shape Addition:	121
4.3.2 Shape parallel copy or Offset	123
4.3.3 Shape Subtraction	125
4.4 Other additive generic rules	129
4.4.1 Copy generic rule	129
4.4.2 Move Generic rule	131
4.4.3 Scale Generic rule	133
4.4.4 Rotate Generic rule	135
4.4.5 Shape subdivision generic rule	136
4.5 Results and reflexions from generic rules	139

4. Generic grammar rules

4. Generic grammar rules

In the previous chapter the case study of three grammars was analysed and discussed.

This chapter reads the results and comments on the discoveries made.

From the extensive set of shape rules (72 from Palladian grammar (Stiny & Mitchell 1978), 99 from the Frank Lloyd Wright Prairie houses grammar (Koning & Eizenberg 1981) and 120 from the Malagueira grammar (Duarte 2001)) many differences were observed. These differences relate mostly to formal issues like:

- 1) generation process and methodology
- 2) conceptual outline and design intent
- 3) spatial layout and functional distribution
- 4) particularities of the language of design
- 5) corpus of solutions generated
- 6) detailing

However, similarities arise and identical patterns flourished. When observed closely some rules appear to be identical within a grammar design set. Others were mere variations of the parameterization which hint to a number of repetitions. For example, the Palladian grammar allowed for 'n' types of porticos and loggias designs where the variation was the number of columns, the position of the space with regard to the main outline, etc. The Prairie houses grammar proposed 4 initial rules that positioned the rectangular living space adjacent to the fireplace.

The similarities did not constrain itself within each grammar, some rules appeared very similar across the three independent grammars, particularly the rules related with preliminary stages of design or the latest stages (detailing).

The odd similarities amongst the case study allowed for the prediction that a generic formulation for shape rules and s might not only be relevant but possible.

It seemed possible that shape rules could be summarized into types of rules that consistently occur in rule sets. In addition, these common rules varied mostly in the range of parameters allowed whilst the graphical representation remained the same. Such little variation led to the hypothesis that shape rules could be illustrated via a generic graphical representation. This generic shape rule could be illustrated in a graphical manner and completed with an algebraic expression to describe the geometric transformation. This expression or schema would encapsulate the transformation leaving specific parameterization to be associated with the language in place.

"Shape grammars can also be given to construct the designs in languages formed by combining or augmenting other languages of design in terms of language theoretic operations. These operations include, for example, set union, intersection and difference: shape union, intersection and difference" (Stiny, 1980 b)

The specific generic shape rules and schemas application will be discussed in the following chapter.

4.1 Introductory Notions

The first observed common rule occupies the starting point of all three grammars and consists in the addition of a polygonal shape as shown in **Error! Reference source not found.**. This polygon, usually a rectangular shape can be specified in different ways with different parameters. This rectangular boundary is usually represented as a parametric shape with certain proportional specifications to suit each language.

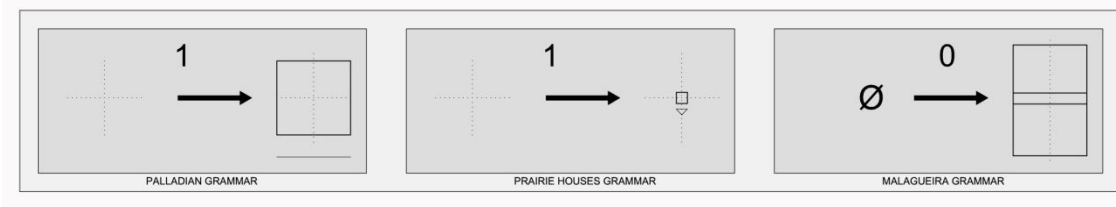


Figure 4.0.1: Rule comparison among the Palladian, Prairie and Malagueira Houses Grammar Addition of an initial rectangle (rule 1 in the Palladian Grammar, rule 1 in the prairie house grammar and rule 0 in the Malagueira grammar)

Although Palladian villas are generated using a grid, the room layout and the first grid cell is represented with a rectangular addition (rules 1 to 10). Prairie houses, that are example of a pure addition grammar, present several different spatial rules to assemble elements such as fireplaces to room layouts (rules 1 to 6). While Malagueira (despite the subdivision nature) proposes to start with a rectangular shaped plot (rule 0/1). These shape rules are illustrated in Fig 4.1.

The formulation or algebraic expression for a rule of this type is simple. One either starts from a blank canvas and adds a first element or starts with an element (point or other initial shape) proposing additions to this existing element.

Other common rules address not the first spatial room in the house but its confinement. Observation has shown that three grammars presented a similar manner to frame the interior layout with a wall system. The process described can be usually found in toolbars of CAD software and is frequently named Offset. It describes a series of operations that allow the parallel copy of an element within a constant distance. Is a sequence of operations including: copy, scale and move. Both the Palladian and Malagueira grammar use it for wall design in respectively rules 11 and rule 2, while Prairie houses grammar uses it more frequently for creating roofs overhangs that establish a cantilevered offset copy from the building perimeter and can be found in rules 75 to 84. The schematic mathematical expression of this operation would describe a scaled transformation occurring on the shape boundary. **Error! Reference source not found.** illustrates this type of rule.

4. Generic grammar rules

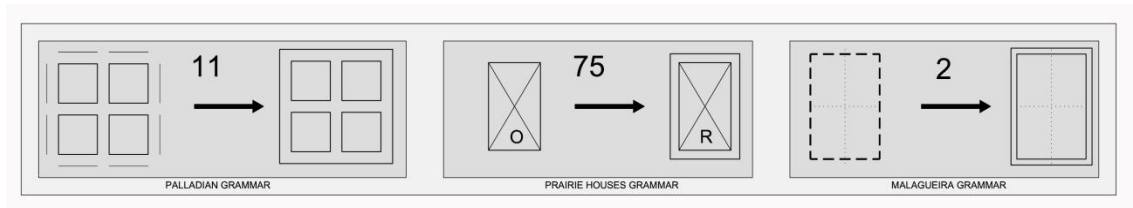


Figure 4 0.2: Offset Rule comparison among the Palladian, Prairie and Malagueira Houses Grammar

Part of complex shape creation for housing design is generated by the operations among adjacent shapes. Often subtractions and additions compete to alter what was once a simple four sided polygon. Subtracting a rectangular shape is therefore a popular operation and that can be confirmed with close observation in the set of rules. The Palladian grammar uses this operation to detail entrances and porticos. Often a rectangular shape is altered by the subtraction of other shapes that create a rhythm of openings, a series of columns or simply intricate elements found in rules 27 to 32. As for the Prairie grammar a similar process takes place in order to design terraces and balconies as described in rules 49 to 53. The Malagueira grammar also shows examples of this operation. In this case it is used for further detailing such as the generation of risers, chimneys or niches as presented in rules 62 to 67 and illustrated in **Error! Reference source not found.**

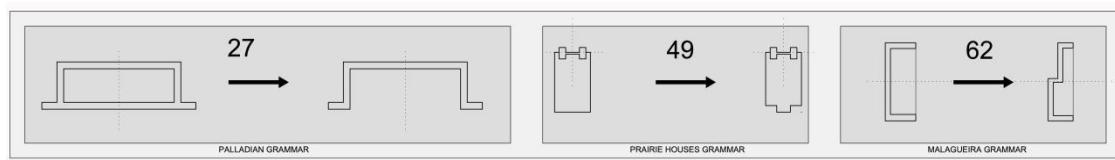


Figure 4 0.3: Subtractive Rule comparison among the Palladian (rule 27), Prairie (rule 49) and Malagueira Grammar (rule 62)

In accordance to what was described in the subtraction rules in order to attain complex shapes, another method is usually put into practice to generate certain complex shapes. This is described often by designers that express their design concept as space merging, space union or concatenation of adjacent spaces. This targets mainly adjacent spaces that share a side or overlap. These elements are unified and its intersecting borders erased creating altogether a unified space with one single perimeter. The Palladian grammar uses this method consistently and even devotes the whole 3rd stage to different shaped room concatenation from elongated rectangles to I, T, or crossed shaped elements (rules 12 to 18). The Prairie and the Malagueira houses grammars are more limited with the application of this rule; however it can be illustrated in rules 12 and 59. In three dimensional terminologies this operation would be entitled Boolean union referring to the process of creating a new element which results from the combination of more than one. However in bi-dimensional representation this will adequately refer to a series of consecutive operations from considering the shapes boundary, identifying the intersection and erasing those overlapping coincident elements keeping only the outline as shown in the following Figure 4.4.

4. Generic grammar rules

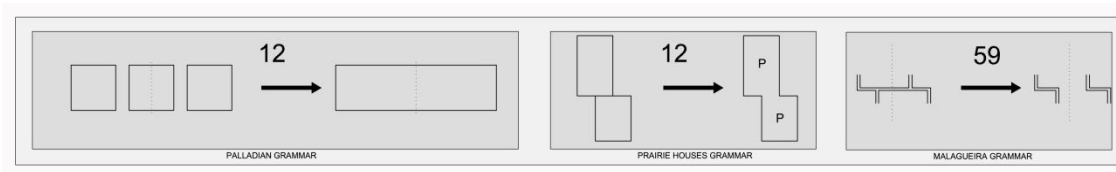


Figure 4 0.4: Concatenation Rule comparison among the Palladian, Prairie and Malagueira Houses Grammar

4.2 Rules Schemas

As an abstraction of shape rules parametric expressions Stiny introduced schema representation proposing simple algebraic expressions to describe these geometric operations (Stiny 2011).

For rectangular shape addition we either start from a blank canvas state and introduce a new element $\emptyset \rightarrow X$ or we depart from an existing shape X and introduce a new shape that can translate a transformation on the initial shape $X \rightarrow X + t(X)$.

Shape subtraction constitutes a common operation for detailing purposes or for creating elaborate geometries. Its formulation can be expressed by the expression $X \rightarrow \text{prt}(X)$ where X represents the initial shape and the transformation occurs taking part of that original shape.

Likewise Offset (that relates directly to the complex succession of three geometric operations, copy, move and rotate) can be expressed by $X \rightarrow b(X) + n \cdot b(X)$, where $b(X)$ stands for boundary of the X shape and 'n' for a rational number multiplied to the boundary in order to scale it up or down.

Space merging or concatenation refers to the unity of two or more elements. This can be described generically by the expression $(X' + X'') \rightarrow \text{prt}(b(X' + X''))$ where X' and X'' stand for two independent adjacent shapes which transformation results into the concatenation of each interior space merging into one. Therefore we consider both boundaries, delete the coincident or overlapping part of the boundary and result with the exterior outline.

These formulations inspired the subsequent work, namely the conception of four generic shape grammars rules. These are part of a high hierarchical meta-grammar that ultimately ramify and generate lower more specialized shape grammars.

These four rules derive from the observation of each similar shape grammar specification and try to combine as much detail and special conditions as possible. The level of detail would be attributed by the specific parameterization that each grammar would be accompanied with.

4.3. Four generic shape rules:

The case study observation of the three grammar rules allowed to identify four basic rules:

- 1) addition
- 2) parallel copy or offset
- 3) subtraction
- 4) concatenation

4.3.1 Shape Addition:

Shape addition refers to the addition of a new shape in the design. Its application might be initialized with a starting point P_1 with coordinates (X_1, Y_1) and a polygon placed from a vertex. If the shape at stake is a polygon, often a rectangle, then the base width, the height and its four end points will be easily placed accordingly and conveniently numbered from 1 to 4 (X_1, Y_1) , (X_2, Y_2) , (X_3, Y_3) , (X_4, Y_4) . A rotation can also be included taking the form of an angle α .

The shape rule can be summarized into an initial stage with a position point P_1 , and the final stage P_1 plus the desired rectangle complying with the established variables.

In a simplistic way the transformation can be explained by the following schema:

$X \rightarrow t(x)$

X represents the initial shape (with the starting point P_1), the arrow the occurred transformation and $t(x)$ the transformation of the shape (the insertion of the polygon x).

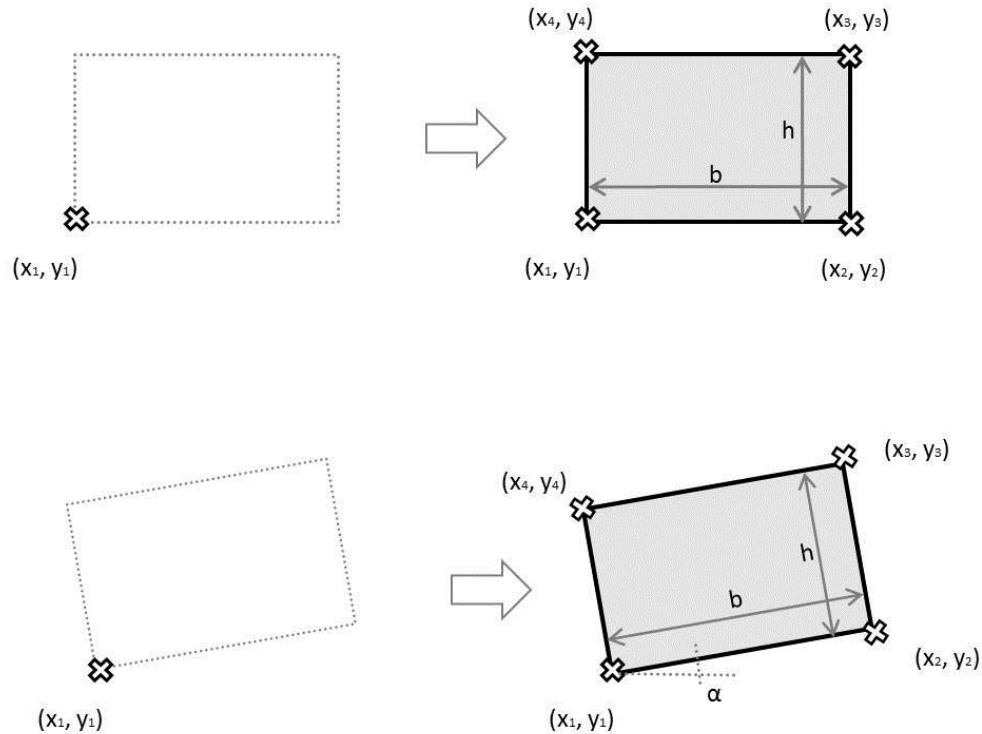


Figure 4.5: Design Rule - Shape Addition

4. Generic grammar rules

This schema can be algebraically expressed by:

$$(X_1, Y_1) \rightarrow (X_1, Y_1) + (X_2, Y_2) + (X_3, Y_3) + (X_4, Y_4)$$

or taking b for base width and h for height:

$$(X_1, Y_1) \rightarrow (X_1, Y_1) + (X_1+b, Y_1) + (X_1+b, Y_1+h) + (X_1, Y_1+h)$$

and finally generalizing for other rotated placements in relation to the Cartesian Axis:

$$(X_1, Y_1) \rightarrow (X_1, Y_1) + (X_1+b.\cos\alpha, Y_1+b.\sin\alpha) + (X_1+b.\cos\alpha - Y_1+h.\sin\alpha, Y_1+b.\sin\alpha + h.\cos\alpha) + (X_1-h.\sin\alpha, Y_1+h.\cos\alpha)$$

4.3.2 Shape parallel copy or Offset

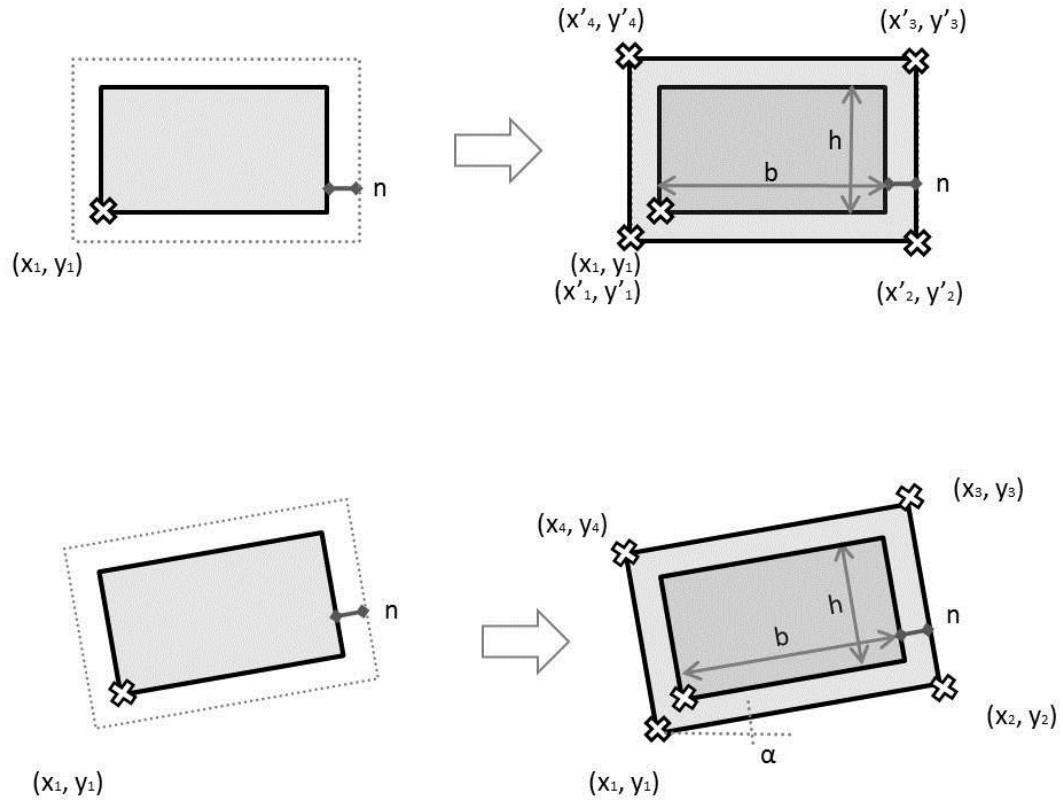


Figure 4.6: Design Rule - Shape Parallel copy or Offset

The offset represents a particular case of a copy operation. Offset consists on a parallel copy combined with an object scaling. Although it constitutes a frequently applied rule has certain restrictions namely it will not enable its use in first instances of generation since it needs a pre-designed shape.

Examples of its application are: the boundary wall on the Palladian villas (stage 2 rule 12), the prairie houses rules ranging from 75 to 64 on the 19th stage design the roof edge projecting it over the overall perimeter and applied on the Malagueira houses on the 2nd rule in order to out copy the plot creating a party wall.

Originally the shape is placed on a specific starting point or vertex conveniently named P_1 with coordinates (X_1, Y_1) , presenting an angle α measured according to the Cartesian Axis, a base width and a height.

As variables are always placed in reference to point P_1 , such as base width b , height h , angle α and the constant distance between the original and copied shape d .

Therefore, the generic schema can be presented in the following expression:

4. Generic grammar rules

$$X \rightarrow b(X) + n.b(X)$$

This compromises an initial shape X which is transformed into the new set of shapes. In the final stage it is considered the boundary of X $b(X)$, it is copied and its size is altered multiplying it by a scaling factor n . When converting it into algebra and taking 'd' as distance between original object and copied, we get:

$$\text{If } R = (X_1, Y_1) + (X_2, Y_2) + (X_3, Y_3) + (X_4, Y_4)$$

Representing the rectangle R with corners $P1, P2, P3, P4$, with coordinates (X_n, Y_n)

$$R \rightarrow R + (X_1 - d, Y_1 - d) + (X_2 + d, Y_2 - d) + (X_3 + d, Y_3 + d) + (X_4 - d, Y_4 + d)$$

Replacing points $P2, P3$ and $P4$ regarding $P1$:

$$R \rightarrow R + (X_1 - d, Y_1 - d) ; (X_1 + b + d, Y_1 - d) ; (X_1 + b + d, Y_1 + h + d) ; (X_1 - d, Y_1 + h + d)$$

And finally generalizing for any rotation α :

$$\begin{aligned} R \rightarrow R + & (X_1 - d.\cos\alpha, Y_1 - d.\sin\alpha) ; \\ & (X_1 + (b + d).\cos\alpha, Y_1 + (b + d).\sin\alpha) ; \\ & (X_1 + (b + d).\cos\alpha - X_1 + (h + d).\sin\alpha, Y_1 + (b + d).\sin\alpha + (h + d).\cos\alpha) ; \\ & (X_1 - (h - d).\sin\alpha, Y_1 + (h - d).\cos\alpha) \end{aligned}$$

4.3.3 Shape Subtraction

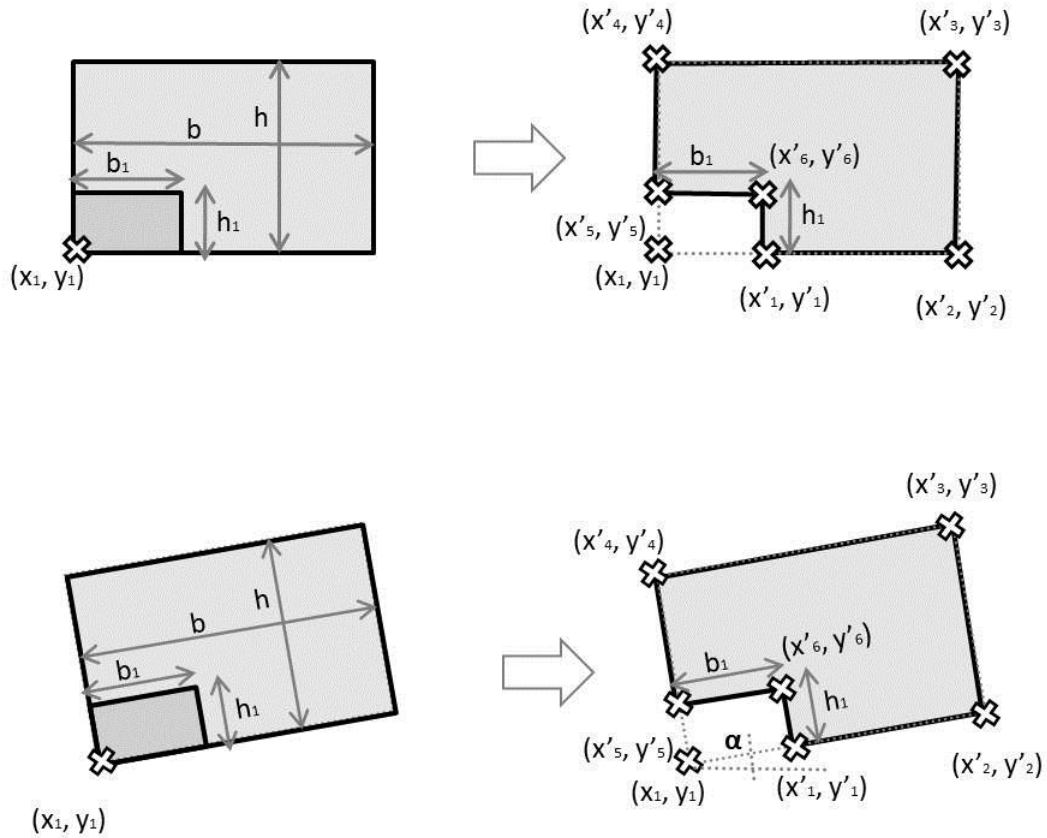


Figure 4.7: Design Rule – Shape Subtraction

Shape subtraction represents another operation that requires the presence of initial shapes in order to occur. It constitutes a detailing operation where the transformation occurred is resultant from the alteration of a shape by an intersection process. Although similar to an intersection, the end result is the removal of one of the shapes in the initial stage with the aim to alter the final geometry as shown in the illustration.

Its associated schema can be explained by the following expression. If we consider an initial shape (or composition of shapes) X , and take into account its boundary ' $b(X)$ ' a transformation occurs regarding just a selected part of that boundary ' prt ', like so:

$$X \rightarrow prt(b(X))$$

If the initial shape X is defined by a polygonal shape like a rectangle formed by different points and if we are subtracting another intersecting polygonal shape we can summarize the following algebraic expression:

4. Generic grammar rules

$$\begin{aligned} & ((X_1, Y_1) ; (X_2, Y_2) ; (X_3, Y_3) ; (X_4, Y_4)) \rightarrow \\ & \rightarrow ((X_1, Y_1) ; (X_2, Y_2) ; (X_3, Y_3) ; (X_4, Y_4)) - ((X'_1, Y'_1) ; (X'_2, Y'_2) ; (X'_3, Y'_3) ; (X'_4, Y'_4)) \end{aligned}$$

$$\begin{aligned} & ((X_1, Y_1) ; (X_2, Y_2) ; (X_3, Y_3) ; (X_4, Y_4)) \rightarrow \\ & \rightarrow ((X'_1, Y'_1) ; (X_2, Y_2) ; (X_3, Y_3) ; (X_4, Y_4) ; (X'_2, Y'_2) ; (X'_3, Y'_3)) \end{aligned}$$

Replacing points P2, P3 and P4 for P1 parametrizing with variables like base width and height, b, h, b1 and h1:

$$\begin{aligned} & ((X_1, Y_1) ; (X_1+b, Y_1) ; (X_1+b, Y_1+h) ; (X_1, Y_1+h)) \rightarrow \\ & \rightarrow ((X_1+b_1, Y_1) ; (X_1+b, Y_1) ; (X_1+b, Y_1+h) ; (X_1, Y_1+h) ; (X_1, Y_1+h_1) ; (X_1+b_1, Y_1+h_1)) \end{aligned}$$

And finally generalizing for any rotation α :

$$\begin{aligned} & ((X_1, Y_1) ; (X_1+b, Y_1) ; (X_1+b, Y_1+h) ; (X_1, Y_1+h)) \rightarrow \\ & \rightarrow ((X_1+b_1.\cos\alpha, Y_1+b_1.\sin\alpha) ; (X_1+b.\cos\alpha, Y_1+b.\sin\alpha) ; (X_1+b.\cos\alpha-h.\sin\alpha, Y_1+b.\sin\alpha+h.\cos\alpha) ; \\ & (X_1-h.\sin\alpha, Y_1+h.\cos\alpha) ; (X_1-h_1.\sin\alpha, Y_1+h_1.\cos\alpha) ; (X_1+b_1.\cos\alpha-h.\sin\alpha, Y_1+b_1.\sin\alpha+h_1.\cos\alpha)) \end{aligned}$$

4.3.4 Shape Concatenation

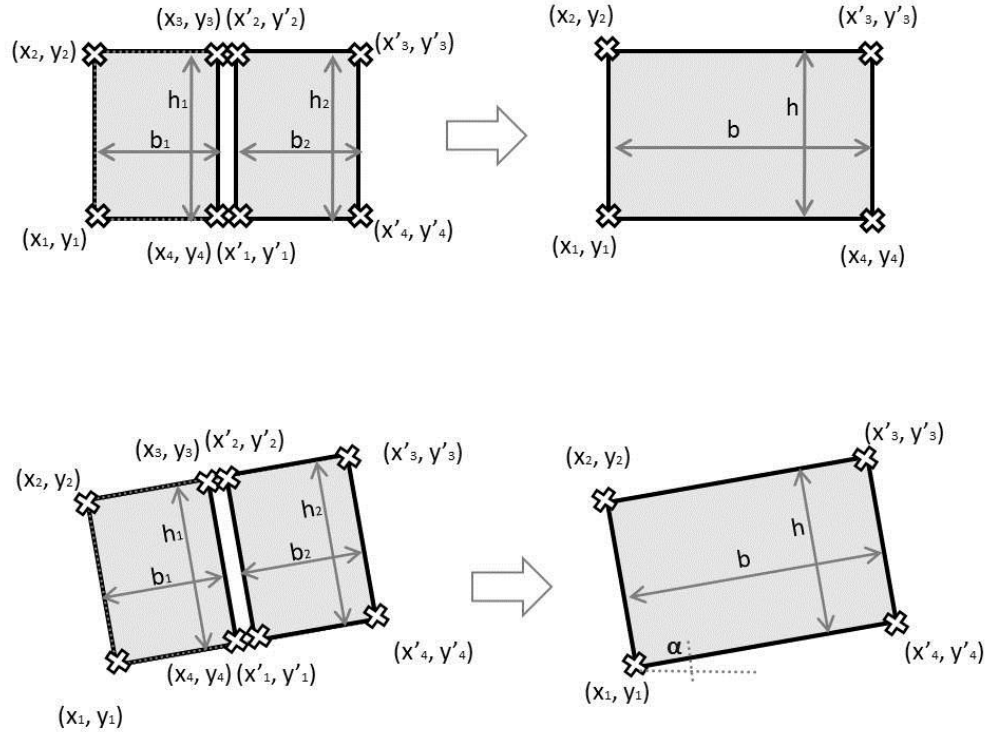


Figure 4.8: Design Rule – Shape Concatenation

The shape concatenation or shape merging is a consequence of additive operations combining more than one simple polygon in order to obtain more complex geometries. The particularity of this operation is combining adjacent spaces and merging them into one. From the combined shapes only external maximal lines are maintained, while coincident or internal segments are deleted. This feature is quite relevant in housing design to create adjoined open spaces with more intricate geometries.

Is important to note that this operation is not more than a combination of additive, subtractive and deletion processes all at once. However every grammar discussed in this paper and others studied like the Buffalo Bungalows or the Queen Ann Houses (Downing & Flemming 1981) (Flemming 1987), propose a stage and several transformation rules with merging or concatenation options. This is therefore relevant for housing design shape grammars such a feature since is more than a sum of operations, but a specific technique of space creation where adjacent spaces with different geometries are combined to create a larger compartment connected in a specific way with other spaces and services.

Likewise is possible to propose a schema to describe the operation.

X' and X'' stand for two (or more) simple shapes or polygons that have at least a coincident maximal line. These two shapes will be transformed by taking their external combined boundary b

4. Generic grammar rules

$(X' + X'')$ (every line but the overlapping ones) keeping this selection and deleting the intersecting parts:

$$(X' + X'') \rightarrow \text{prt}(b(X' + X''))$$

If we consider two four sided polygons X' and X'' , then:

$$[(X'_1, Y'_1); (X'_2, Y'_2); (X'_3, Y'_3); (X'_4, Y'_4)] + [(X''_1, Y''_1); (X''_2, Y''_2); (X''_3, Y''_3); (X''_4, Y''_4)] \rightarrow \\ \rightarrow ((X'_1, Y'_1); (X''_4, Y''_4) (X'_3, Y'_3); (X'_2, Y'_2))$$

Taking dx for distance measured parallel to the X axis and dy distance measured parallel to the Y axis and replacing R'' values for variables of R' , then:

$$((X'_1, Y'_1); (X'_1 + b_1, Y'_1); (X'_3, Y'_3); (X'_4, Y'_4)) + ((X''_1, Y''_1); (X''_2, Y''_2); (X''_3, Y''_3); (X''_4, Y''_4)) \rightarrow \\ \rightarrow ((X'_1, Y'_1); (X'_1 + b_1 + dx, Y'_1 + dy + h_2) (X'_1 + b_1 + dx, Y'_1 + h_1); (X'_1, Y'_1 + h_1))$$

Generalizing this expression including an angle α : $b_1 \cdot \sin \alpha + h_1 \cdot \cos \alpha$

$$((X'_1, Y'_1); (X'_1 + b_1, Y'_1); (X'_3, Y'_3); (X'_4, Y'_4)) + ((X''_1, Y''_1); (X''_2, Y''_2); (X''_3, Y''_3); (X''_4, Y''_4)) \rightarrow \\ \rightarrow ((X'_1, Y'_1); (X'_1 + b_1 + dx, Y'_1 + dy + h_2) (X'_1 + b_1 + dx, Y'_1 + h_1); (X'_1, Y'_1 + h_1))$$

4.4 Other additive generic rules

Apart from the previously enunciated generic rules other shape rules can be added to complement the notion of generic additive rules. In fact following our criteria, these can be reviewed as particular cases of addition rules demonstrated by the similarities between the represented schemas as follows. The formulation of the following generic shape rules is not imperative to the success of a shape grammar but can expedite results and contribute for a more elegant grammar. The use of these rules is not transversal across the majority of the grammars studied but are certainly present in more than one case and its utility is understandable.

4.4.1 Copy generic rule

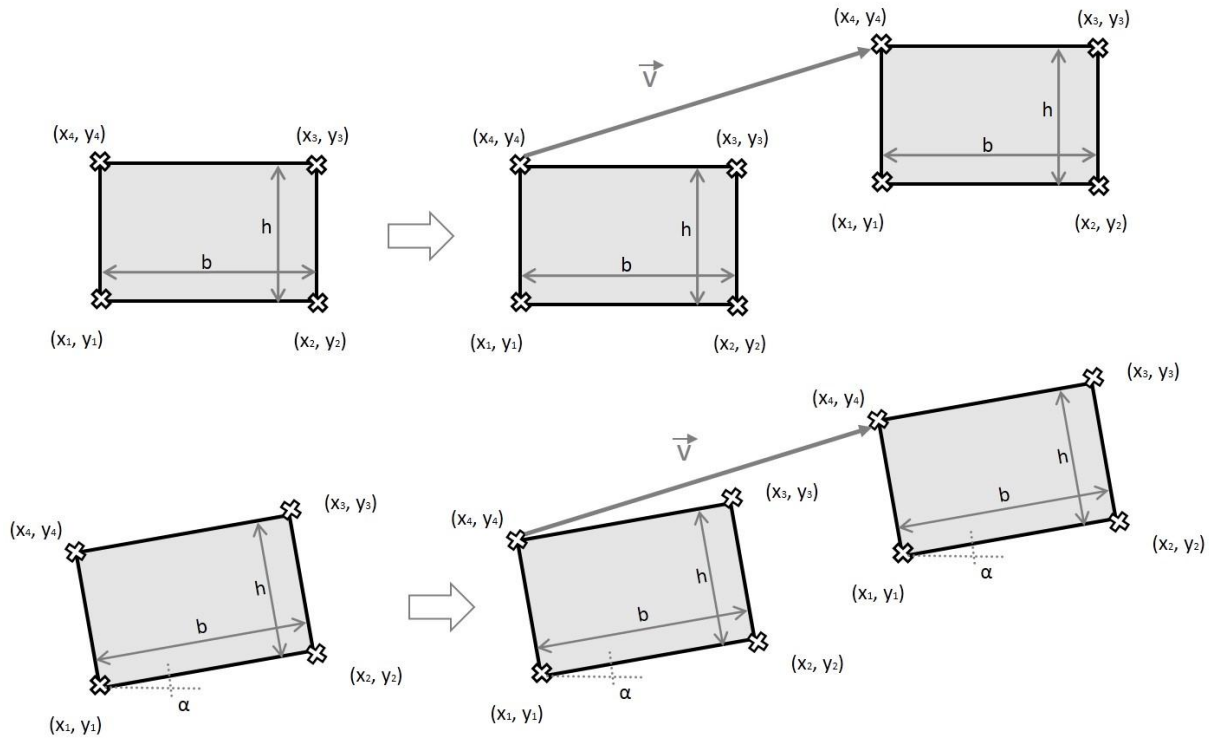


Figure 4.9: Generic shape rule – Copy

Previously it was demonstrated how an addition of a shape (polygon) can be schematically and then algebraically expressed:

$$X \rightarrow X + (t) X$$

$$\text{or } (X_1, Y_1) \rightarrow (X_1, Y_1) + (X_1 + b, Y_1) + (X_1 + b_1, Y_1 + h) + (X_1, Y_1 + h)$$

4. Generic grammar rules

This rule (as it is expressed) designs a simple four sided shape, or if we like, transforms a point (X1, Y1) into four points that are distanced from each other a b width and a h height.

In order to design a similar shape one had to repeat the rule applied to one of its existing points. Other transformation rules can be applied with that intent. Move, Copy and Scale are frequent tools available in algebra and currently used in CAD software such as AutoCAD, Microstation, Rhino, Revit, ArchiCAD to name a few. It was only relevant to attempt to generalize the schemas to these notions.

Copy as a paradigm or special case of an addition rule would resemble the following schema:

$X \rightarrow X + X'$ or $X \rightarrow X + (t) X$ Where X' symbolizes a transformation of X , or $t(X)$ a transformation of X

Looking closely to the geometric procedure at stake, an 'n' shape is copied to another location by a translation applied through a vector 'v'. This vector is expressed by two points the origin and the end point (P1, P2). So in fact we are adding to the first shape a vector (and its coordinates) and placing an identical shape at the other end:

$$X \rightarrow ('v' + X) \Rightarrow ((P_2 - P_1) + ((X_1, Y_1) + (X_1 + b, Y_1) + (X_1 + b_1, Y_1 + h) + (X_1, Y_1 + h))) \Rightarrow$$
$$(((X_2, Y_2) - (X_1, Y_1)) + ((X_1, Y_1) + (X_1 + b, Y_1) + (X_1 + b_1, Y_1 + h) + (X_1, Y_1 + h))) \Rightarrow$$
$$X + ((X_2 - X_1, Y_2 - Y_1) + ((X_1, Y_1) + (X_1 + b, Y_1) + (X_1 + b, Y_1 + h) + (X_1, Y_1 + h))) \Rightarrow$$
$$X + ((X_2, Y_2) + (X_2 + b, Y_2) + (X_2 + b, Y_1 + h) + (X_2, Y_2 + h)) \Rightarrow$$

Copy constitutes a particular case of the addition rule but its formulation is time effective since it can be summarized in a more economic expression.

4. Generic grammar rules

4.4.2 Move Generic rule

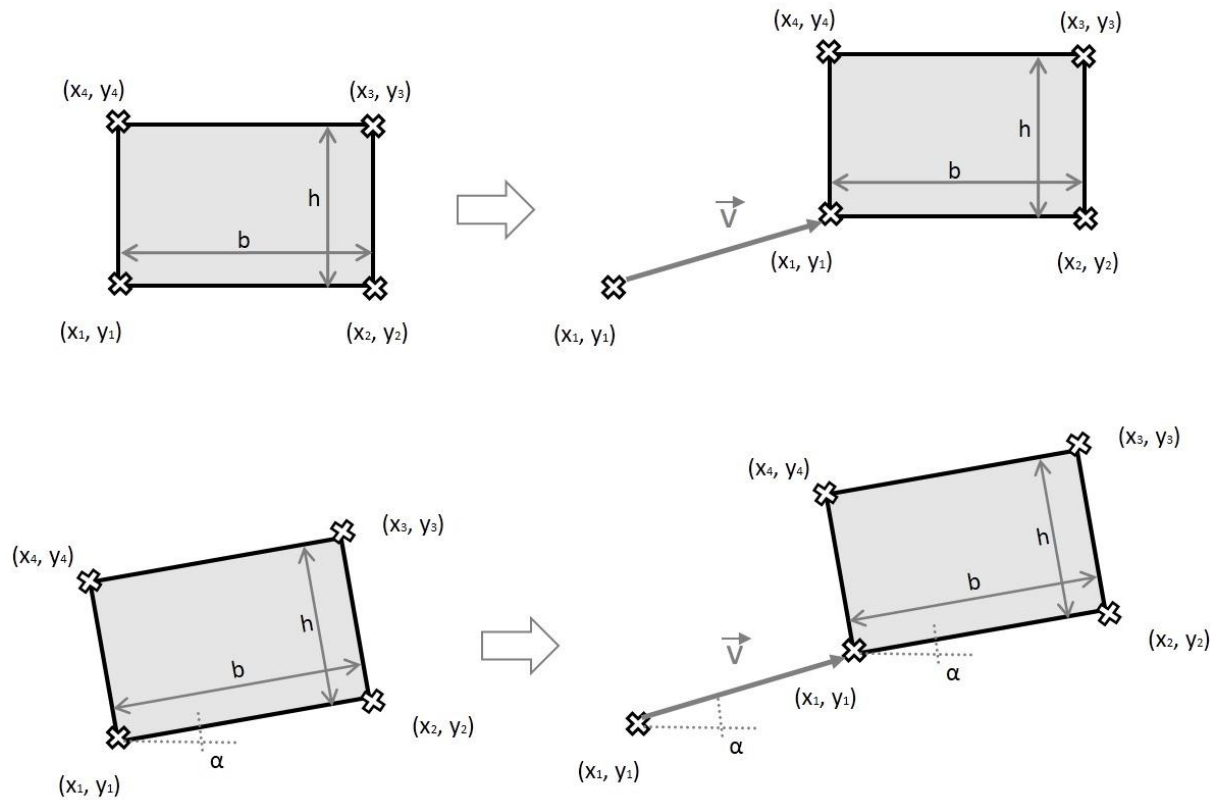


Figure 4.10: Generic shape rule – Move

Also part of the so-called transformation rules move as a simple translation by application of a vector allowing a new location or relative position, it is a special case of copy. This could be explained by two points of view:

- Move is a transformation where a given shape is repeated in a different location and the original shape is then erased
- Copy is a transformation where move takes place and no deletion occurs

Move can be also explained as a special condition of addition where an original shape is erased and a new shape (similar to the previous) is added in a new location

Therefore in the move transformation a simpler schema can be formulated:

$X \rightarrow X'$ Where X and X' are similar objects but with different locations and coordinates.

Or $X \rightarrow t(X)$ Where X represents the original shape and $t(X)$ represents an occurred transformation towards the shape (X).

Algebraically the expression increases slightly in complexity, however the principle remains simple at core: To a given shape X is applied a vector ' v ' defined by $P1$ and $P2$. $P1$ represented the starting point or origin and $P2$ the endpoint. The given shape has a certain ' b ' width and an ' h ' height:

4. Generic grammar rules

$$X \Rightarrow X + 'v'$$

$$[(X_1, Y_1) + (X_1 + b, Y_1) + (X_1 + b, Y_1 + h) + (X_1, Y_1 + h)] + (X_2 - X_1, Y_2 - Y_1) \Rightarrow$$

$$((X_1, Y_1) + (X_1 + b, Y_1) + (X_1 + b, Y_1 + h) + (X_1, Y_1 + h)) \Rightarrow$$

$$((X_2 - X_1, Y_2 - Y_1) + (X_1, Y_1) + (X_1 + b, Y_1) + (X_1 + b, Y_1 + h) + (X_1, Y_1 + h)) \Rightarrow$$

$$((X_2, Y_2) + (X_2 + b, Y_2) + (X_2 + b, Y_2 + h) + (X_2, Y_2 + h))$$

Effectively what happens to the expression above in comparison with the original was a simple substitution of the indices related to point P1 (origin) to the point P2 (endpoint).

Mathematically speaking this transformation is a pure addition between a shape X and a vector 'v' as expressed in the first expression and easily proven.

Therefore we can conclude that this operation is a particular case of the addition rule previously demonstrated.

This operation constitutes intuitively a modification rule and geometrically the need to exist simplifies the understanding of some design processes used at times.

4.4.3 Scale Generic rule

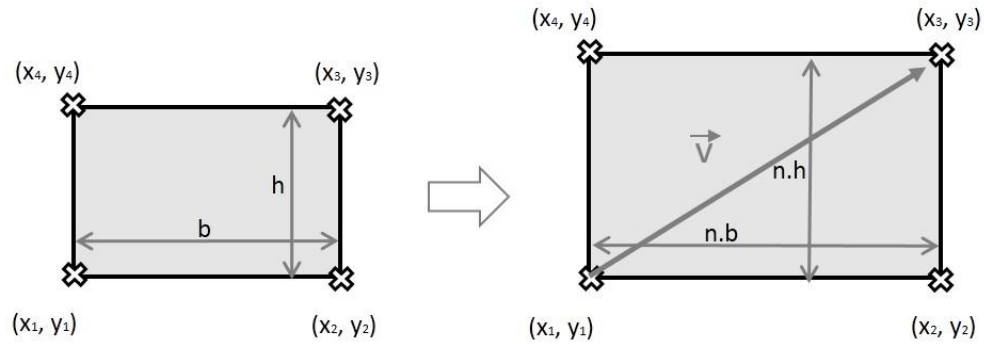


Figure 4.11: Generic shape rule – Scale

Alike the previously demonstrated alterations/modification generic rules scale targets alterations to an original shape. However the type of alteration performed changes the very essence of the shape changing its proportional ratio and therefore the overall location of all (or most of) its original boundary points. This is performed by multiplying each dimension or each side by an increment factor (or decrement factor) responsible for enlarging (or reducing) the overall size of the shape whilst maintain the proportion ratio and internal angles. The final shape is similar to the original but usually on a different location and size.

Schematically his can be expressed by the Schema:

$X \rightarrow t(X)$

Where X represents the original shape and $t(X)$ represents an occurred transformation.

Or

$X \rightarrow X'$

or even $X \rightarrow n(X)$ Where X and X' are similar objects but with different proportional ratio, and $n(X)$ represents the original X shape transformed by a scaling factor 'n'.

$X \rightarrow n(X) + 'v'$

Geometrically what occurs in this transformation is a vector operation where a vector based in points $P1$ and $P2$ is applied to an initial point and its dimension ' n ' = $(P2-P1)$ represents this incremental/detrimental factor. The vector origin determines the starting point of the transformation while the endpoint the destination. With polygonal figures this vector is applied recursively in all points to determine the final shape silhouette.

If we express this into an algebra, considering vector ' $v1$ ' that could be:

$$((X_1, Y_1) + (X_1 + b, Y_1) + (X_1 + b, Y_1 + h) + (X_1, Y_1 + h)) \Rightarrow ((X_2 - X_1, Y_2 - Y_1) + (X_1, Y_1))$$

4. Generic grammar rules

$$[((X_1, Y_1) + (X_1 + b, Y_1) + (X_1 + b, Y_1 + h) + (X_1, Y_1 + h))] + ((X_2 - X_1, Y_2 - Y_1)) \Rightarrow \\ [(X_2, Y_2) + (X_2 + b, Y_2) + (X_2 + b, Y_2 + h) + (X_2, Y_2 + h)]$$

Or if we omit the vector nomenclature and introduce an incremental value n:

$$n \times [(X_1, Y_1) + (X_1 + b, Y_1) + (X_1 + b, Y_1 + h) + (X_1, Y_1 + h)] = \\ = [(X_1, Y_1) + ((n'.X_1 + n'.b, n''.Y_1) + (n'.X_1 + n'.b, n''Y_1 + n''h) + (n'.X_1, n''.Y_1 + n''h))]$$

To simplify the expression initial and final shape should coincide in one point (point (X_1, Y_1)), therefore in the initial expression and the transformation its coordinates remain the same. n' and n'' are respectively projections of the transformation vector in the horizontal x axis and in the vertical y axis, therefore do not represent the real dimension of n (scale factor).

4.4.4 Rotate Generic rule

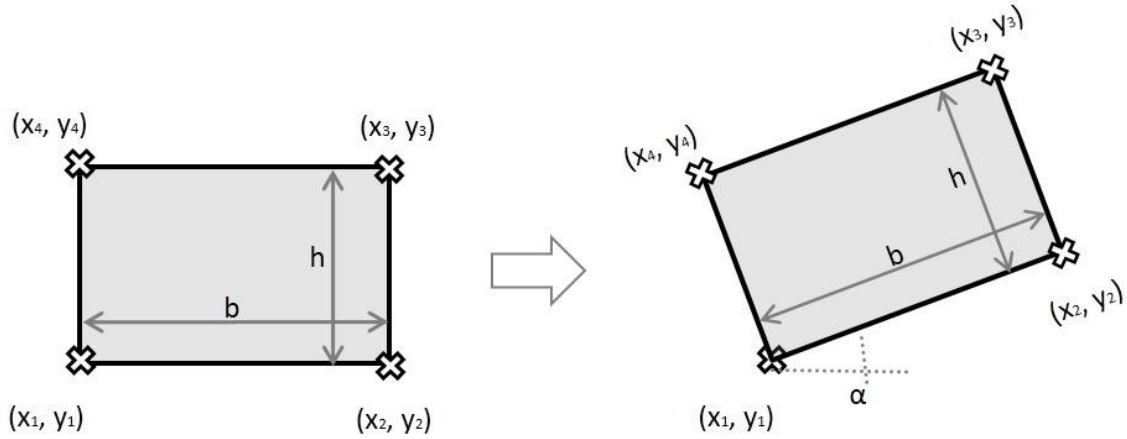


Figure 4.12: Generic shape rule – Rotate

The last generic transformation rule to be presented is the rotation rule. This rule encompasses the relative movement of an original object by varying the general position of the shape bounding points while maintaining its internal spatial relations. This variation results from a revolve process around an axis (z) that is perpendicular to the plan of work (xy).

Similarly to the rules presented, this could be seen as an additive rule. This addition would not be a pure or simple addition process, but an addition associated to a deletion of the original shape, only to replace it for a similar shape that differs in location. Therefore the schema allied to the rotation rule can be enunciated as:

$$X \rightarrow X'$$

Where X stands for the initial shape and X' the final shape with a similar nature, or:

$$X \rightarrow t(X)$$

Where X stands for the initial shape and t(X) for the resultant shape after applying a transformation 't'.

Equally, the algebraic expression to represent the rotation taking into account the coordinates of the bounding shape, would be:

$$(X_1, Y_1) + (X_1+b, Y_1) + (X_1+b, Y_1+h) + (X_1, Y_1+h) \rightarrow (X_1, Y_1) + (X_1+b.\cos\alpha, Y_1+b.\sin\alpha) + (X_1+b.\cos\alpha - Y_1+h.\sin\alpha, Y_1+b.\sin\alpha + h.\cos\alpha) + (X_1-h.\sin\alpha, Y_1+h.\cos\alpha)$$

Not surprisingly the expression resembles in part what was stated for the generalization of the additive rule. This was formulated to suit any positioned shape (allowing shapes not orientated along the Cartesian axis and therefore rotated) which explains the similarities.

The rotation rule allows the inclusion of a level of complexity that usually is allowed in contemporary architecture (not so much in classic canons) hence the need to include it. As a geometric transformation it allows the production of a series of shapes that otherwise would only be possible to design in an angular manner.

As some of the previously described rules it is a feature of several CAD platforms and a useful design tool.

4.4.5 Shape subdivision generic rule

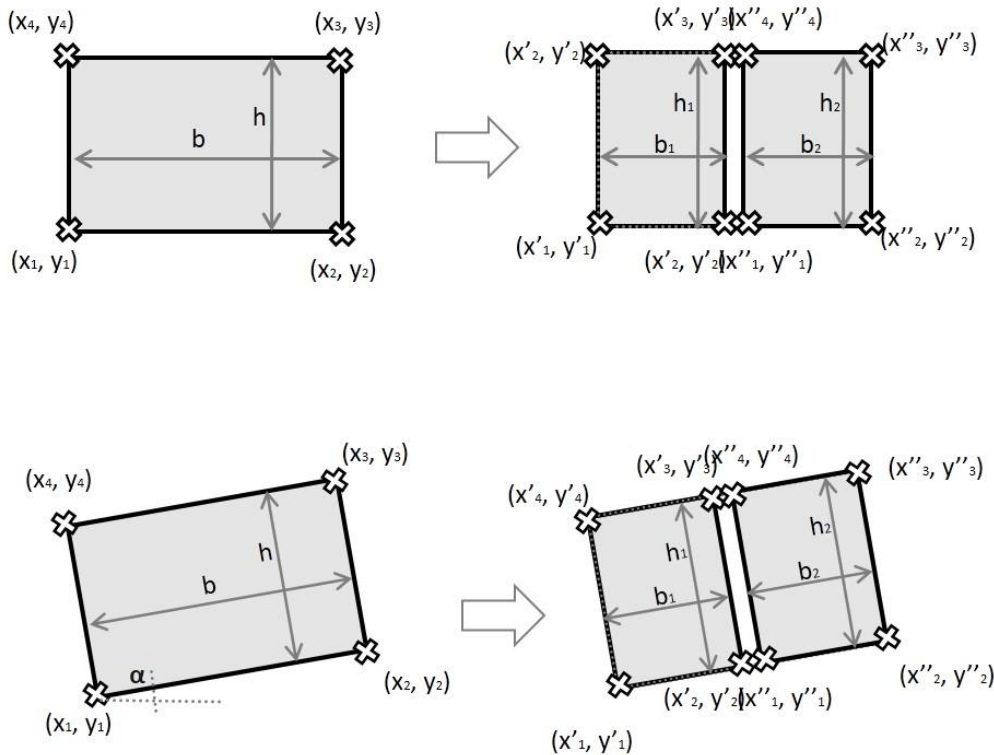


Figure 4.13: Design Rule – Shape Subdivision

The shape subdivision, contrastingly to shape concatenation or merging, is a consequence of a subdivision operation which separates one simple polygon into two or more polygons. Allegedly could be considered the inverse operation of the concatenation process. However, its utility is significant since allows the manipulation of internal polygons and their proportions without altering the overall aspect ratio of the boundary. This property is particularly useful in designs with a Top-bottom approach.

This rule was mostly observed in grammars such as the Malagueira grammar that recur to this operation throughout in stage 2 to stage 5 and allowing for multiple shape rules of this nature. In

4. Generic grammar rules

fact the grammarian has indented 14 types of subdivision rules labelled from A to N, varying in type of subdivision and shape as illustrated in image 4.17. The inclusion of this high concentration of subdivision helps classifying the grammar as a subdivision. This was not the first grammar recurring to subdivision rules. The Ice-ray also discussed previously showcases 4 out of 5 shape rules of its set dedicated to subdivision of polygons (Stiny 1977).

Other grammars such as the Hepplewhite chairs grammar also use subdivision throughout, demonstrating once more its relevance in design (Knight 1980) or even the De Stijl paintings grammars of Vantongerloo and Glarner (Knight 1989).

Alike the concatenation rule, is possible to propose a schema that enables to describe the operation. If X stands for the original polygon, X' and X'' stand for two (or more) simple shapes or polygons that have at least a coincident maximal line. X will be transformed by subdividing the inside space of its boundary b(X) dissecting it into two or more parts:

$$X \rightarrow (X' + X'')$$

$$\text{prt}(b(X)) \rightarrow (X' + X'')$$

If we consider one original shape X and two resultant four sided polygons X' and X'', then:

$$((X_1, Y_1); (X_2, Y_2) (X_3, Y_3); (X_4, Y_4)) \rightarrow \\ [(X'_1, Y'_1); (X'_2, Y'_2); (X'_3, Y'_3); (X'_4, Y'_4)] + [(X''_1, Y''_1); (X''_2, Y''_2); (X''_3, Y''_3); (X''_4, Y''_4)]$$

Considering b, b1 and b2 as base width and as height h, h1 and h2 for the original and final polygons, then:

$$((X_1, Y_1); (X_1+b, Y_1) (X_1+b, Y_1+h); (X_1, Y_1+h)) \rightarrow \\ ((X_1, Y_1); (X_1+b_1, Y_1) (X_1+b_1, Y_1+h); (X_1, Y_1+h) + ((X_1+b_1, Y_1); (X_1+b_1+b_2, Y_1) (X_1+b_1+b_2, Y_1+h); (X_1+b_1, Y_1+h))$$

Since $b = b_1 + b_2$

$$((X_1, Y_1); (X_1+b, Y_1) (X_1+b, Y_1+h); (X_1, Y_1+h)) \rightarrow \\ ((X_1, Y_1); (X_1+b_1, Y_1) (X_1+b_1, Y_1+h); (X_1, Y_1+h) + ((X_1+b_1, Y_1); (X_1+b, Y_1) (X_1+b, Y_1+h); (X_1+b_1, Y_1+h))$$

Generalizing this expression including an angle α : $b_1 \cdot \sin \alpha + h_1 \cdot \cos \alpha$

4. Generic grammar rules

$((X_i, Y_i); (X_i+b.\cos\alpha, Y_i+b.\sin\alpha); (X_i+ b.\cos\alpha- Y_i+h.\sin\alpha, Y_i+b.\sin\alpha +h.\cos\alpha); X_i-h.\sin\alpha, Y_i+h.\cos\alpha))$

\rightarrow

$((X_i, Y_i); (X_i+b_1.\cos\alpha, Y_i+b_1.\sin\alpha); (X_i+ b_1.\cos\alpha- Y_i+h.\sin\alpha, Y_i+b_1.\sin\alpha +h.\cos\alpha); X_i-h.\sin\alpha, Y_i+h.\cos\alpha)) + ((X_i+b_1.\cos\alpha, Y_i+b_1.\sin\alpha); (X_i+b.\cos\alpha, Y_i+b.\sin\alpha); (X_i+ b.\cos\alpha- Y_i+h.\sin\alpha, Y_i+b.\sin\alpha +h.\cos\alpha); X_i-h.\sin\alpha, Y_i+h.\cos\alpha))$

4.5 Results and reflexions from generic rules

The delineation of generic shape rules is the first step towards the construction of a generic grammar. In the past grammarians started their formulations by observing different design examples of each language and illustrating them by analogy using simple shape rules.

These shape rules would be then parameterised and variables attributed in order to replicate the existing design solutions. These rules could be applied recursively or singularly in accordance to the grammar structure and following a sequence that could be more flexible or more restrictive.

The construction of a generic grammar follows similar principles. To prompt this novel generic grammar it was required the inference of singular generic shape grammars. This was achieved by the collection of shape rules that consistently occurred consistently in the study case grammars and simultaneously in other grammars consulted.

Four rules were extracted:

1. addition
2. parallel copy or offset
3. subtraction
4. concatenation

These rules summarize a myriad of geometric operations that can be represented by shape rules.

To add to the above mentioned generic rules other were illustrated to complete the set:

1. Copy generic rule
2. Move generic rule
3. Scale Generic rule
4. Rotate Generic rule
5. Shape subdivision generic rule

In addition, another important aspect was retained. Apart from the graphical representation of the shape rule, a schema can also illustrate the rule in a more abstract way. This happens because most graphical representations are read in a very literal way which can be restrictive and limit its application. Whilst a schema can represent and suggest several operations without getting trapped in a specific geometric representation.

Important to note that at this stage we are not concerned with language related parameterizations. The abstraction provided by a given schema allows for an enlarged application. Parametrization will convey specificities of style language as further chapters will discuss. The schemas represented for each of the generic rules captures the essence of the shape transformation without expressing the particular ratio or spatial proportion. This level of abstraction could be particularly useful in early stages of design or grammar use. The schema describes an operation. The parametrization will be style specific.

4. Generic grammar rules

The four generic rules inferred and the additional proposed constitute a tool for future shape grammar sets.

These generic rules constitute the bricks and mortar of a generic grammar as future chapters will illustrate. Moreover the generic rules demonstrated allow grammarians to enlarge or simplify the rule set by illustrating more than one viable way of performing geometric operations (as chapter 6 will illustrate with an alternative grammar). These can be applied in a myriad of ways or as proposed further on in chapter 7 where a generic grammar for single housing is proposed.

5.	Concept of Housing Function	141
5.1	Shape and Function	141
5.1.1	Palladian villas	143
5.1.2	Prairie Houses	151
5.1.3	Malagueira houses	156
5.2	Shape and Function findings	158
5.3	Preliminary results and conclusions	172

5. Concept of Housing Function

5.1 Shape and Function

The concept of function in a housing typology is fundamental in the study of housing grammars. Is directly linked with form, particularly within house typologies both in single housing or housing units within a block.

Form and function act as a dynamic duo where shape, proportion and area play an important part. There are numerous variations and processes for assigning function to spaces.

Overall function assignment varies with: stylistic currents, with cultural and social aspects, with geographical or climate requirements and with periods of construction.

The following chapter addresses the functional rules within each grammar of the case study. This rule research will allow to determine how grammarians assigned functions within a housing grammar and how these functions could be represented with a shape rule. From three grammars chosen only two promote functions for the spaces created. The Palladian villas refrain from functional assignment. The remainder grammars, Prairie houses and Malagueira assign functions differently reflecting the Bottom-up and Top-Down approach as we will discover.

The method used to study the use of functions is twofold:

1. By analysis of the grammar adjacency graph
2. By analysis of the grammar derivation graph

Whilst the adjacency graph refers to the spatial distribution of spaces and the quantity/quality of connections created, the derivation graph will allow to assess at which stage of conception each function is assigned. Typically the derivation graph will be the same for all corpus of solutions within the grammar universe of designs. However, the adjacencies (although very similar) might allow variants within families. Nevertheless, the principals of design should be easily identifiable.

Observation will also assist with the proposal of function assignment generic design rules.

As discussed, the Palladian villas do not provide functional rules. Rare references are made to function of singular spaces and their outline is to some extent misleading. One of the key features is the central main room of the design which represents the social core of the villa and the largest space. This space communicates either directly with other living spaces or through smaller circulation nodes such as halls, ante-chambers, or vertical circulation. The space is *'per se'* scenic or monumental and only purpose is to provide entertainment to the house guests.

The contemporary example of Siza's Malagueira is easily grasped from one glance over the floorplan and Prairie houses showcase a clear separation between social and service areas.

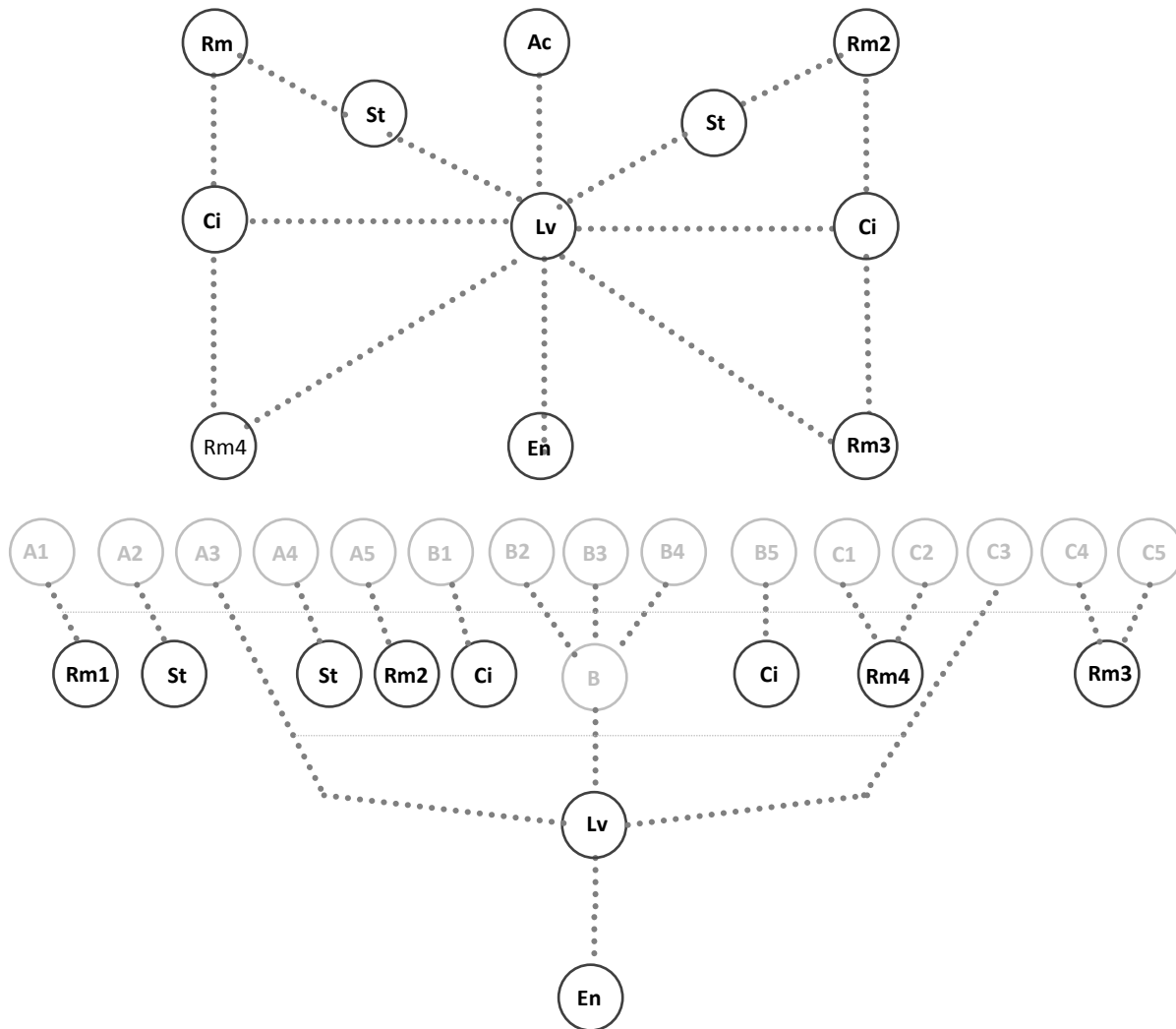
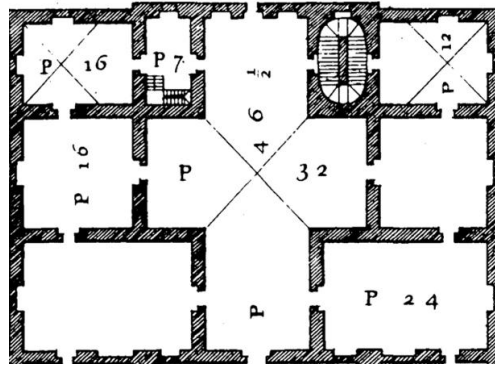


Figure 5. 1: Palladian villa Malcontenta Adjacency Graph Key: Rm1- drawing room, Rm2- study, Rm3- formal ining, Rm4, Ci- circulation/ante-chamber, St- stair, Lv- Living space, En- Entrance, Ac-access

Palladian villa Malcontenta Functional Derivation Graph Key: A1 to C5 Grid cells numbered by column and labelled alphabetically by row: Rm1- drawing room, Rm2- study, Rm3- formal dining, Rm4, Circulation/ante-chamber, St- stair, Lv- Living space, En- Ent

5.1.1 Palladian villas

Visits to the existing villas and analysis of the floorplans allowed to determine the uses or functions of each space. To represent that two diagrams were prepared. An adjacency diagram for villa Malcontenta and a derivation diagram for the Palladian villas. The shape of the diagram illustrated in 5.1 is not important and it reflects the floorplan boundary, but what makes it characteristically different is the type of connections. Due to the spatial constraints circulation is optimized to the minimum. Only vertical circulation occupies a significant portion of the floorplan to allow upper floor access. The remaining spaces are navigated by adjacencies and without intermediate spaces.

An important issue identified in the Palladian villas graphs is the number of connections between nodes or spaces. While the Palladian villas show a great inter-communicability of spaces, Malagueira houses are often a succession of spaces with one to one connections.

Palladian villas maximum depth (calculated from the number of rooms needed to access from the main entrance to the destination is 5 as demonstrated in la Rotonda, and villas Sarego, Thieni, Trisini, Valmarana and Moncenigo.

The maximum depth is a characteristic of each design language. Each of the languages/grammars described showcased a particular depth.

The picture identifies what was described above when applied to villa La Malcontenta (Figure 5.1). The greater central space is the best connected. This is simple to explain due to its centrality and the high number of connections that are established. This central space connects often with more than 4 other adjacent spaces in north, south, east and west direction. This radial distribution is reflected on the floorplan and often even diagonal connections are established providing access to central cells. Three zoning tags were determined, E for entertainment, L for living and C for circulation. This analysis was based only on the raised ground level available (*piano nobile*). Once more, villa la Malcontenta was selected to illustrate this functional study. Using Stiny and Mitchell's grid grammar was possible to show the referred inverted tree.

The design process starts with an 'n' number of cells corresponding to the original

Malagueira showcases a different system. Typically, the inside access is provided from the yard or street side. This connects immediately to the living areas and to the vertical circulation. The living then connects to the dinning and this to the food preparation or kitchen. The kitchen might also provide a pantry area for food storage. On the upper level a small circulation space provides access towards the bathrooms and sleeping areas and from these to small terraces and storage facilities.

A methodology was put in place to analyse the housing functions and spatial organization:

1. For each house of the case study an adjacency graph was generated
2. To complement the first graph, a spatial derivation graph was created illustrating the spatial assignment progressively
3. Results were drawn and key spatial relations identified
4. The spatial/shape rules were inferred and illustrated

Figure 5.1 illustrates the adjacency graph for villa la Malcontenta, followed by the derivation graph.

The derivation graph conveys three levels of information that the adjacency cannot explain:

- the sequence used for spatial creation
- the number of instances required to design each space
- the spatial creation through division or concatenation

In Palladian villas the number of cells is a result of the multiplication of the number of rows for the number of columns. In order to simplify these cells/rooms where named recursively A1, A2,..., An, B1, B2, ..., Bn,..., etc. The first derivation stage is merely an enumeration of the different cells available. On the second stage these cells begin a process of concatenation which constitutes the first step into zoning. In the Malcontenta example the 3x5 grid central cells merge to combine A3, B1, B2, B3 and C3. These 5 cells transform into one forming a cruciform shape that is a regular enclosed polygon centred and symmetrical with its centre lying on top of the house's symmetry axis. This space due to its importance is the social area destined to entertainment. Other spaces of relevance result as concatenation of two cells forming a larger rectangle with proportions that are commonly $\frac{1}{2}$ or $\frac{2}{3}$. In this case and respecting the bilateral symmetry two large rectangles are placed in opposite wings of the house facing the main façade. These rooms combine respectively cells C1 - C2 and C4 - C5. The smaller sized rooms derive directly from the original cell sizes such as A1 or A5 turning into a living rooms or studies due to its proximity to the façade. Other small rooms such as A2 or A4 turn into service areas or an ante-chambers for circulation. Similarly, B1 and B5 located in the central bay of the floorplan and in an East-West axis are assigned the functions of circulation. These cells for its strategic position arranged between the central space and two corner rooms in the north and south constitute the perfect candidates for a transition space, which obviously adds some dramatic effect to the guest that is visiting the succession of spaces.

The number of rooms also varies with the subjacent grid. Usually smaller grids have proportionally a smaller number of rooms:

- 3x3 grids vary in the number of rooms from 8 to 11 like villas Angarano (8), Ragona (9) and Emo (11)

- 5x2 grids like the one illustrated in 'il Quattro Libri' villa Godi allows for 10 rooms. The most used grid type which is 5x3 varies from 8 to 10 rooms, examples can be found in villas Sarraceno, Zeno, Badoer Malcontenta, Pisani and others
- 4x3 grids Villas Montagnana, Cornaro and Barbaro vary in terms of rooms from 9 to 10
- 5x4 grids Villas such as Gazzotti and Sarego allow for a larger number of divisions ranging from 10 to 16, can be considered more palaces than villas

From the table prepared and illustrated in 5.1, Palladio seems to have an approximate ideal number of rooms allowed for each villa. According to Stiny's work on grid grammars where villas with 3x3 and 5x3 prove to be the most frequent scenario, from an arithmetical point of view the maximum number of rooms (strictly derived from the number of cells) allowed for each type would be 9 for the first case and 15 for the latest. However, and after counting Palladio's built examples the conclusion taken is that regardless of the number of cells or grid the ideal number of rooms for any villa is comprehended between 8 and 11. Surprisingly the smaller grids can present larger number of rooms such as villa Emo with 11 rooms based on a 3x3 grid exciding in number many of the 5x3 examples. From the sample cases illustrated the ideal number appears to be 9 based on recurrence for both grids as exemplified in villas Ragona, Pisani e Malcontenta. The most atypical villa and considered by many as the master piece, villa rotunda, exhibits 13 rooms.

The Palladian villas functional assessment differs from what was discussed for the Malagueira example. The derivation diagram extracted from the Palladian cases resembles an inverted tree as seen in 5.2. The reason being the way that Stiny and Mitchell (Stiny & Mitchell 1978) constructed the grammar using a grid which uses an agglomeration of cells. Each cell corresponds to one different space allowing the combination of more than one cells to create larger rooms. Using this process, the minimum space available is the one that corresponds to a singular cell. Therefore, the first derivation step includes an 'n' number of different rooms, far larger than the final number of rooms designed for one house.

Space and function are usually interconnected. Palladio rarely describes function or tags its floorplans with a room description (often annotates the room proportional ratio), leaving the task to identify and classify each space. In a conventional floorplan most of the spaces are easily identified. However, these houses were in many ways scenic, prepared to entertain and receive guests. Vital house functions such services (food preparation, pantries and sleeping areas) were placed in different floors respectively in the lower ground and upstairs floor. Most Palladian illustrations feature only the main floor the '*piano nobile*', both the representation of Palladian villas obtained in the four books or the villas that were actually built, remain to our days and were surveyed. This main floor is the foremost important for Palladio whom does not linger into the illustration of other levels in his architectural treaty. For those reasons the functional analysis was based on

observation rather than information provided by the designer. The methodology used was based on correlations between the different spaces, their areas, proportions, number of interconnections and relation with the façade or number of fenestrations. The four books helped adding some extra knowledge with regards to the design intent of Palladio. Other assumptions were made upon visits to 8 of the villas illustrated in the four books which allowed direct observation (the visited villas floorplans are illustrated in 5.1).

The spatial assignment methodology will be furthermore developed in the next section.

The last derivation step represented in the diagram (Figure 5.1) is the definition of access and the associated area. Most likely is one of the features that a designer focus first when designing a dwelling, the position of the entrance. Shape grammars do not attempt to reproduce a design methodology or the chronology of the design process used by a given designer. Stiny and Mitchell simplified the house composition designing the house inside out with the entrance materialized into a portico or loggia added last, terminating the functional assignment. The entrance is carefully placed, as exemplified by Palladio coincident to the main symmetry axis, facing south and connecting directly with the central social space. This has one objective, to direct the guests to the main entertainment space of the house without obstacles. The social aspect of the Palladian villas is emphasised by this feature recurrent in so many villas such as Angarano, Badoer, Zeno, Emo, Pisani, to name a few. Others such as villa Sarraceno that accesses the social area straight from a covered external space confined within the rectangular border, which is a variation of what was described. Still reflecting on the positioning of the external access, Palladio also allowed for a secondary entrance to be placed. This secondary entrance also occupies a position on the north-south symmetry axis opposite to the main entrance accessed from the back/north. There is an explicit attempt to emphasise the main entrance by a succession of hierarchical features. This is incorporated into the architecture with an impact into the form. The main entrance is generally accessed by a main monumental staircase that allows the access to the raised ground floor. This staircase is often constructed with straight or splayed wide steps or a play of two symmetrical branches in L or Z shape that meet the façade in the same spot. All of these elements are further on expressed and framed into the main façade by a series of columns that are celebrated and meet the roof by an ornamented entablature, cornice and pediment. The façade itself at the point of entrance can be flushed, brought forward using a recessed to allow a covered outside space.

In comparison, the adjacency and the derivation graph have little in common. The first reveals a substantial amount of information regarding the architecture used on the play of spaces and their connections. The derivation shows an almost clinical reading of the generation process. Actively shows how spaces are put together and how they derive from one another. However, if the same exercise was put into practice using a different Palladian grammar (such as the alternative grammar later presented in the next chapter) the results attained are quite different and this derivation graph would resemble Malagueira's derivation.

The Palladian adjacency graph illustrates the following spatial features:

- Entertainment or Living space always occupies a central location
- Entertainment space is highly connected with circulation spaces, access in first degree
- Access and Entertainment are always aligned with the central bay
- Circulation surrounds the entertainment room always occupying internal bays
- Larger rooms occupy the corners of the house facing the exterior and with lower degree of

integration

Each derivation step can be enumerated:

1. Creation of independent cells
2. Spatial assignment for peripheral cells with living/dining and study uses
3. Concatenation of central cells for generation of entertainment core space
4. Assignment of access space to the external cells of the central bay

The Palladian grammar spatial derivation however describes a different scenario:

- Generation observes an inverted tree diagram
- All rooms are originated from cells with similar features
- More isolated/less integrated cells originate usable spaces such as drawing rooms, dining spaces and studies. These spaces usually combine one or more cells
- Singular cells within internal bays often generate circulation
- Central cells are combined into more complex spaces and generate entertainment
- Terraces, entrances, and loggias are associated to the entertainment spaces

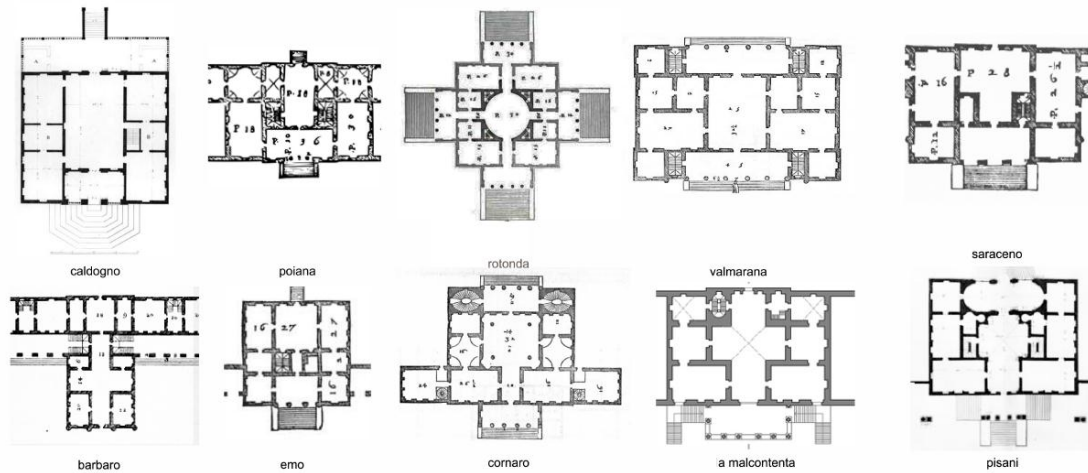


Figure 5. 2: Palladian villas (from top right to bottom left): Caldogno, Poiana, Rotonda, Valmarana, Sarraceno, Barbaro, Emo, Cornaro, Malcontenta and Pisani

villa	grid	number of rooms	Core room
Angarano	3x3	8	I
Ragona	3x3	9	I
Emo	3x3	11	I
Godi	5x2	10	I
Montagnana	3x4	10	T
Cornaro	3x4	10	T
Barbaro	3x4	8	X
Rotonda	3x5	9	X
Sarraceno	5x3	8	T
Malcontenta	5x3	9	X
Pisani	5x3	9	T
Poiana	5x3	10	I
Sarego	5x4	12	T
Gazzotti	5x4	16	T
Mocenigo	5x4	13	I
Valmarana	5x4	17	I

Table 5. 1: Palladian villas floorplan information by grid, number of rooms and main room typology

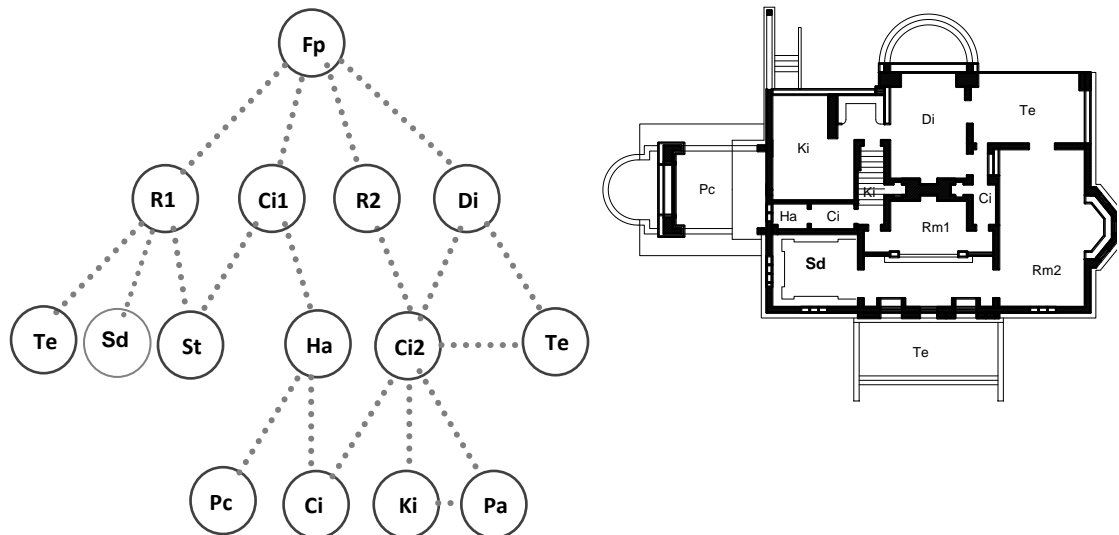


Figure 5. 3: Winslow houses Functional Diagram Key: Fp-Fireplace, R-Room, Ci-Circulation, Di-dining, Te-Terrace, Sd-Study, St-Stair, Ha-Hall, Ki-Kitchen, Pc-Porte-Coucherre or Porch, Pa-Pantry

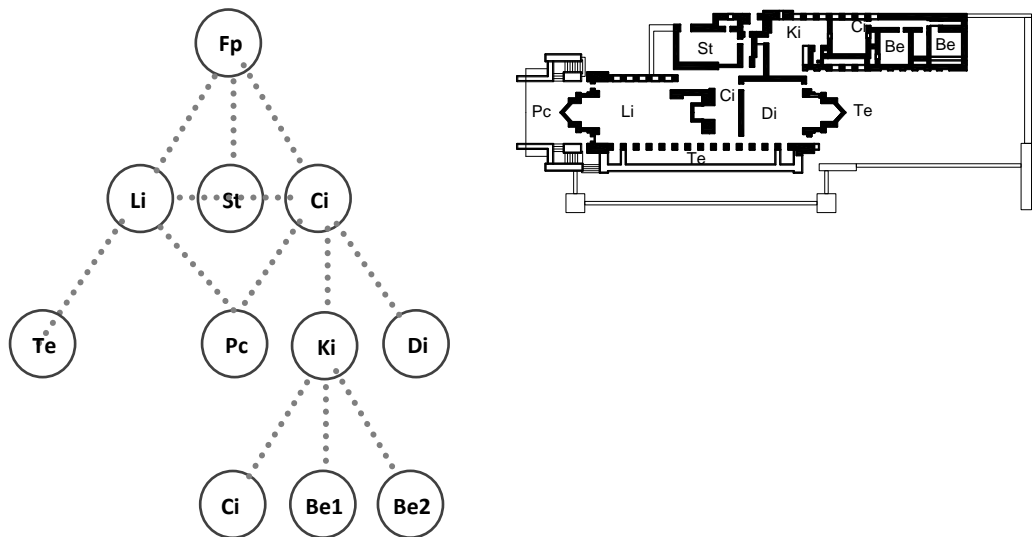


Figure 5. 4: Robie house Functional Diagram Key: Fp-Fireplace, Li-Living room, Ci-Circulation, Di-Dining, Te-Terrace, St-Stair, Be-Bedroom, Ki-Kitchen, Pc-Porte-coucherre or Porch

5.1.2 Prairie Houses

The prairie houses reflect a different housing concept. The adjacency graphs shown in 5.4 and 5.5 are both typical tree diagrams. There is a clear spatial distinction between back of house and front of house. That reflects the type of house user and their status, and unlike Malagueira it is a very different typology of single family housing stepping away from the affordable house reality. This has implications on the functional scheme and sequence of this house type. A similar approach is identified in the Palladian villas, where main floor is dedicated to host and impress, called the '*piano nobile*' or the main noble floor, while the top floor is private and destined to the family, whilst the back of house servants facilities are located in the lower ground.

Is relevant to note Wright's starting point, explains how the house concept progresses and its spatial solution. The key element and focal point is the fireplace. Wright describes it as the house core. Around it all functions array. The living spaces and family room are immediately adjacent. The dining room occupies the nearest position around the fireplace. Linked to the dining area the kitchen is placed. From the kitchen an array of services and function rooms such as servants' facilities, back of house circulation and secondary access, pantries, stores and the like. In the upper floors a similar arrangement takes place. From the circulation arrays a sequence of bedrooms, bathrooms, closets and walking closets and frequently outdoor spaces linked to the bedrooms like terraces and porches.

Not surprisingly the resultant adjacency diagram is a tree diagram.

The derivation functional diagram showcases stage by stage the creation process using the Koning Eizenberg grammar. Each space is carefully added to the previously added space (Koning & Eizenberg 1981a). This generates a succession of spaces from the social core of the house to the private quarters as illustrated in Figures 5.3 and 5.4.

This is related to the design process which is based on an additive method. This was previously identified by Steadman and March that identified a similar functional profile for three independent and time segregated houses designed by Wright.

Each space added, is added to the previous in sequence creating a conglomeration of shapes. The grammar uses the addition of three-dimensional shapes, very much alike Stiny's Froebel building block's grammar (Stiny 1980) (A Economou 1999). This grammar described how a children's building blocks game (similar to Lego) using a limited number of different blocks allowed the endless creation of different solutions. The Prairie houses grammar is a more sophisticated version. To the three-dimensional forms is added a functional label that describes the spatial features. So spaces are intelligently added by shape/spatial rules that foresee likable spatial interactions and avoid undesirable spatial connections. These rules enable the grammar to produce consistent results that satisfy the criteria imposed by the style and the logic of the space. These rules establish

what is acceptable and as a result the adjacency diagram is consistent from Prairie house to prairie house.

Another conclusion can be drawn from this analysis. There is a clear spatial distinction between living spaces and services. Similarly, to Palladio the target user for these type of dwellings could afford a certain life style. If in the Palladian villas the segregation is more drastic where all the services facilities and back of house are pushed to the lower ground level and the main raised ground hosts the noble functions, Wright is more subtle, but also demonstrates segregation. A clear line can be drawn in each house between front of house and main access and back of house and back yard. These specificities illustrate the differences among housing types and the affordability of the house. However, it remains clear is what functions occupy certain spaces despite differences.

Figures 5.3 and 5.4 show a sequence that can be summarized as:

1. Insertion of Fireplace F_p
2. Addition of first social space or room (R1): F_p+R_1
3. Addition of other social spaces such as Living and dining rooms (Li and Di): $F_p+R_1+Li+Di$
4. Insertion of circulation areas (Ci and St): $F_p+R_1+Li+Di+Ci+St$
5. Addition of services such as Kitchens and back of house areas (Ki, Se):
 $F_p+R_1+Li+Di+Ci+St+Ci+Ki$
6. Insertion of external spaces like terraces, '*Porte-Coucherres*' and halls (Te, Pc, Ha):
 $F_p+R_1+Li+Di+Ci+St+Ha+Te$

Adjacency graph and derivation graph can be read as one in the Prairie house grammar since the spatial allocation coincides with the spatial generation. The sequence followed for spatial assignment takes is made by adding a new space to a recently created space. These spaces become naturally adjacent forming a network of connections. This adjacency network can be illustrated by the very same derivation graph since each space connects to its predecessor.

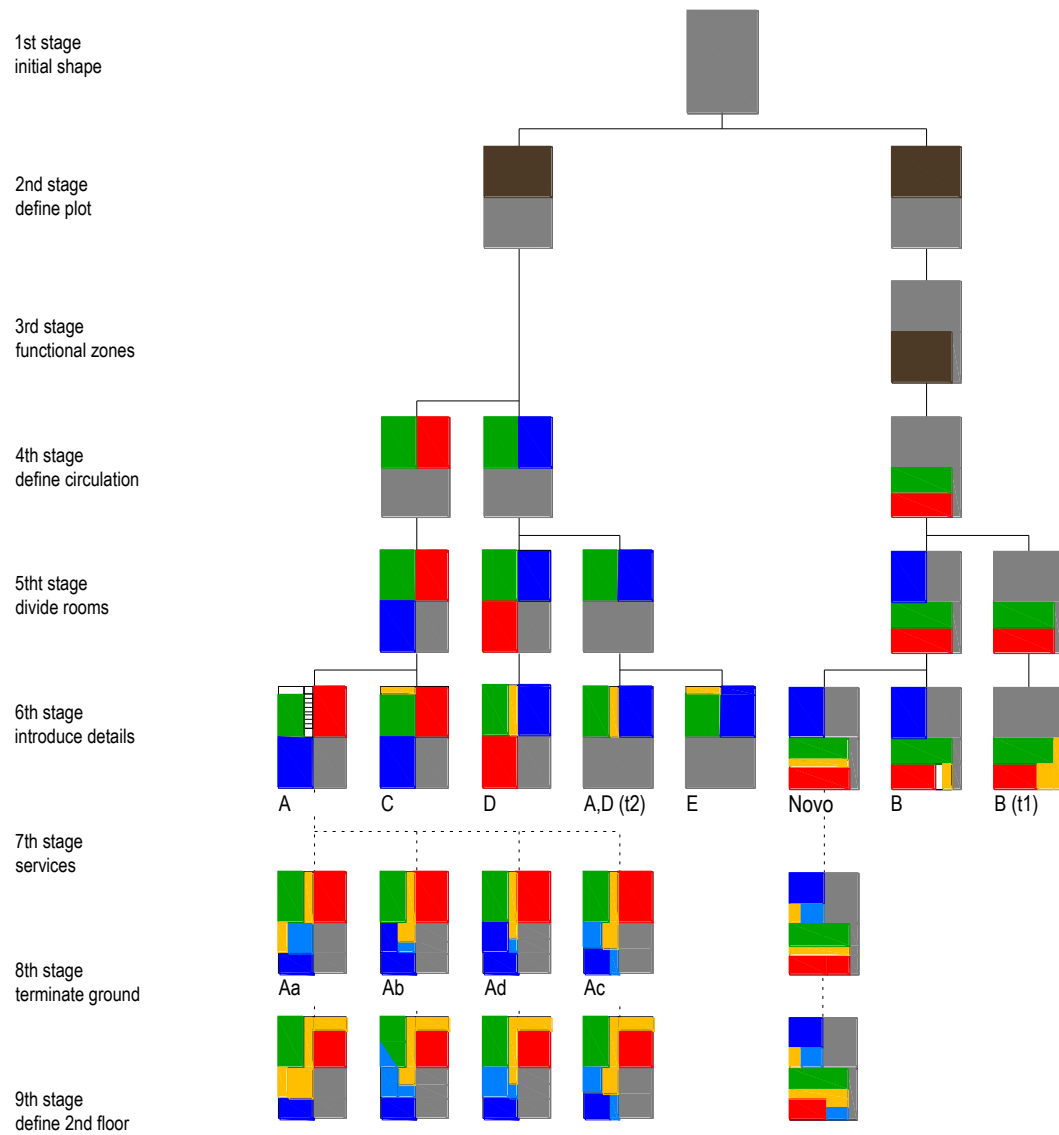


Figure 5. 5: Malagueira functional tree diagram adapted from (Duarte 2001a)

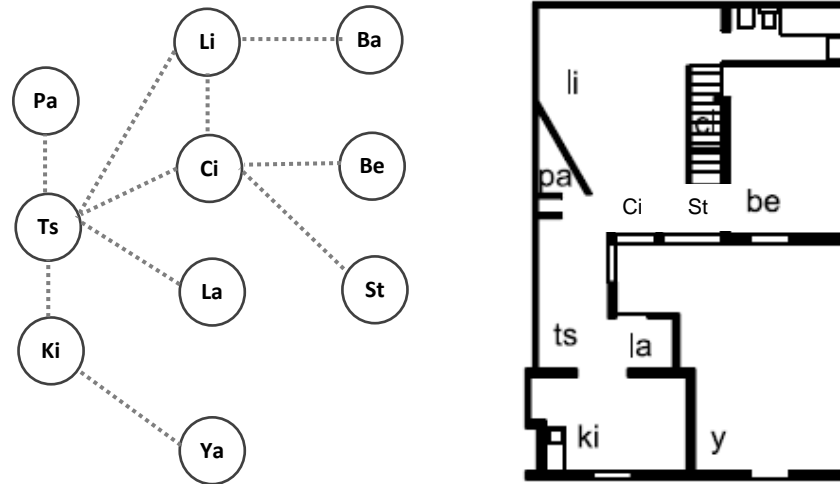


Figure 5. 6: Malagueira house Ab Adjacency graph Key: Ya- Yard, Li- Living, Be- Bedroom, Ts- Transition space, Ki-Kitchen, Pa- Pantry, La-Laundry, Ba-Bathroom, St- Stair

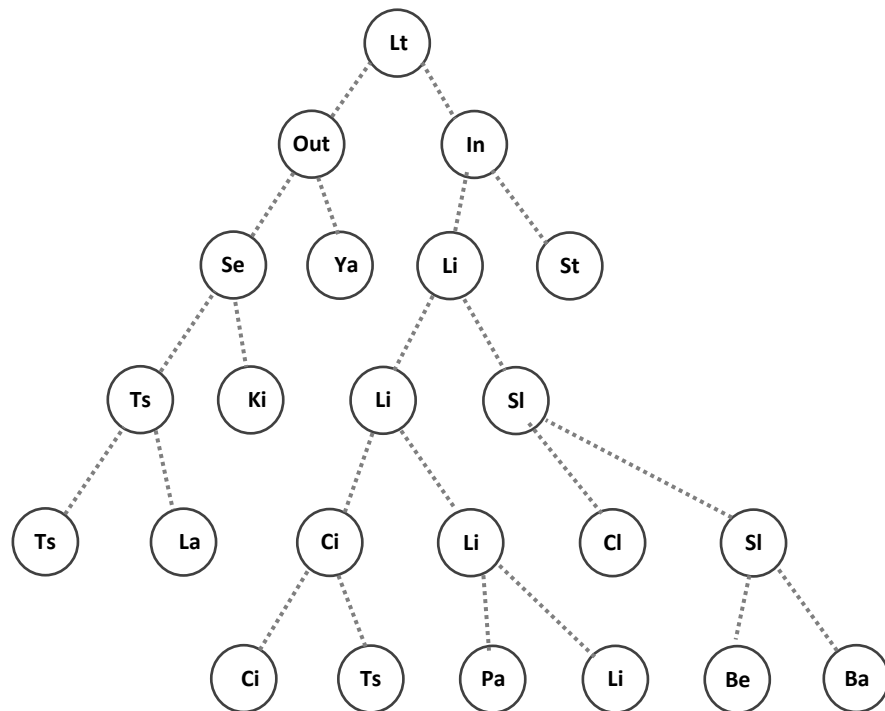


Figure 5. 7: Malagueira house Ab functional Derivation graph Key: Lt- Lot, Out-Outside, In- Inside, Se- Service, Ya- Yard, Ci- Circulation, Ts-Transition space, Ki-Kitchen, La-Laundry, Cl- Closet, Li- Living, Sl- Sleeping, Be- Bedroom, Pa- Pantry, Ba-Bathroom, St- Stair

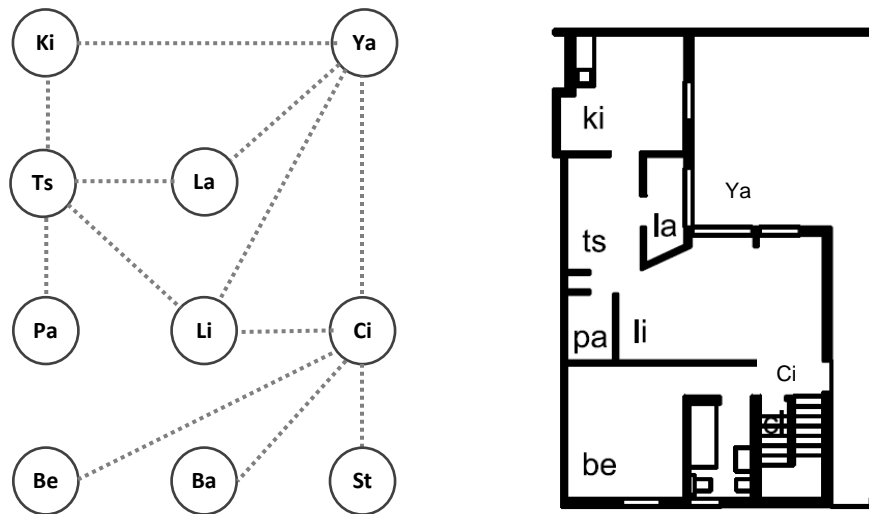


Figure 5. 8: Malagueira house Bb Adjacency graph Key: Ki-Kitchen, Ya- Yard, Ts-Transition space, La-Laundry, Pa- Pantry, Li- Living, Ci- Circulation, Be- Bedroom, Ba-Bathroom, St- Stair

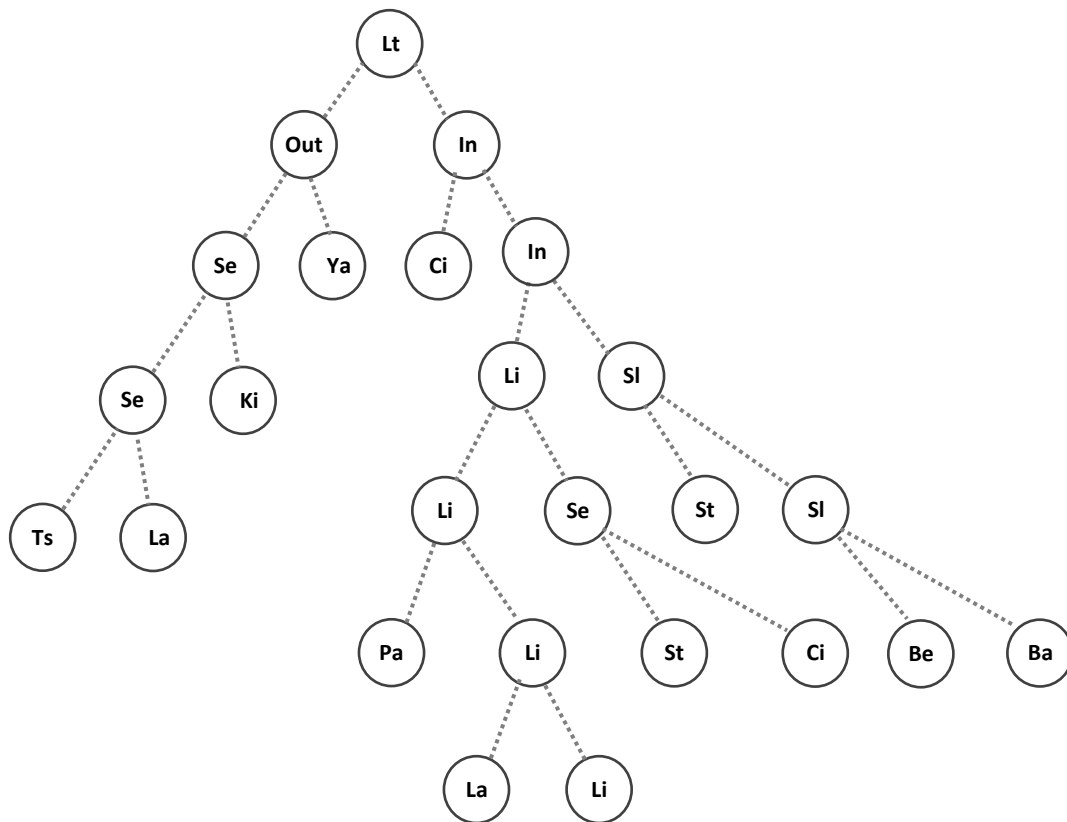


Figure 5. 9: Malagueira house Bb functional Derivation graph Key: Lt- Lot, Out-Outside, In-Inside, Se- Service, Ya- Yard, Ki-Kitchen, Ts-Transition space, La-Laundry, Ci- Circulation, Li- Living, Sl- Sleeping, Be- Bedroom, Pa- Pantry, Ba-Bathroom, St- Stair

5.1.3 Malagueira houses

The Malagueira adjacency graph resembles an irregular spider web of connections. This is based on a simple connection diagram where spaces connect one by one in an economic manner. This is clearly illustrated in 5.7 to 5.8 for type Ab and 5.9 to 5.10 for type Bb.

We have explained in previous chapters how houses generation process can be illustrated through derivation. To demonstrate functional assignment we propose the same analysis. For that a diagram can be underlined showing the progressive steps of creation. Often this diagram takes the form of a tree diagram either a conventional or an inverted one. This links directly with the type of generation performed, either a top-down or a bottom-up approach. Often top-down approaches derive into a typical tree diagram, whilst bottom-up into inverted genres as showcased in the Palladian and Prairie houses grammars

Another observation that can be drawn is that in the top-down, typical functional tree diagrams there is an initial focus on zoning. By zoning we mean the assignment of spatial significance into an area that will be further on detailed and defined into different spaces.

The Malagueira houses exemplify this paradigmatic use of sub-division to attain spaces (shape and function). The first step is the definition of the building lot (Figure 5.5). This lot is then immediately divided into interior dwelling space and outdoor area either yard or garden. This is the first phase of the so called zoning. The second step is the house layout. The assigned indoor space is then divided into two. This division reflects the separation between living space and services. Living is then furthermore divided into living room and storage. Storage is divided finally into circulation space and pantry, whilst in an upper branch of derivation services is subdivided into circulation, bathroom and service spaces. This functional derivation tree diagram was developed to illustrate house type Ab but similar results were achieved for houses like Aa and Bb to give some examples. All of these present similar two to three first steps and sharing many other branch commonalities despite the different spatial layouts and adjacencies/communicability.

When the derivation graph for house Ab and its adjacency graph are compared we can conclude that they do not share any comparable feature. This shows that the process used to design the house does not explain how functions are organized, despite addressing functions from the initial point. Therefore, the adjacency graph is foremost important to illustrate how the different spatial features of the house connect. The usefulness of this exercise on putting into shape a derivation diagram serves to illustrate the power and reach of shape grammars and explains how function is assigned in chronologically.

Observation of adjacency diagrams 5.7 and 5.8 for houses Ab and Bb show the convergence of one main space in each house, the living room. Regardless of the size of the house or the size of this particular space various rooms connect to this one. This is difficult to observe directly from the

floorplan where the living room occupies a position, space and hierarchy not with the same relevance of other rooms in other houses. But when the number of connections to this space is determined then it seems clear. The living room connects with the stair, the main access or circulation space, the yard and an additional room that could vary from a bedroom to a kitchen.

The less connected spaces are usually bedrooms or service spaces that show one degree of connection with the space that gives it access.

The derivation graph, on the other hand, illustrates a different aspect the space creation sequence. This has particular relevance for subdivision grammars that tailor their spaces by first zoning and then subdividing it into smaller spaces. In this case the distribution observed can be summarized in:

1. Creation of the Lot
2. Division of the lot into two areas, Indoors and Outdoors: $\text{Lot} = \text{In} + \text{Out}$
3. Outdoor space becomes the Yard (Ya)
4. Division of the indoor space into zoning, Services versus Living (Se and Li) : $\text{In} = \text{Se} + \text{Li}$
5. Division of Service area into Kitchens (Ki), Bathrooms, Pantries (Pa) or undescriptive Service spaces (Se): $\text{Se} = \text{Ki} + \text{Ba} + \text{Pa} + \text{Se}$
6. Division of Living areas into Living and Circulation : (Ci) $\text{Li} = \text{Li} + \text{Ci}$
7. Division of Living spaces into Living (Li), Dining (Di) and Sleeping: (Be) $\text{Li} = \text{Li} + \text{Di} + \text{Be}$
8. Division of Circulation into corridors (Ci), halls, stairs (St) and transition spaces (usually between the kitchen and the living area: (Ts) $\text{Ci} = \text{Ci} + \text{St} + \text{Ts}$

5.2 Shape and Function findings

Regardless of the spatial concept and macro-scale distribution there is a logic of space connectivity that overcomes boundaries of geographical, stylistic or budget differences. The main key space functions: living, dining, sleeping, food preparation and personal hygiene are hosted in every housing unit. The main differences are related to circulation, connectivity and the shape/area/proportion.

When compared these three types of houses that constitute the case study, their differences emerge clearly on their adjacency graphs.

Palladian villas reflect on their adjacency graph the radial nature of the design. The core space destined to entertainment is clearly positioned in the centre. From this highly connected space arrays a series of other spaces. It is not hard to understand that the graph presents a radial structure. It is also heavily centred and well hierarchically defined.

Differently the Prairie houses adjacency diagram presents a typical tree configuration, where from the focal point – the fireplace- irradiates a series of living areas that will irradiate services and circulation.

Malagueira showcases an apparently random adjacency graph but an ordered spatial derivation similar to the Prairie houses graph and despite the type of grammars (addition vs subdivision). Whilst Prairie houses adds a space and function progressively, Malagueira dissects the existing plot sequentially and assigns a function as it goes.

Proportional ratios occupy the conceptual background on classic architectural languages. Specific shaped rooms have specific uses. Classical architecture often proposed regular polygonal shapes to host spaces of praise as temples and churches. Vernacular architecture often used rectangular graphical representations to illustrate housing. Palladio addresses it in his treaty illustrating with real examples of projected and built villas how to formalize spatial relations, symmetry, harmony of proportions and how to compose a plan and a façade. He shows proportional ratios for each room. Ratios can be derived from the numerous examples. Many art historians and experts have defined those including the studies of Freedman and Hersey on the 'Possible Palladian villas' (Hersey 1992) which also quote Wittkower 'Principles in the age of modernism' (Wittkower 1998), Rowe's 'The mathematics of the ideal villa'(Rowe 1976) or even Howard's work on Palladio and musical scales.

Palladio states that most house rooms destined for living should have square footprints or alternatively these should have golden ratios up to a double square proportion.

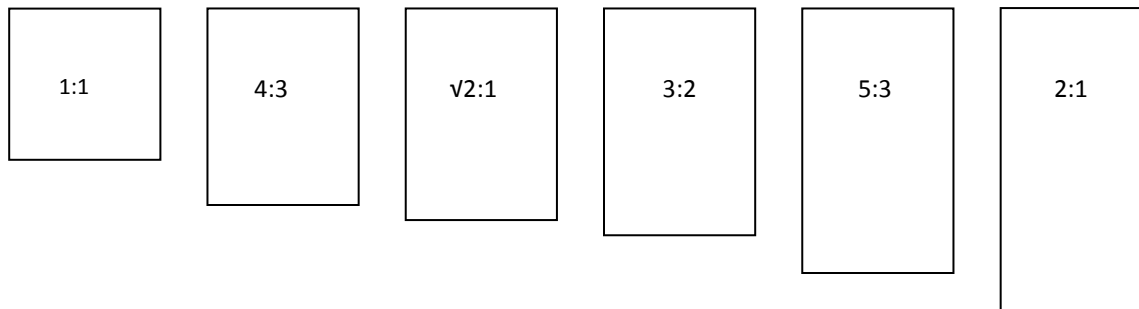


Figure 5. 10: Palladian rooms' ratios according to Hersey (Hersey 1992)

This can be explained by the fact that an elongated rectangle gives a dynamic feel and a sense of movement while a more regular enclosed geometry invites permanency. In addition proportions should be simple divisions resulting from additions of square root proportions derived from the golden ratio in intervals comprehended from the 1:1 proportion to the 2:1 such as: 1:1, $\sqrt{2}:1$, and 2:1 (Hersey 1992). Proportions such as 4:3, 3:2, and 5:3 are also included in Palladio's work and are approximations of rectangular ratios to the golden ratios. Often villas are illustrated in the 'Quattro Libri' with square proportions such as in villa Barbaro. However the illustrated proportions do not add up. So Palladio himself seems to adulterate his own treaty when required as some square proportions when measured are not really square but an approximate measurement. The only spaces that break this rule with a long and narrow footprint and proportion larger than the 2:1 are the circulation spaces. Circulation spaces resemble regular enclosed polygons, but differ from living areas through its size. These smaller circulation spaces accommodate spiral staircases, antechambers and are $\frac{1}{2}$ to $\frac{1}{4}$ of the size of the larger squared rooms. As also noted by Stiny and Mitchell originally there are other shapes used by Palladio. These regular symmetrical polygons vary from cross shape 'X' to 'I' and 'T'. Their characteristic configuration determines a very specific occupation, entertainment 'E'. Due to its unique use these rooms also have a special configuration that justify the particular use. Proportion wise they present the larger footprint when compared with all other rooms, occupying 2 to 5 cells concatenated. In this case shape describes function allowing space to accommodate a larger number of people, complexity to provide interest and an array of different spots or environments and centrality to facilitate access. The concatenation process and shape rules are illustrated in the following Figure 5.11.

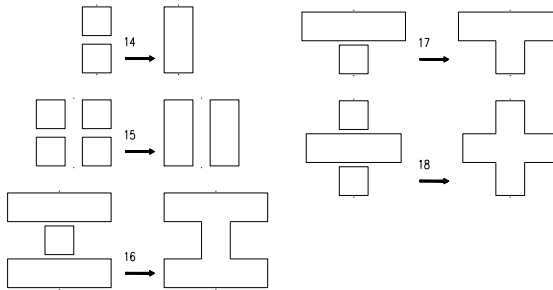


Figure 5.11 Palladian X, I, T, rooms illustrated as shape rules from the Palladian grammar (adapted Stiny & Mitchell 1978)

Figure 5.12 illustrates the previous derivation diagram coloured, the derivation of villa la Malcontenta using the same colour coding and the adjacency graph. Four colours are shown:

- Grey – symbolizes the spaces that have not been assigned with a function
- Red – accommodation and social spaces such as living, studies, dining areas
- Green – central entertainment space
- Yellow – circulation spaces such as halls, corridors, stairs and ante-chambers

From the diagram the following sequence is highlighted:

1. The first spaces to be created are the accommodation and living spaces.
2. Circulation occupies the next step. This is indicated with the colour yellow. This space mediates between the living areas and the main central space
3. This is followed by the main central space which is created through a series of concatenation in steps 3 and 4.
4. Once the spaces are assigned the adjacencies and spatial connections are placed by the inclusion of door openings that will create the final process to take place. This constitutes step 8.
5. The functional layout is finalized by the introduction of the access space.

A generic spatial sequence can be drawn from this: Core (Et) \rightarrow Li + Ci \rightarrow En + Te

As for the Prairie houses a very different language is taken into place. Wright was more concerned with the dynamics of the dwelling, the expression of horizontality and the asymmetry of the composition. However the houses derive from basic Euclidean solids that are conglomerated in an ordered manner. A group of piled boxes carefully stacked on top of each other and showing cantilevers is the most common image. The first box is placed and identified as fireplace. This box with specific proportions (chimney like with an extended un-proportioned height and contained width and depth) is placed roughly in the centre of the construction. A large sized box (10 to 20 times the width or depth) is placed adjacent to this. This box shows in plan a proportion contained between a square and a double square and is destined as living space. Additional box shaped rooms are positioned in a cross shaped effect. The closest to the fireplace the most social the space. The furthest and segregated the most likely to be a service area.

Malagueira houses derive from a very particular form/function relation. All houses are limited to a 12x8 m fixed footprint lot regardless of the number of rooms or house type. This has its tolls on the overall footprint of the internal space and in the size of the garden/yard. However some of the grammar rules help describing this important form/space relation. Siza as a confessed admirer of Alvar Aalto, frequently used some proportions famously preferred by Aalto such as the 1:2 and 2:3. Careful observation over the lot dimensions conclude that the preferred ratio is a 2:3 ($8/12 = 4/6 = 2/3$). The first division rule from the grammar encompasses the placement of the first functional zones of the house (interior and exterior). These divisions as described by Duarte can take place in two manners, either splitting horizontally the rectangle in 2:3 or 1:2 of its length. Taking into account the 2:3 proportion is once more the ratio selected it leaves us with an external yard of rectangular footprint and 4x8 m (a 1:2 ratio) and an internal space to be detailed of a squared 8 x 8 proportion and a 1:1 ratio. The exterior space is then recursively divided to create services spaces adjacent to the main corpus of interior space. To attain this a vertical division splits the yard space into two creating two squared proportions of 2 m sides. This creates a kitchen space. Let us remind that these houses constitute affordable dwellings and these sizes are just about the minimum efficient area for a compact kitchen.

$$\text{Lot} = \text{Out} + \text{In}$$

$$\text{Lot} = \text{Out} / 3 \times 2.\text{In}/3$$

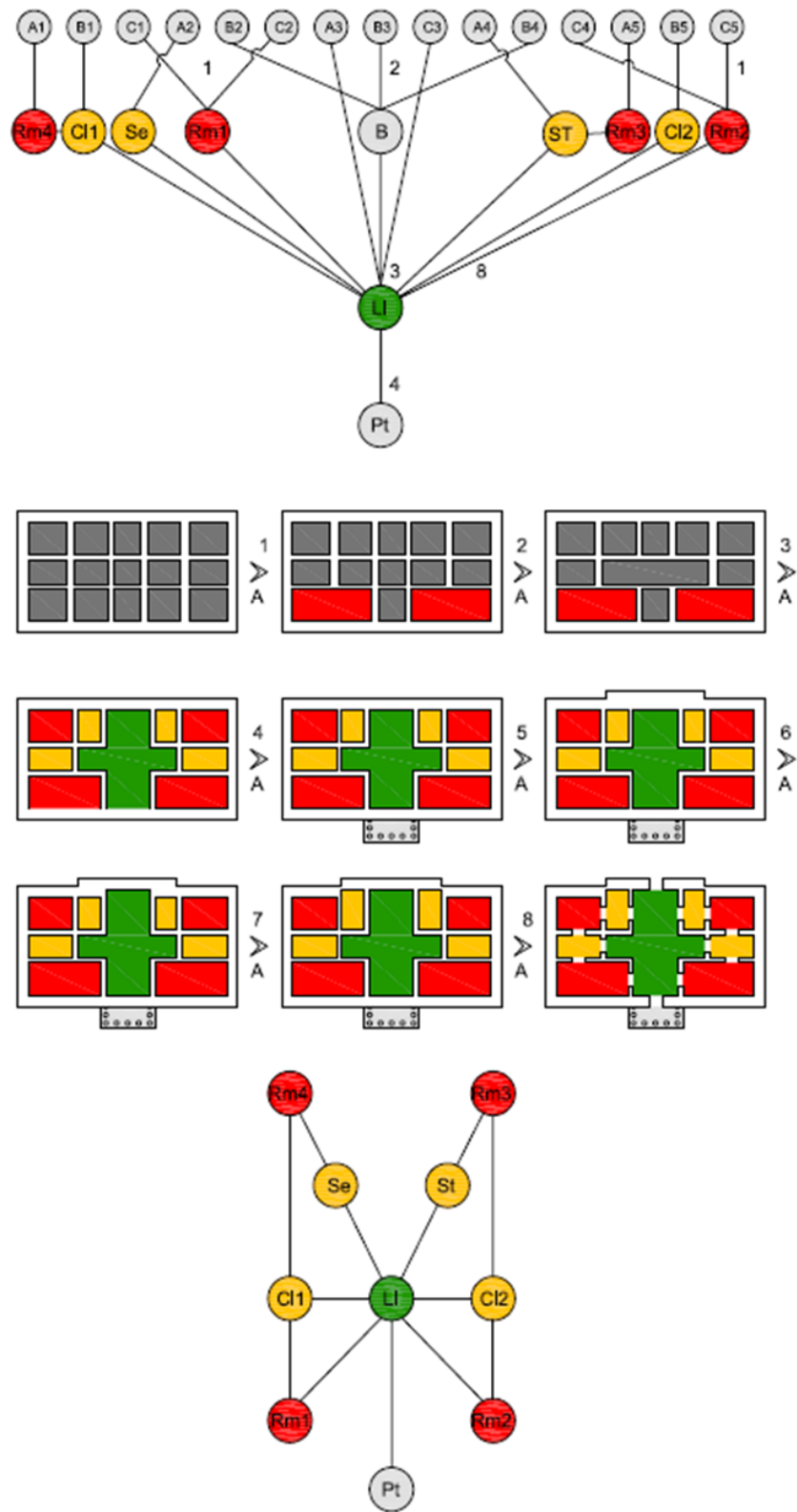


Figure 5. 12: Palladian villas Functional Diagram

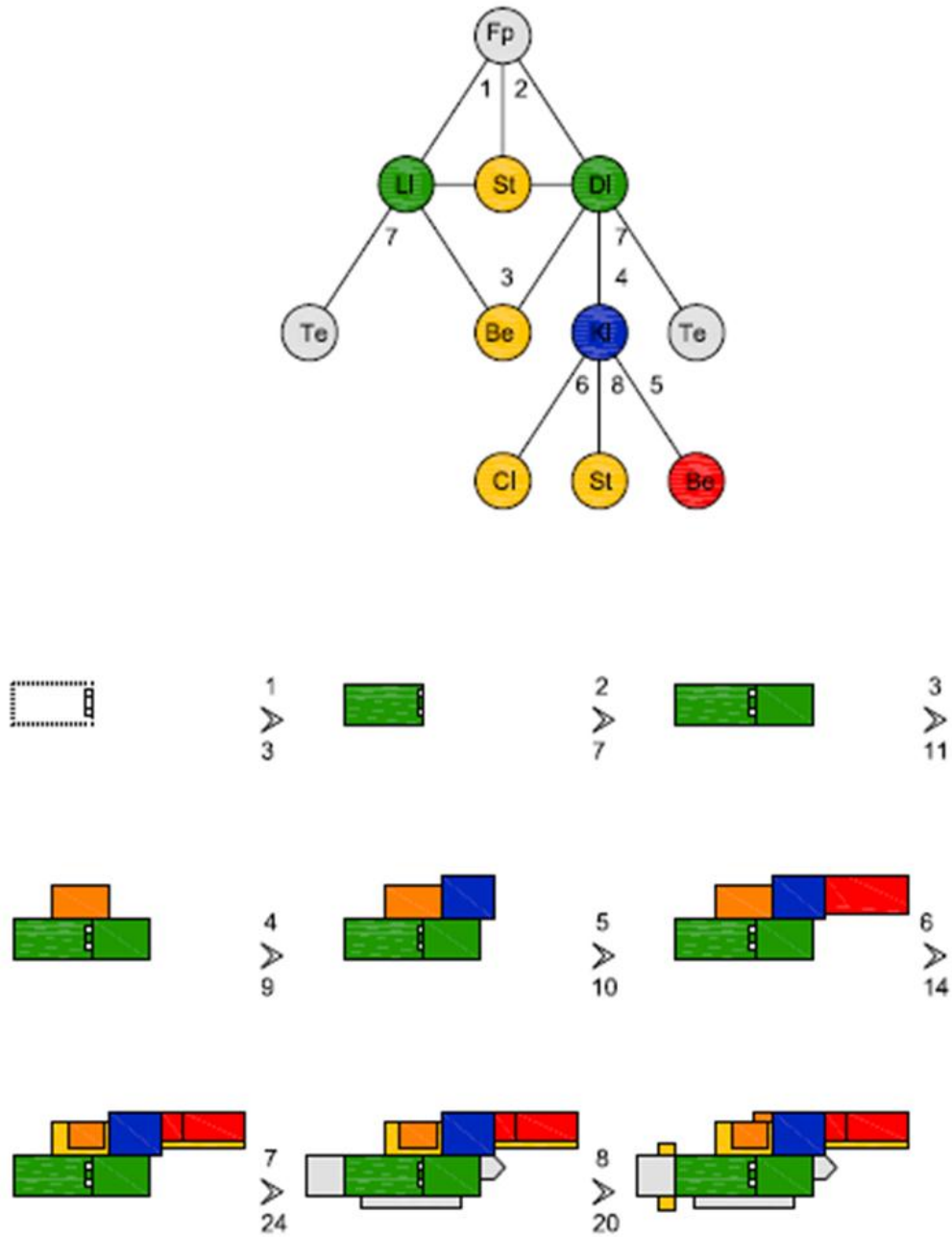


Figure 5. 13: Prairie houses Functional Diagram

The derivation diagram is shown in Figure 5.13 followed by the Robie house floorplan derivation. The derivation of this house using the Koning/Eizenberg grammar illustrates clearly the additive process that takes place. Each step of the derivation a new shape and function is added, this way:

1. Addition of fireplace
2. Inclusion of fireplace adjacent to the fireplace
3. Confinement of the fireplace with additional social space opposite to the first space
4. Insertion of a smaller room of less social nature such as a study or bedroom
5. Design of service areas such as kitchen
6. Addition of back of house areas such as staff facilities
7. Insertion of circulation
8. Completion with other external areas such as terraces and porches

The derivation diagram shows:

- A tree diagram prompted by the main social area adjacent from the fireplace
- Spaces irradiating from the main central space
- Inter-connectability of spaces bottom-up
- Highly connected spaces identified by the Depthmap occupy the top of the tree
- Poorly integrated spaces occupy bottom positions at the diagram and showcase one singular connection
- A generic spatial sequence can be drawn from this:
 - i. $Fp \rightarrow Li + Di + Ci \rightarrow Rm + Te + Ci \rightarrow Se + Ci$

The sequence of spatial assignment is illustrated in table 5.2:

space	1 st floor	2 nd floor	roof
1 st stage	Fireplace		
2 nd stage	Living room 1		
3 rd stage	Living room 2		
4 th stage	Living extension		
5 th stage	Social area function		
6 th stage	Corner space extra		
7 th stage	Adding porches		
8 th stage	Interior detail		
9 th stage	Basement outline def		
10 th stage	Terrace outdoor		
11 th stage	Exterior details		
12 th stage	Entrance portico		
13 th stage		Easing labels	
14 th stage		Bedroom level	
15 th stage		Bedroom extension	
16 th stage		Bedroom details	
17 th stage		Double height rooms	
18 th stage			Erasing labels
19 th stage			Definition of roof line
20 th stage			Definition of balcony
21 st stage			Erasing labels
22 nd stage			Roof formation
23 rd stage			

Table 5. 2: Grammar functional generation by stages in Prairie houses grammar

The Prairie grammar as an addition grammar proposes a functional arrangement that derives straight from the shape rules placement.

The basic schema for these shape rules is based on the shape rule addition schema:

$$\emptyset \rightarrow X$$

To an existing shape X and you introduce a new shape that can translate a transformation on the initial shape, or even:

$$X \rightarrow X + t(X)$$

This schema sums the spatial assignment typical on an addition grammar such as the Prairie houses grammar.

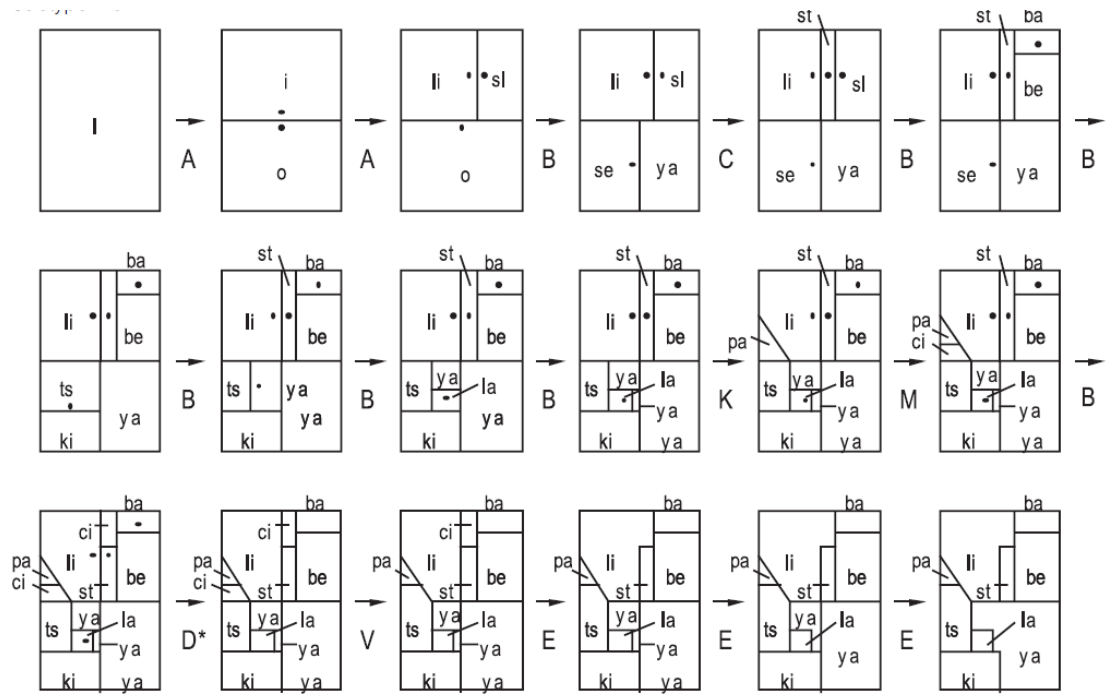


Figure 5. 14: Malagueira house Ab functional Derivation

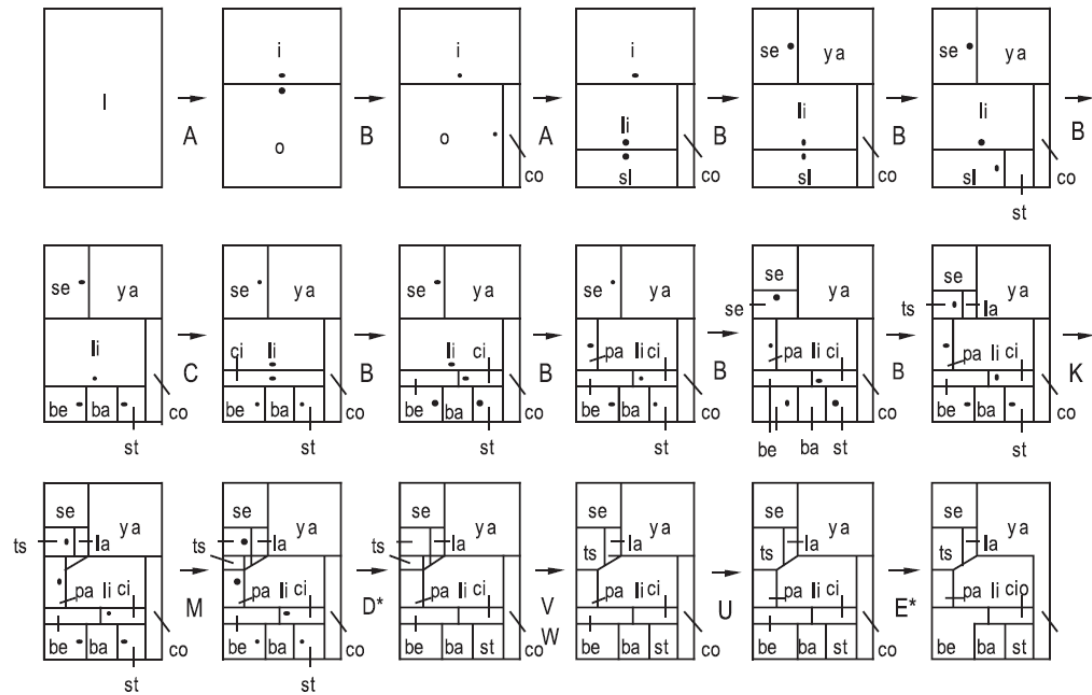


Figure 5. 15: Malagueira house Bb functional Derivation Key: Lt- Lot, Out-Outside, In-Inside, Se- Service, Ya- Yard, Co- Corridor, Ci- Closet, Li- Living, Sl- Sleeping, Be- Bedroom, Pa- Pantry, Ba- Bathroom, St- Stair

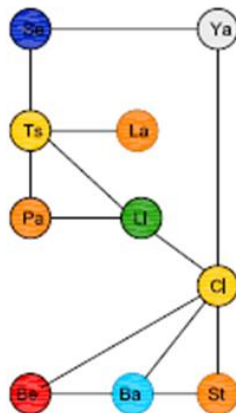
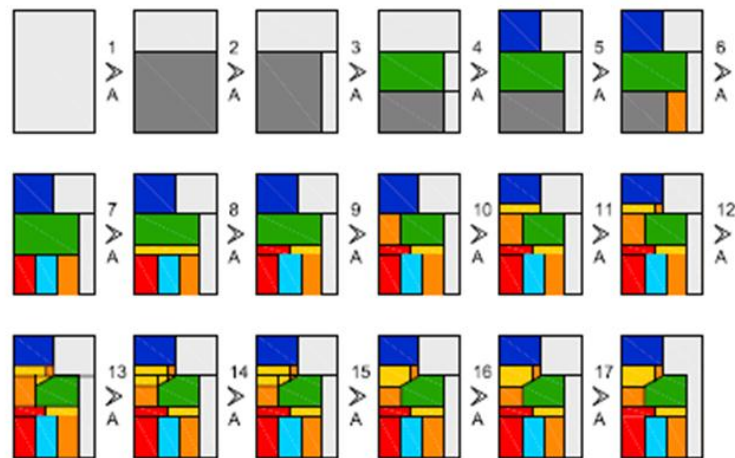
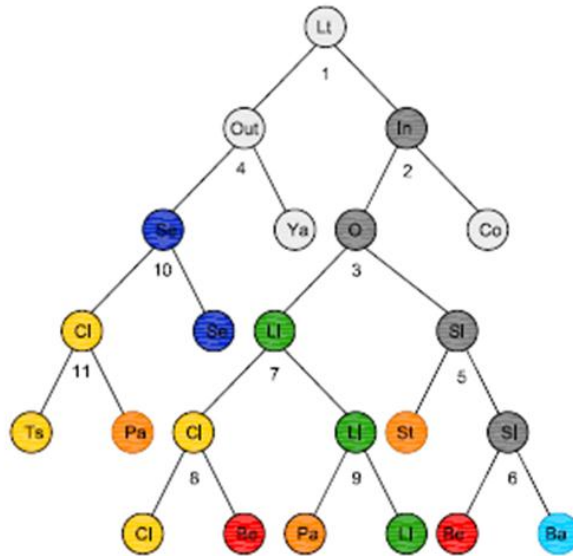


Figure 5. 16: Malagueira house Bb functional Derivation Key: Lt- Lot, Out-Outside, In-Inside, Se- Service, Ya-Yard, Co-Corridor, Cl- Closet, Li- Living, Sl- Sleeping, Be- Bedroom, Pa- Pantry, Ba-Bathroom, St- Stair

Figure 5. 17: Malagueira houses Functional Diagram

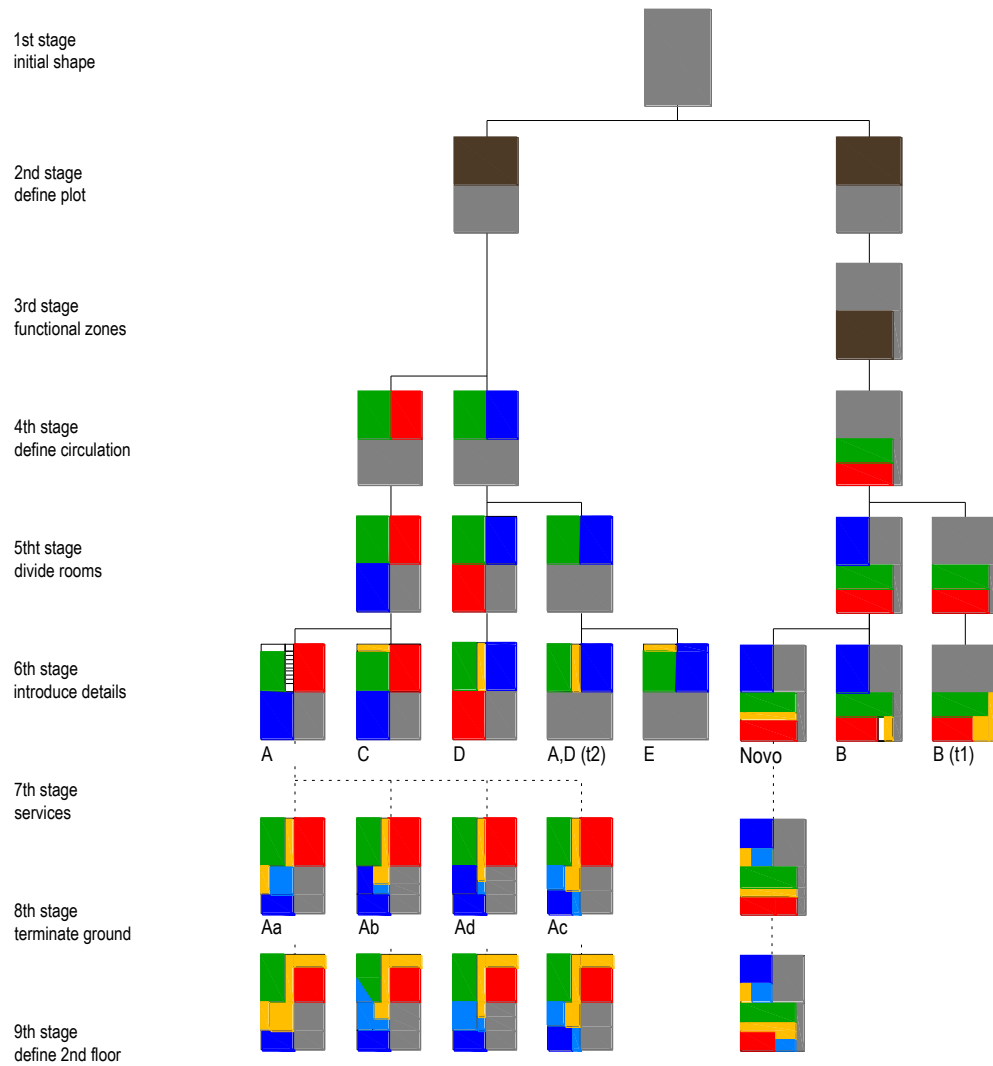


Figure 5. 18: Malagueira tree diagram with functions (Duarte 2001a)

GRAMMAR	STAGE											
	1: DEFINE FIRST FLOOR						2: DEFINE SECOND FLOOR				3: DEFINE TERRACE	
F1	S	Z	C	R	D	O	T					
F2	↓	↓	↓	↓	↓	↓	↓	S	Z	C	R	D
F3							↓	↓	↓	↓	↓	↓
E												

Table 5. 3: Functional definition by stages in Malagueira houses - adapted from (Duarte 2001b))

The table 5.3 above shows the chronology of functional assignment for the Malagueira houses grammar. Function and shape are generated simultaneously where no space is designed without the assigned function. At times there is the specific definition of function but zoning which will later lead to specific rooms, justifying the grammar top-down approach.

As shown above there are up to three stages linked to number of storey of each single-family house. The first mandatory stage corresponding to the first storey with all the basic house functions are included from the generic zoning (z).

These results were then compared with the functional derivation diagram illustrated in Figure 5.18. This derivation diagram showcases the functional assignment for house Bb stage by stage as described below:

1. Creation of the lot by setting the outline
2. Division between inside and outside
3. Sub-division of the internal space into living and circulation
4. Further sub-division between two living spaces
5. Division of the yard area into outside and service area
6. Further subdivision of internal areas to integrate bedrooms, bathrooms, pantries, and laundry spaces
7. Concatenation of adjacent spaces to form different shaped rooms
8. Creation of connections among adjacent spaces by placing door openings

The sequence of stages described can be perceived both through the derivation of house Bb and through the derivation tree diagram in Figure 5.18. The tree diagram illustrates a particularity. Unlike the adjacency diagram (with a net like form of spaces that occupy central positions and connect to a myriad of other spaces) the derivation diagram shows a typical tree diagram. In this tree diagram, space by space is sequentially added to the previously designed space. This results from the subdivision process, to each lot is applied a subdivision, followed by the creation of inside and outside space. To the inside space another subdivision is created. This process can be described as a generic spatial sequence that can be expressed by:

Lt → In + Out

→ In → Service + Living

→ Service → Bathroom + Kitchen

→ Kitchen → Kitchen + Pantry

→ Living → Living room + Bedroom

→ Living room + circulation

→ Out → Yard + Service

→ Yard + Circulation

This generic spatial assignment can be further expressed by a rule schema. This rule schema can be expressed by: $X \rightarrow \text{Part}(x)$ or

$$X \rightarrow (X' + X'')$$

or

$$\text{prt}(b(X)) \rightarrow (X' + X'')$$

This schema defines generically the type of spatial assignment that occurs on a typical subdivision grammar. Each grammar will advise on the particular adjacency circumstances.

5.3 Preliminary results and conclusions

From the architectural grammars exposed in this study there is a clear distinction between the formulations used to describe each corpus of designs. Three big groups arise: 1) the Addition, 2) Grid and 3) Subdivision grammars:

From the first group the Prairie houses grammar and the Queen Anne Houses (Flemming 1987) are suitable candidates. The grid grammars are clearly represented by the Palladian grammar which is the first architectural grammar to be put into practice. The Bungalows of Buffalo (Downing & Fleming 1981) are arguably a good candidate for a grid grammar since are derived from a self-contained rectangular shape that is somehow converted into an orthogonal matrix. But this could also join Malagueira into the group of Subdivision type grammars.

In terms of shape rules used there is a clear standing out of four basic groups of shape rules. From these rule types we highlight: the rectangular addition, the rectangular offset, the concatenation and the subdivision. Regardless of the grammar type or process used these rule types find representation in all of the grammars described. Additionally, these rules are illustrated more than once in the grammars studied as an important design process.

The study of functional derivation graphs allowed two conclusions:

1. Grammars resulting from a Top-down approach usually refer the functional assignment to later stages of design focusing first on the geometrical generation
2. Grammars resulting from a Bottom-up approach ally the spatial creation with the functional assignment prompting it simultaneously. The space is created one by one with a purpose from the genesis

In addition the study of the set of available design rules for each study case grammar one can find three types of functional generation rules:

1. A subdivision function rule usually associated with a Top-Down approach, with a schema type of:

$$X \rightarrow (X' + X'')$$

2. An additive function rule usually associated with a Bottom-Up approach, with a schema type of:

$$X \rightarrow X + t(X)$$

3. The concatenation function rule also associated with Bottom-up approaches, with a schema type of:

$$(X' + X'') \rightarrow X$$

The functional subdivision rule occurs frequently in grammars such as the Malagueira or the Buffalo houses and results from a process of zoning or division of logical spaces and recursive subdivisions into functional rooms.

The additive rule was observed in additive grammars such as the Prairie house and the Queen Ann Houses. Is a simple 1+1 type of rule and allies the spatial creation to the functional assignment.

The functional concatenation rule presents the reverse of the subdivision rule, proposing the merging of two spaces to allow for one singular function. Despite the lack of functional identification the Palladian villas grammar recurs to this mechanism to generate space with a specific use and therefore it cannot be ignored. Nevertheless this could be either substituted by the alternate form of the subdivision rule or by a variant of the addition rule.

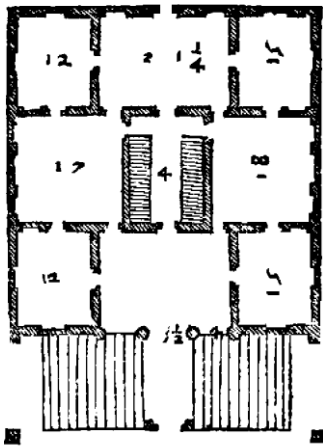
The observation of the functional derivation of each grammar corpus allowed us to conclude that each family proposes a specific functional derivation that is common. However the adjacency graph revealed that the adjacencies show variants between corpus of solutions. Nevertheless the spatial articulation between spaces, the depth of the social spaces versus private quarters is maintained and observed. Alike geometry spatial distribution allows parameters and variants however there are intervals and conditions that rule each language.

For the Palladian villas an express number of rooms is limited as the examples show. Likewise the depth of the 'Entertainment', 'Living' and 'Circulation' spaces is similar amongst built examples. The adjacency graph is consistently radial from the 'Entertainment' space. Prairie houses showed

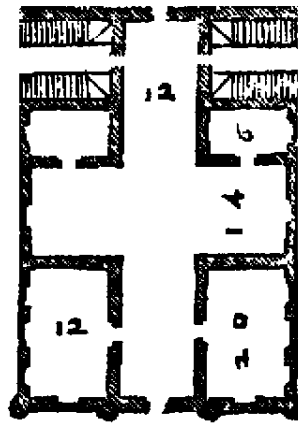
similar relations. The depth of the 'social' areas versus the 'services' was consistent and the adjacency graph a typical tree diagram which corresponds with the derivation diagram proposed. The Malagueira derivation diagram also is represented as a tree diagram, however the adjacencies are shown as net like system with some complexity and variation from house to house. This reflects the variety proposed for spatial distribution over the 200 examples designed and built. Nevertheless the main relations between social spaces, services and circulations are well recognisable and defined by Duarte's grammar.

Future chapters will associate the generic spatial grammars above mentioned within the set of design rules and propose an articulation for the spatial assignment. Practicality seems to point towards the use of the most common functional assignment rule, the subdivision.

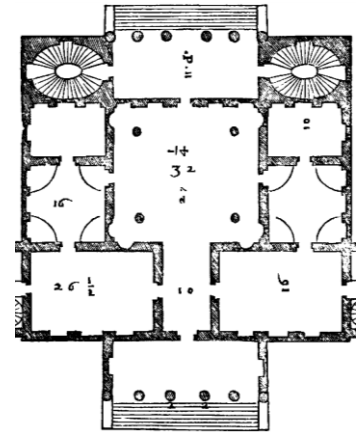
6.	An Alternative Grammar	179
6.1	Motivation	179
6.2	Original Palladian Villas Grammar vs. Alternative Grammar	187
6.3	Alternative Grammar type and classification	192
6.4	Rule System	194
	6.4.1. Wall placement:	195
	6.4.2. Door placement:	198
	6.4.3. Window placement:	200
	6.4.4. Staircases:	202
	6.4.5. Loggias:	203
	6.4.6. Colonnade:	203
6.5	Palladian Function Rules	205
6.6	Villa Derivation	222
6.7	New Designs	225
6.8	Computer implementation	234
6.9	Chapter results and conclusions	235



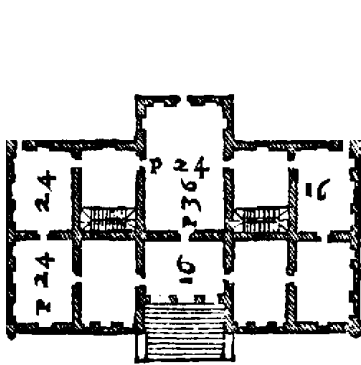
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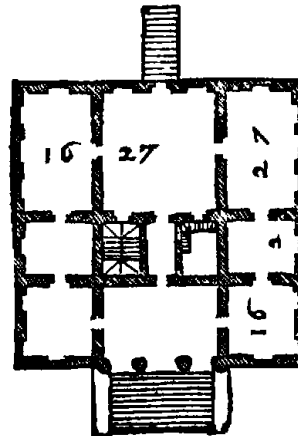
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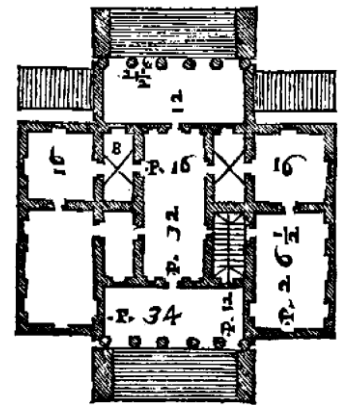
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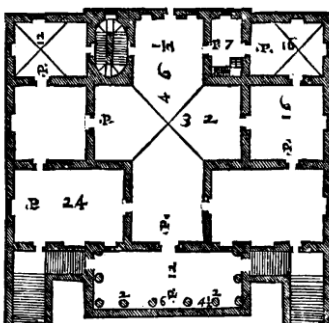
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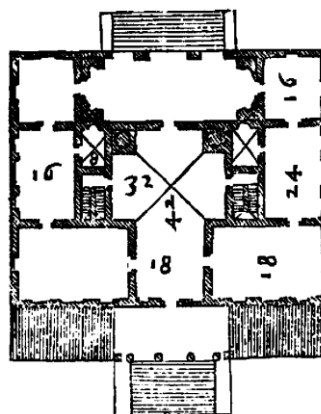
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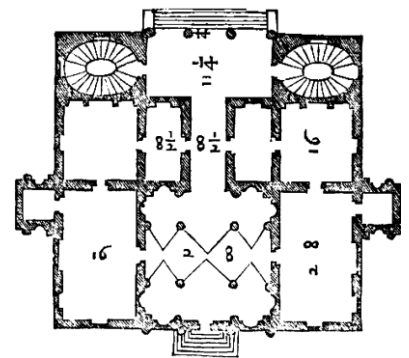
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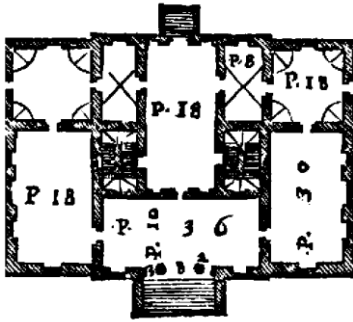
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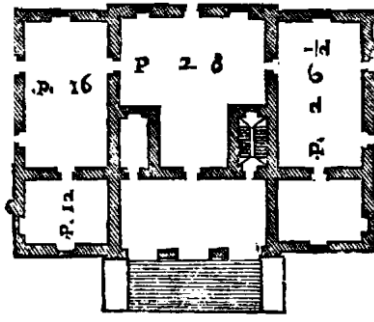
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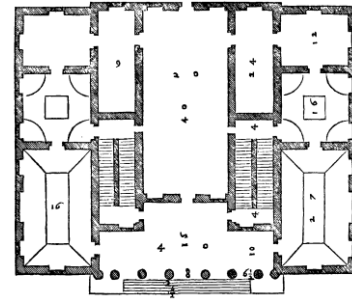
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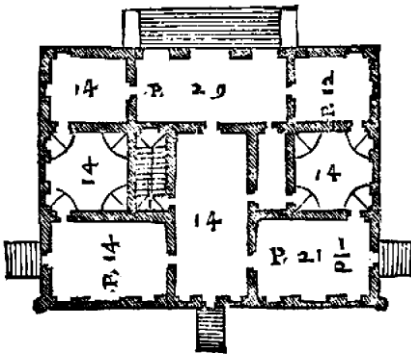
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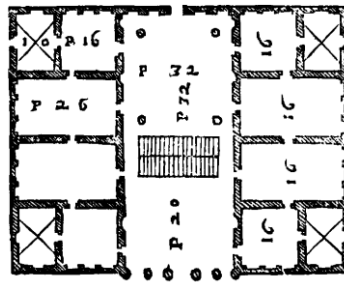
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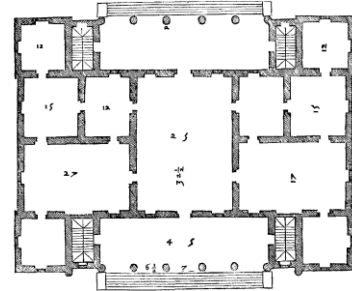
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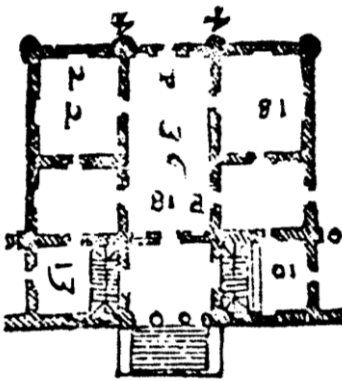
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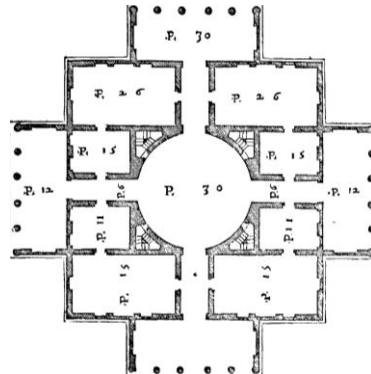
Villa Mocenigo



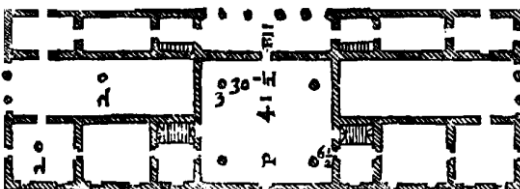
Villa Valmarana



Villa Angarano



Villa Rotonda



Villa Thiene

Figure 6. 1: Palladian villas as pictured in 'Il Quattro libri' (Palladio 1997)

6. An Alternative Grammar

This chapter describes a shape grammar that recreates Palladio's villas. A Palladian grammar was previously proposed by Stiny and Mitchell (Stiny & Mitchell 1978). However, this alternative grammar uses different parametric shape rules and methodology to test the hypothesis that different grammars can generate the same corpus of designs. Throughout the sections the chapter describes the motivation to recreate an alternative grammar for the Palladian villas, the rule system deployed, the grammar formulation and its computer implementation.

Often the best solutions are the simplest ones and that is the effort attained with this work.

To attain a simpler solution a rule system based on the subdivision generic rule was implemented as discussed in previous chapters. This allowed for a reduction of the number of rules applied. In addition, the implementation of rules schemas and its associated parametrization allowed for a more compact and economic alternative grammar as the below describes.

6.1 Motivation

Stiny and Mitchell's grammar creates a corpus of solutions that was clearly identified by the authors in works as "Counting Palladian plans" (G Stiny & W J Mitchell 1978b) in which a finite number of possible solutions was illustrated as solutions derived from the grammar. Within this corpus a smaller number of solutions were identified as original Palladian designs as reproduced in "Il quattro libri" or as built by the master. The proposed alternative grammar here exposed allows for the creation of the same Palladian designs and the Mitchell and Stiny's new corpus. The formalism is then implemented using a computerised design tool. The grammar includes subdivision rules that allow for a more economical formulation.

This constitutes the first task towards a wider research aimed at formulating a generic housing grammar as described in future chapters.

This section describes the methodology used to recreate Palladio's design signature using shape grammars. Stiny and Mitchell (Stiny & Mitchell 1978) illustrated the range of shape grammar and its first architectural applications. This work replicates what was presented forty years ago, using an alternative grammar incorporated into a computerised tool.

The concept of generic grammars was also applied to urban design to develop a partial generic urban grammar (Beirão et al 2011). The novelty of this research lies in the hypothesis that more than one grammar can generate the same design solutions. Moreover, if two separate sets of rules can produce the same design solutions, this can be represented by generic shape rules. The concept of a generic grammar then seems feasible.

A shape grammar as a structured formalism incorporates a limited number of shape rules to describe geometric operations. According to Knight as quoted by Prats there are three types of shape grammar processes: grid, addition and subdivision (Prats 2007). Grid grammars propose a form of design generation based on a matrix. The design is derived from the grid using cell

merging and subdivision processes. Subdivision creates a design confined by a boundary and the solution is achieved through a succession of divisions. The original Palladian Grammar used a grid process to address the symmetrical rectangular floor plan.

This alternative grammar uses a subdivision process. The reasoning for this selection is threefold: firstly, it represents a more intuitive way of designing, secondly, it allows for fewer shape rules hence is more economical, easier to use, and thirdly it proposes a different process, thus proving that two distinct formulations can generate same results.

The rule extraction process was based on observation of different existing villas, examining each floor plan, number of rooms, proportions, geometries, spatial adjacencies and communicating rooms. The layout of the floor plan for each villa uses 9 basic rules in contrast with the 25 used in the original grammar and all the shape rules have a parametric formulation. The initial rules define the boundary setting, whilst the rules that follow propose subdivisions. Other rules allow for the merging of consecutive cells by erasing borders. The design is completed by allocating centrally aligned openings in accordance with the original specifications (Palladio 1997).

Functions were not addressed in the original grammar, but were incorporated in this study. It was possible to identify the functions in unlabelled sixteenth century villas by observing spatial adjacencies, using comparison as a method and taking proportion, shape and area into account. Notions of shape syntax integration and segregation were also considered (Hiller and Hanson 1984).

Additionally, site visits, photographs taken, surveys and local observation were key in the study of the villas.

A computerised tool was created, developed in AutoLISP. The routine created provides non-shape grammar users with a visual platform for exploring design. The script is easily accessible to non-proficient CAD users and allows for real time 2D design generation and 3D model visualisation.

Previous studies have proved that an extensive corpus of designs can be produced from limited design rules (Stiny and Mitchell 1978b). The original grammar allowed for the production of 20 villas for 3x3 grids (as shown below) and 220 different solutions for grids of 5x3 cells.

The motivation for this occurred while analysing the original Palladian grammar. The formulation, compared with other grammar types, showcased a complex group of shape rules that carefully allowed the proportional aspect of the overall geometry. While doing so the grammar consisted in a set of rules with the only purpose to generate a base grid. Once generated, this grid would be manipulated by a set of shape merging rules and by grid lines deformation rules. Other grammars seemed to use more resolute manners of achieving the basic layout even if less accurate in the generation of proportions.

The alternative grammar chapter follows the work developed with grammar analysis. This constitutes the next step in order to determine if more than one set of rules can generate the

same design corpus. This would allow to conclude how hermetic a grammar can be and if more than one grammar can generate the same results. It is believed that one shape grammar enables the generation of one specific family of results. 'Each shape grammar defines a language of designs' (Stiny 1980b). Up to this point no two grammars allowed for the generation of the same corpus of results. There is no data to demonstrate this. However, in shape grammar teaching there are many examples of this, for some reason this was not documented to this point.

The four books famously illustrate 18 villas as the figure above shows. There are other edifications that also feature the second book however its dimensions and typology hardly fits the criteria. Omitted were all the palaces and urban dwellings (Palladio 1997). As Palladian villas were considered only the villas within a rural setting and with the features as described in chapters 4 and 6. The villas are ordered accordingly to its underlined grid and size, from the smallest villas such as Angarano and Ragona (3x3), to Barbaro and Cornaro (3x4), Godi (5x2) and the most famous, Emo, Badoer, Malcontenta, Pisani Bonetti, Pisani Montagnana, Poiana, Sarraceno, Sarego, Zeno all exhibiting a 5x3 grid and the largest villas such as Mocenigo, Valmarana both with a 5x4 grid, Rotonda a 5x5 and Thiene standing singularly with 7x3.

This group of examples also showcases different variations within the style. This echoes both the period on which they were built varying from the early 1500's to the third quarter of the sixteenth century. All of the examples showcase five basic feature spaces within the villa typology: the main centre room with special geometry, the loggia, the external staircase, the circulation spaces and the rooms placed by the facades.

The main room ranges from a simple rectangular shape as in Ragona, Godi, Emo, Badoer, Poiana, Sarego, Zeno, Moncenigo Valmarana and Thiene, to 'I' shape as in Pisani Montagnana, 'T' shape like Cornaro, Sarraceno, 'X' as represented in Barbaro, Malcontenta, Pisani Bonetti, and circular as in Rotonda.

Other features are noticeable such as the orthogonally, the array of rooms symmetrically distributed east to west, the door/window alignment and the entrance and rear entrance placed opposite creating simultaneously the north/south symmetry axis.

Stiny and Mitchell's grammar reproduced most of the above-mentioned villas. The question at stake was if a different grammar could generate the same results. An experiment was conducted in order to collect some data. The validity of theory that more than one grammar can reproduce the same family of results needed to be tested.

In a workshop developed in April 2013 entitled 'Re-inventing ceramic tiles patterns' the students were challenged to develop a shape grammar by inferring the shape rules from an example of a tile pattern. Two separate groups were given the same pattern (Fig. 6.2 and 6.3) and they were asked to draw the inherent shape rules (Eloy, Benros, Duarte; 2014). The two separate groups developed a set of independent rules that enabled the generation of the original pattern.

This was achieved using different rules. The result is two different grammars that are operative and respond to the exercise goal.



Figure 6. 2: Ceramic tiles pattern as provided to Team A and B

Figure 6. 3: One of the teams inferring shape rules and its graphical representation

Group 1 methodology started by analysing all the different shapes allowed by the system and identifying them. Secondly they identified the different spatial relationships allowed between the identified shapes. The rules proposed illustrated the spatial relations and shapes identified. (Fig. 6.4). Group 2 proposed a more sophisticated approach. After identifying the shapes present they discovered that some of the spatial relationships were basically a mirroring or a simple copying of shapes. Therefore, instead of transcribing all the different relationships in independent rules they created a simple square shape to symbolize the tile and a dotted label. This label was positioned in the tile corner and its final location (same spot, opposite corner or diagonally opposite corner) would symbolize the operation.

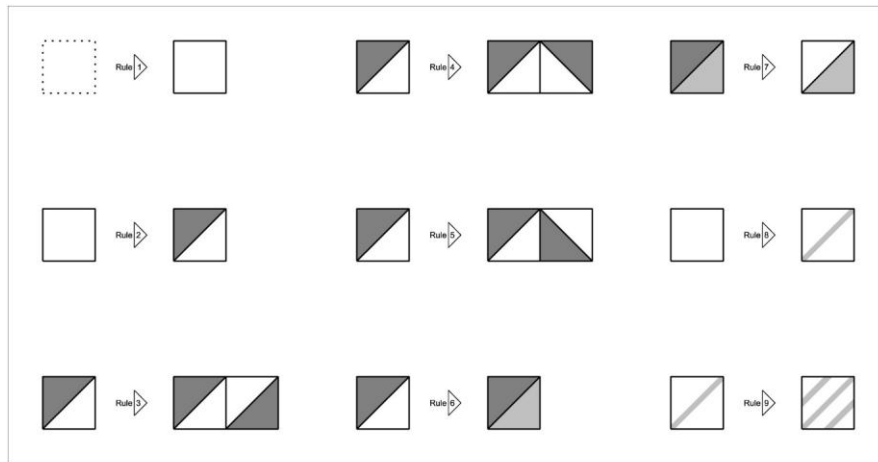


Figure 6. 4: Team 1 inferred shape grammar

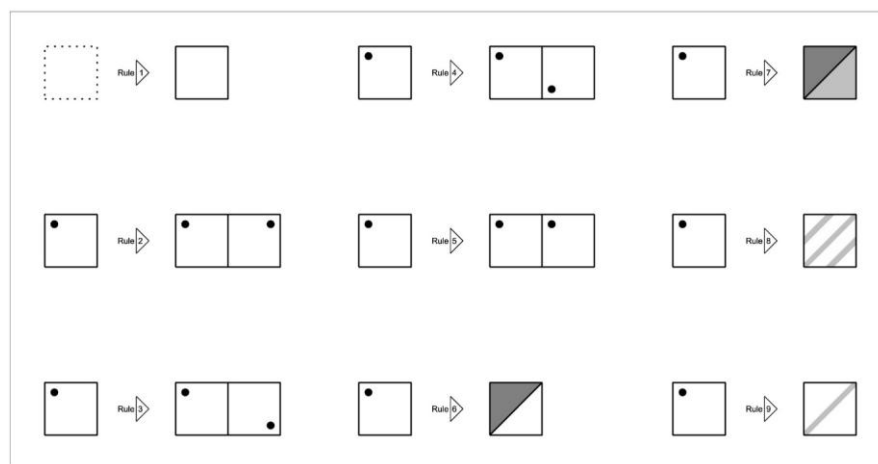


Figure 6. 5: Team 2 inferred shape grammar

This allowed the same body of results with fewer rules. Group 2 showed a more elegant grammar. This was also a more unpredictable grammar since the shape could be substituted easily and generate a wider set of examples. Additionally, the universe of solutions would be greater. (Fig. 6.5)

The main conclusion of this experiment is the fact that two grammars can generate the same results. However, and even though there is a common set of solutions, there is also a set of results that might be particular to one or two of the grammars and not common to both.

Nevertheless, the experiment show that there is more than one solution for one problem, and there is more than one grammar that enables to generate a set of results.

Likewise, the notion of two independent grammars generating the same corpus of solutions was put to practice with the Alternative Palladian grammar, to prove that a new set of rules, structure and grammar type could develop the same corpus of design that the Palladian grammar enabled to.

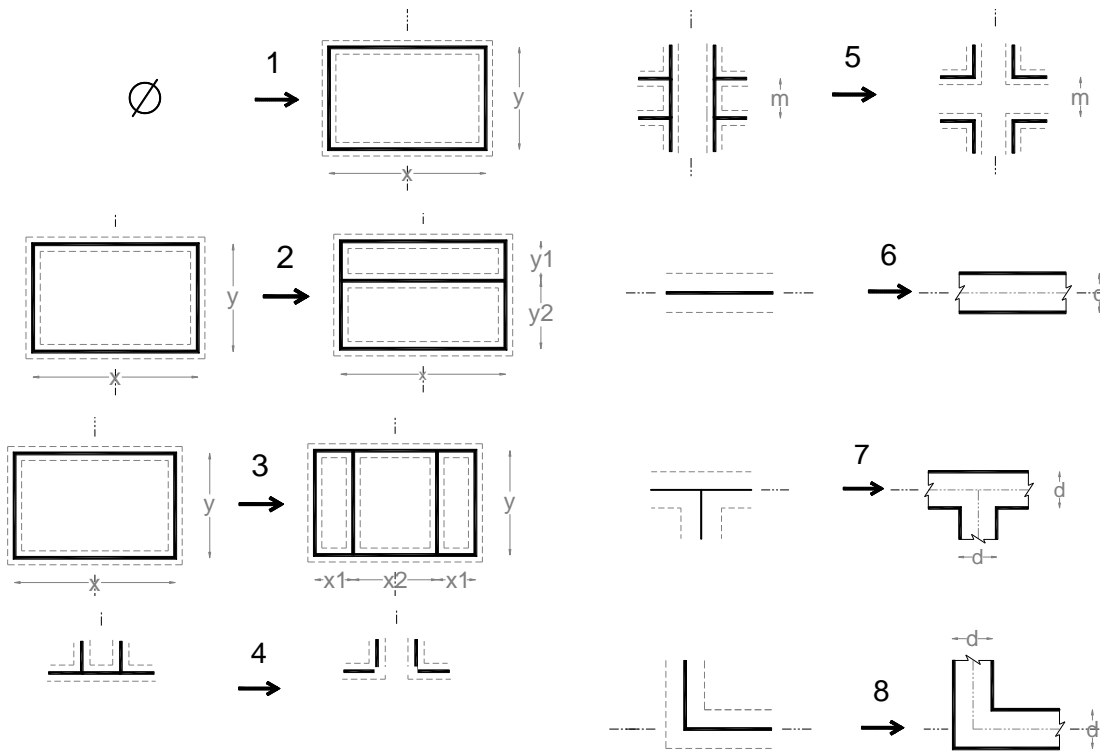


Figure 6. 6: Alternative set of rules for generating a Palladian grammar

A new approach was taken and a subdivision grammar was proposed. This subdivision grammar used the previously formulated generic shape rules as part of the shape rules set.

This contained for layout rules a small set of eight rules (Figure 6.6):

1. This shape grammar would start with the addition of the generic rectangular shape of a typical floorplan. This first rule would be an additive process characterized by the addition of a simple shape. The proportions of this initial shape would be driven from Palladio's original designs taking into account the proportional ratios used by him. Therefore, it would be a parametric shape rule conditioned to certain proportion ratios.
2. The second stage would vary from the first extracted grammar stepping into a subdivision process. Two rules would be incorporated at this stage. The first one (rule 2) proposes a parametric horizontal division perpendicular to the villa's symmetry axis.
3. The second subdivision rule or rule 3 had to keep the symmetric aspect of the design. It proposes a double vertical cut parallel to the symmetry axis. Both division rules (horizontal and vertical) allow embedding. This means that the rule can either be applied on the external peripheral rectangle, on a rectangular cell or even on a combination of cells as long as the requirements are matched such as the relative position of the symmetry axis. Both subdivision rules can be applied recursively and in a desired number of times until the cell size reaches the minimum desired area for a suitable housing compartment.

4. Rule 4 initiates the third stage which allows the adjustment of the layout by altering some of the border lines already designed. At this stage the rigid matrix of rectangular shaped cells can be altered to receive crossed, T and I shaped cells patent on Palladio's designs. This stage consists of two basic rules that address horizontal and vertical borders deletion.
5. The fifth shape rule allows the deletion of one horizontal cell border. This will contribute for a future possible creation of a merged cell area of T, I or X shape. The rule eliminates one cell border at a time so in order to come up with a more complex layout it has to be recursively applied. The grammar 5th rule allows the double deletion of two vertical walls confined within two cell borders and parallel to the axis. This way a larger more elaborate symmetric shape can be introduced leading to a cross shape. Similarly, if we erase a horizontal cell border (perpendicular to the axis) will come up with a T or I shape.
6. Features like the axial label and the maximum number of rows and columns on the plan design will suffice to manage the symmetric geometry of the shape merging process and avoid unwanted shape merging. On the original Palladian grammar the fourth stage allowed the wall realignment aiming towards a matrix detached from the original fixed, regular, orthogonal grid. However, in this proposal the grid does not constitutes a starting point and the alignment of internal compartmentation results directly from the design process of laying down the subdivisions designed. Therefore, it seems redundant in this case to allow realignment or shifting of any kind. Another difference constitutes the insertion of walls, both exterior and interior in a later stage. The simplicity and limited number of shape rules are enforced in order to allow embedding as mentioned before. It is important that the layout can be as changeable and permeable until the solution is clearly frozen. To impose wall definition beforehand would increase the complexity of shape rules, so design is left bordered by a fine line that will be later on replaced by the insertion of walls. Instead In this alternative grammar the fourth stage is destined for wall assignment. Each borderline is swapped for a wall placed with its axial centre coincident with the borderline and offsetting its limits inward the room or exterior. The wall thickness established follows what Palladio determined and also what was prescribed on the original Palladian grammar and is two vincentine feet, roughly by today's metric system 0.75 m. The proposed rules cover the possible situations from straight line layouts to T, L or crossed situations. The fifth stage is responsible for the functional assignment addressing zoning and functional distribution.
7. The seventh rule is responsible for the detailing T shaped wall junctions
8. The eight and last rule addresses corner wall edges.

The comparison, between the set of rules that constitute an alternative grammar for the design of Palladian villas and the original grammar raises important issues. Both generate the same corpus of designs, the original villas and the new set of design solutions. However, by exploring the universe of pre-existing examples some other design concepts were encapsulated and it seemed relevant to introduce these nuances on this new grammar. So it was possible to extend the universe of solutions in relation to the original grammar. Despite the solutions produced by Stiny and Mitchell's grammar a new corpus of solutions was introduced as shown in the following chapter. In addition, radio-centric villas like 'La rotonda' are also reproduced by the alternative grammar, which was one of the famous handicaps of previous grammars and design rule based systems (Hersey, G., 1992). Another issue noticed was a limited example of villas with smaller sized grids ranging from 2x3 to 3x4. These were also contemplated by this grammar. The villas generation process is replicated in the derivation process shown in Figure 6.1 where villas Angarano, Poiana, Pisani and Sarraceno are recreated using the set of 9 rules proposed. The previous grammar seems to omit these grids with even bays.

The design process and the succession of stages remains in principle very similar with minor corrections of stages positions (for instance the later insertion of the wall design).

Nevertheless, the greatest difference noticed was the set of design rules and ultimately the type of grammar used to describe the style. The subdivision process constitutes a more intuitive choice when facing an orthogonal geometry even when presenting an underlined grid. The subdivision allows a freer placement of a matrix that in a way resembles a grid but with less repetition and more flexibility.

It also seems evident that if a design language as Palladio's can be encapsulated by not one but two distinct grammars a higher hierarchical grammar might contain both discussed grammars and furthermore pointing out towards a generic grammar.

6.2 Original Palladian Villas Grammar vs. Alternative Grammar

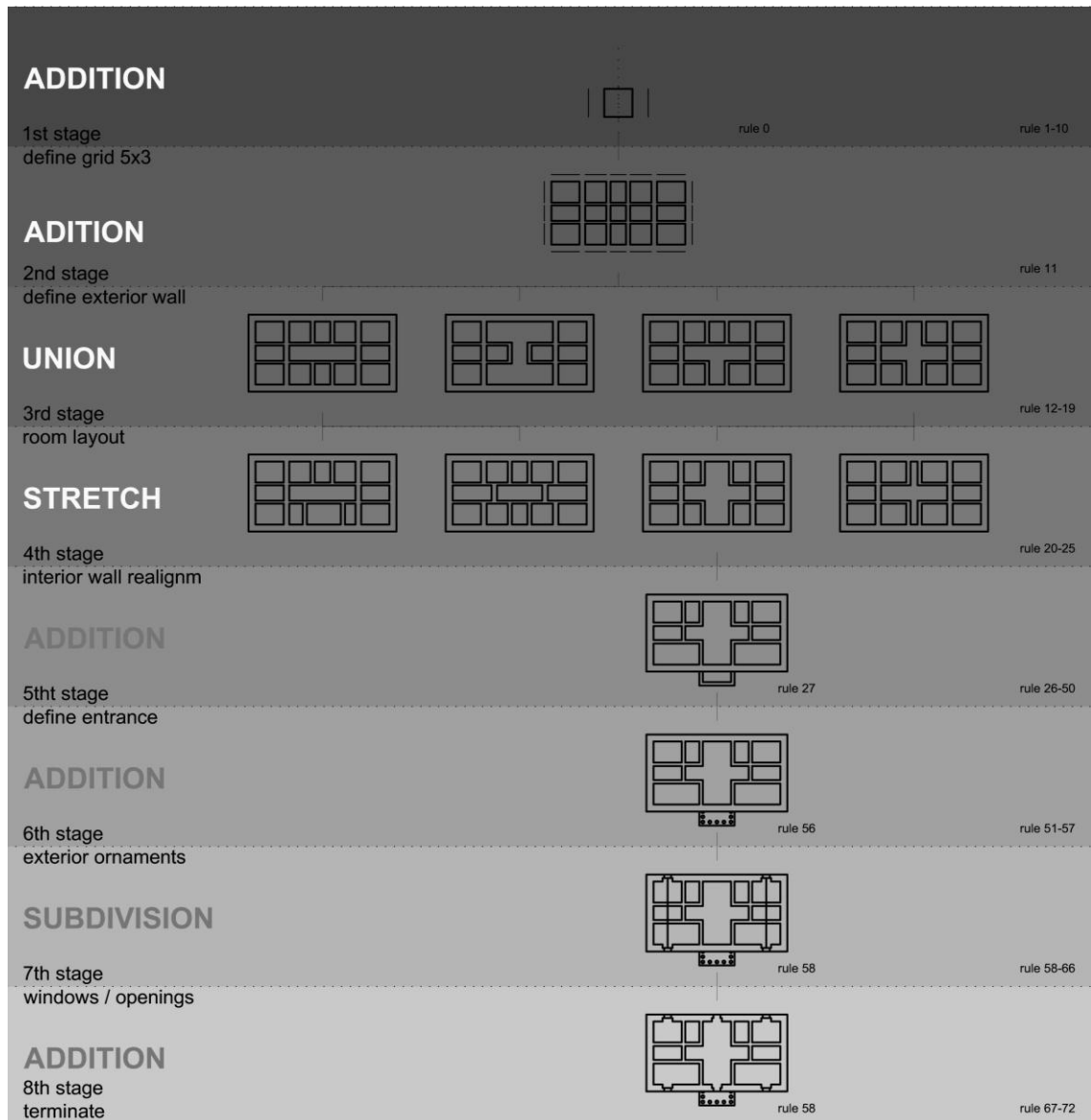


Figure 6. 7: Original Palladian Grammar Derivation Tree

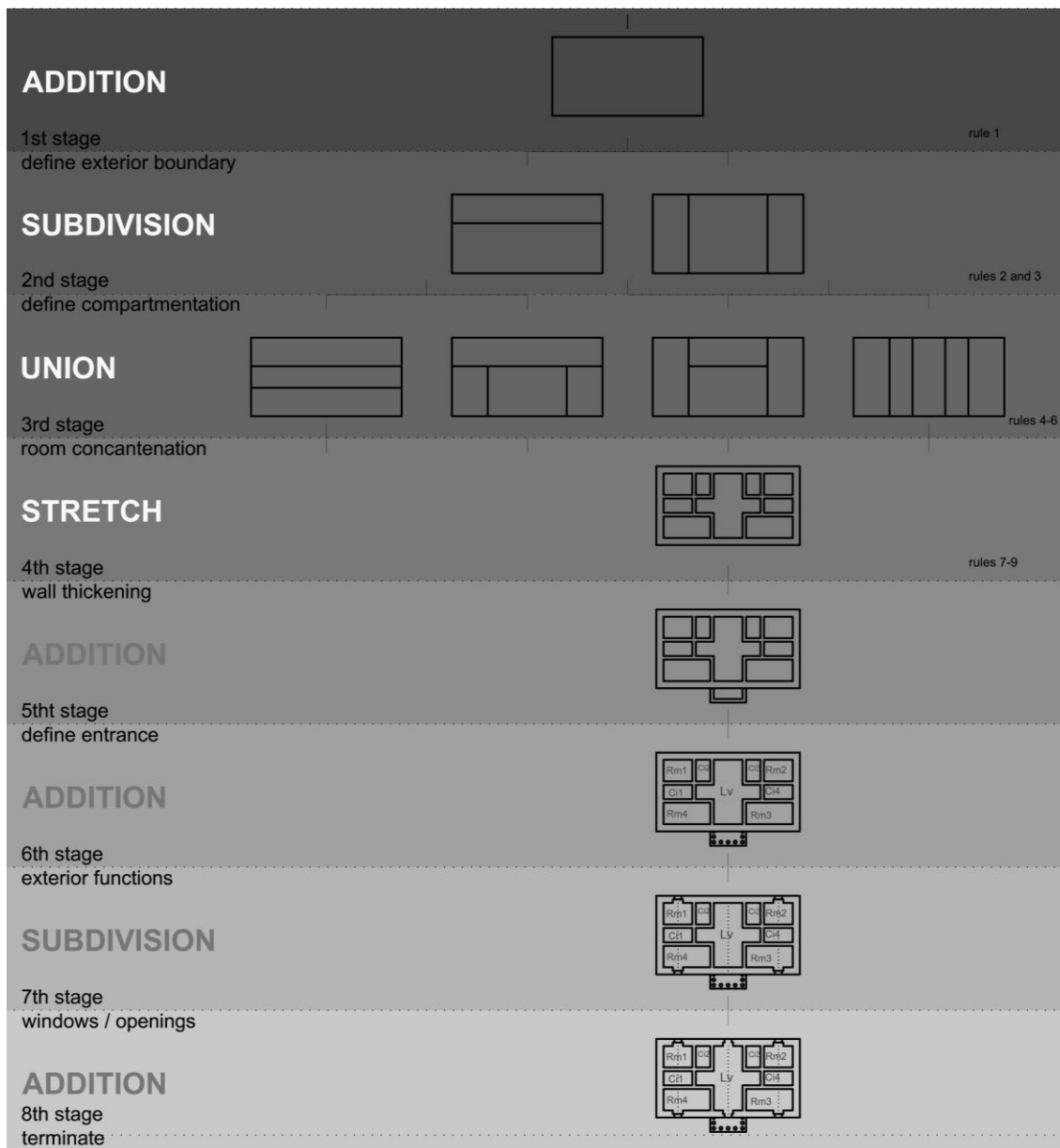


Figure 6. 8: Proposed Palladian Grammar Derivation Tree

The original scheme proposed 8 design stages (Figure 6.7) as does the proposed scheme (Figure 6.8).

The original grammar adopted a bottom up approach to design the basic layout the latter a top-down. This approach compromised an elaborate set of addition rules. These addition rules were responsible for:

1. First the design of a singular cell. The addition of the first cell constituted the first stage.
2. The second stage prompted the creation of the grid of cells. The first cell was then replicated by a series of additive processes of copy, mirror and translation in order to recreate a grid.
3. The third stage allowed for the concatenation of cells to design the entertainment core space. This is tagged in the tree diagram above as a conglomeration of union processes.
4. The fourth stage allowed the very rigid grid to be deformed and manipulated in accordance to the preferred proportion ratios proposed by Palladio.
5. The fifth stage introduces the portico, loggia or colonnade area. This constitutes an addition process.
6. The sixth stage creates the external ornamentation exemplified here by the colonnade.
7. The seventh stage proposes the inclusion of openings by a combination of subtraction rules. These subtraction rules allow door and window openings to be pierced and punctured within the internal walls and external envelope respectively.
8. The eighth stage is concluded with the deletion of labels.

The alternative grammar proposes a different strategy and formulation.

The structure of the formulation is diagrammatically presented in figure 6.9. The strategy behind the design of this alternative formulation was to allow the grammar a top-down approach. The villa design would start with the general and from that the progressive carving of detail would take place:

1. The initial stage sets the rectangular boundary (more complex shapes are also allowed by addition processes).
2. The second stage encompasses subdivision rules. It groups together the two subdivision shape rules for horizontal and vertical formulation. Vertical subdivision is performed in accordance with the symmetrical nature of the design, therefore introducing two division lines.
3. The third stage encompasses room concatenation for the creation of the main central space
4. Stage Four, which involves wall thickening, makes the basic two-dimensional outline. This stage deviates from the original grammar since in the latter the wall thickness was derived from the gaps between cells, whereas this grammar generates an abstract layout then makes it two-dimensional.

5. The fifth stage prompts the functional assignment. Adjacencies are created, spaces are named and labelled. This constitutes a novelty for a Palladian grammar.
6. The detailing with introduction of the entrance, consisting of a section extending from the core outwards to create an obtrusion constitutes the sixth stage. Whereas the oblong spaces are mainly circulation areas, the rectangular and square spaces opposite the façades are the living areas, and the central space is the social area.
7. The seventh stage involved the subtraction of volumes for openings. As previously stated, the windows are arranged in even numbers and the doors in uneven numbers.
8. Stage eight terminates the design

This is believed to be a more intuitive and expedite solution to recreate the Palladian villas. The problem of proportions, to be discussed in the following sections, was overcome by the imposition of parametric shape rules that condition the inclusion of unwanted proportions. That allowed the detachment of the underlined grid used in the original grammar which served as a guarantee of proportional standard. Without the complex set of shape rules to create the grid and manipulate its proportions the design of a viable solution is believed to be faster.

The first shape to be designed, the house outline, is based on the fixed proportions of either 1:1, 2:3, 4:3, 3:5, 4:5, 3:7. This subtly underlines an interior with a number of subdivisions allowed. The number of bays East to west is always an odd number. The sub-consequent rooms will maintain proportions that are seen in Palladio's work 1:1, $1:\sqrt{2}$, 2:3, 4:3, 1:2.

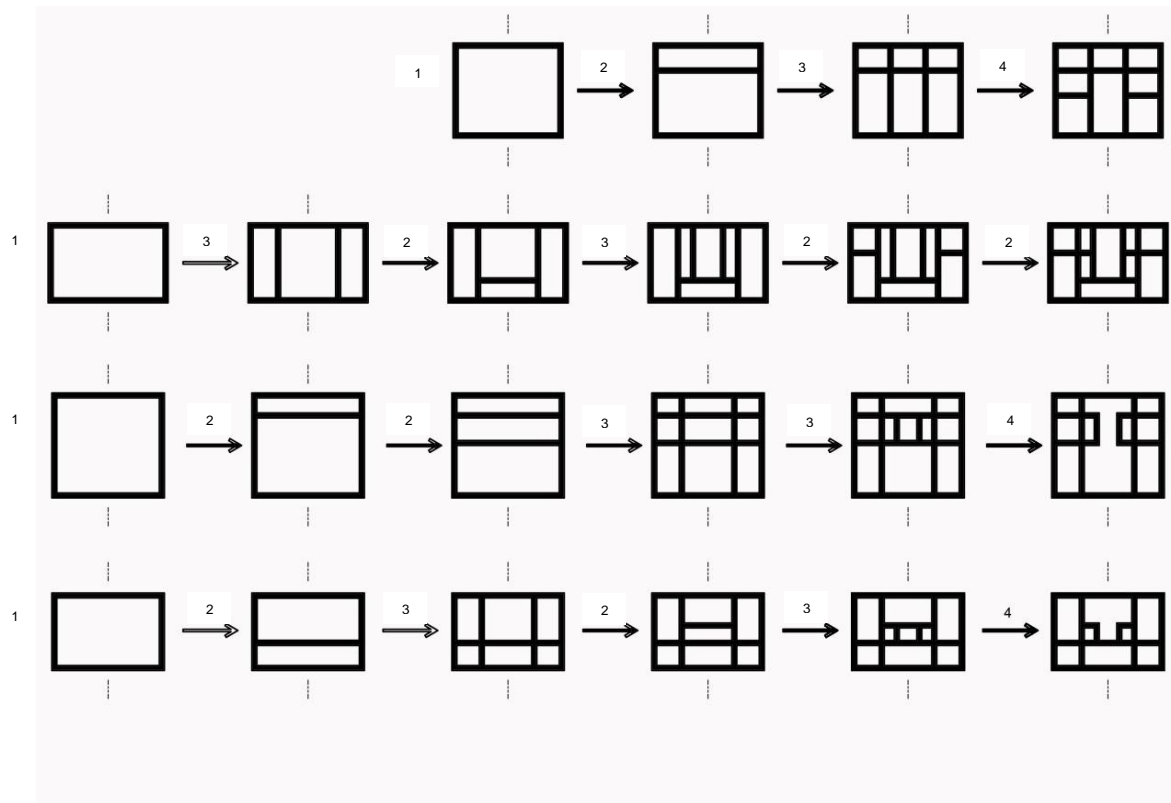


Figure 6. 9: Alternative Palladian grammar Derivation for Villa Angarano, Poiana, Pisani and Sarraceno

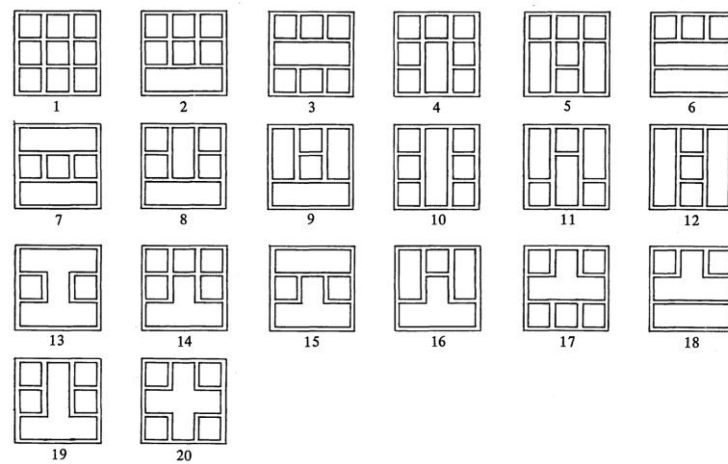


Figure 6. 10: The Palladian grammar universe of solutions for grids 3x3 in (G Stiny & W J Mitchell 1978)

6.3 Alternative Grammar type and classification

The design process begins with the overall shape and evolves to further detailing using a top-down approach. The self-contained boundary is set and a series of tasks are carried out to refine the design. The starting point for the proposed alternative grammar was the exploration previously inferred grammars. Using Knight's grammar classification, a case study was selected using a selection of Palladian villas that also composed the reflection of previously inferred grammars (G. Stiny and W Mitchell, 1978a, 1978b). The grammar structures were compared and the results led to a proposal that addressed the basics of housing design and was related to Palladio's signature design.

The existing villas were observed, visited and material gathered from different types. Some of the villas studied are shown in picture 6.1. Certain generic features describe the family of designs known as Palladian villas: a rectangular envelope constitutes the boundary and the villas are composed of a raised ground floor which contains the main social areas. A lower level ground floor contains the service areas, pantries, kitchens and servants quarters. The upper level contains living and bedroom areas for the house owner and family. The main floor is raised by steps and has more headroom. This is the only floor plan usually illustrated in Palladio's writings. The Palladian floor plan has been studied by art historians and is easily identified. The symmetry between the east and west wings is created by the north/south axis. The larger core area in the geometric centre of the construction has a complex shape that is the result of cell merging. Windows (like the eyes in the human face) are arranged in even numbers whereas doors are centred and arranged in uneven numbers.

The main entrance is defined by a portico supported by columns which enhances classical features of the façade that are typical of Renaissance/Maneiristic architecture. The preferred proportions for the design of rooms, service and circulation areas are usually rectangular or oblong spaces with $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ ratios, whereas the main rooms are square or regular symmetrical polygons or rectangular areas with a $\frac{2}{3}$ ratio (a proportion frequently adopted by established architects such as Aalto and Siza). The Malcontenta, Angarano, Badoer, Zeno, Pisani, Emo, Sarraceno, Ragona and Poiana villas were used as case studies for rule extraction. Except villa Angarano, they all have a 3x5 grid. The proportional ratio of these villas is shown in Figure 6.11.

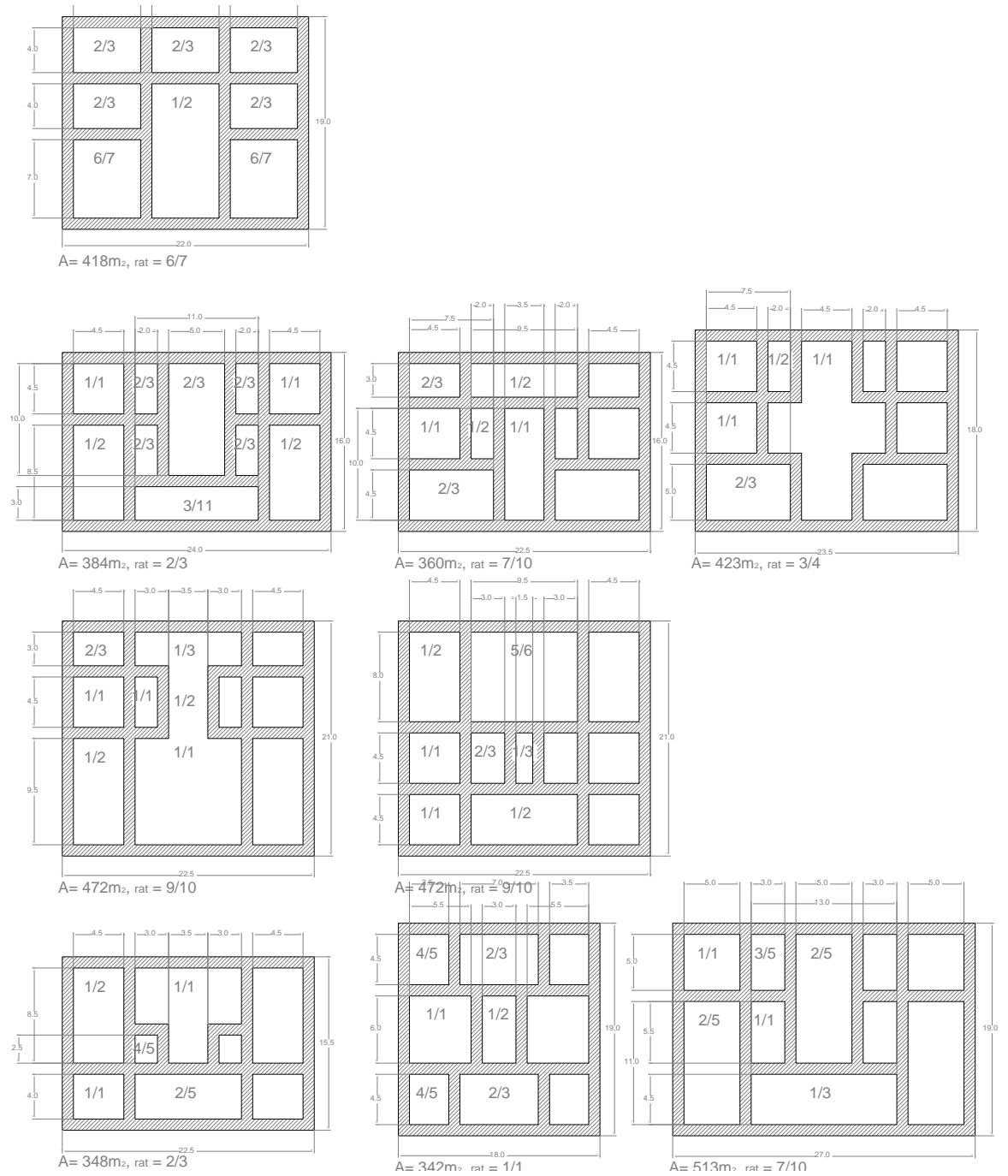


Figure 6. 11: Original Palladian Villas and their internal proportional ratios

6.4 Rule System

The rule system proposes three types of rules: addition, subdivision and adjustment (figure 6.12). The first type is represented by the inclusion of the outline. This rule proposes the addition of a rectangular overall shape, with parameterization based on the minimum and maximum acceptable areas.

Subdivision rules include vertical and horizontal rules, as exemplified in Rules Two and Three. These parametric rules represent the main operative process, allow for embedding (applied repetitively inwards or outwards) and can be used recursively (applied a number of times consecutively until the minimum usable space is attained). Subdivision rules respect the symmetrical design, hence the duplication of division in Rule Three.

Concatenation rules follow subdivision and, although independent, are intrinsically related. Once the division has been executed, concatenation is used to arrange larger, complex spaces connecting adjacent rooms. To avoid unwanted space merging, these are applied only to borders, taking the adjacencies and symmetrical relations imposed by labels into consideration. The resulting space is displayed as an enclosed polygon. The west and east wings will be symmetrical. Deletion rules allow for the creation of complex spaces or a regular polygon boundary. Border deletion is achieved by using the forth (horizontal) and fifth (vertical) rules. Wall thickness conforms to Palladio's specifications as observed in site visits, roughly 1200mm for external walls and 900 for internal masonry walls. The stone masonry construction used load bearing sheer walls as a tectonic method for envelope enclosure. The thickness of these walls is based on know-how and a generous safety coefficient that matches the construction grid lines and units. A depth of two Vicentine feet is prescribed for the wall thickness (roughly 750 mm using the metric system). The process is described in Rule Six, using an offsetting method. The desired wall centre will sit on top of the abstract layout. Wall intersections are addressed in Rules 6 to 8, which propose the thickening of T, X, L combinations. Other addition rules add details, such as the entrance feature (porticos or loggias). These sets of rules are followed by other addition rules involving the inclusion of functions. The last set of rules provides for alterations using volume subtraction to include openings. They involve the positioning of windows, which are arranged symmetrically, or the positioning of doors. Uneven numbers of external doors are centred in the façade and internal doors are created to connect rooms.

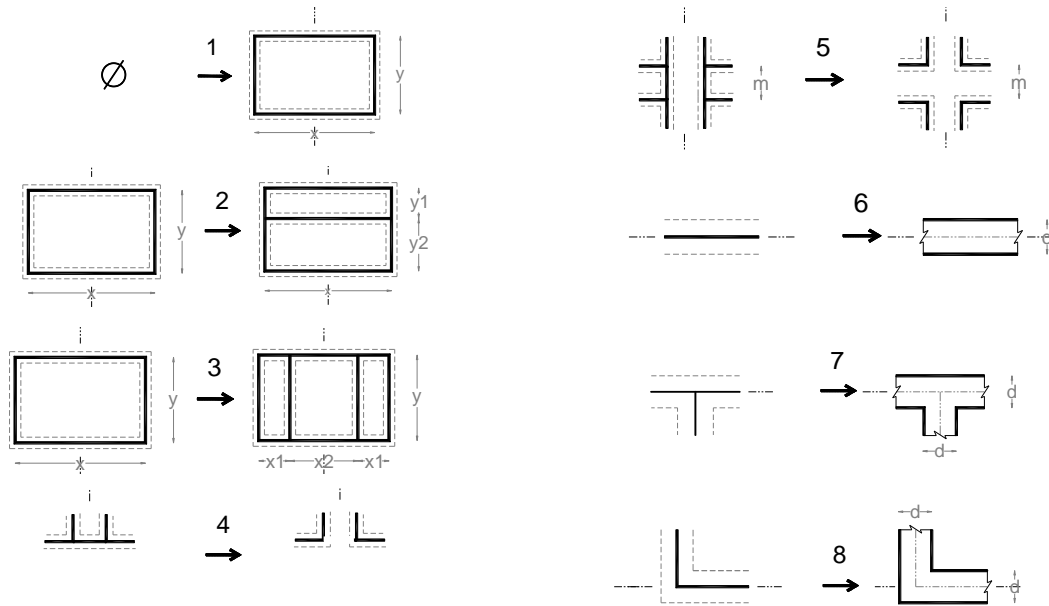


Figure 6.12: Proposed Shape Rules

6.4.1. Wall placement:

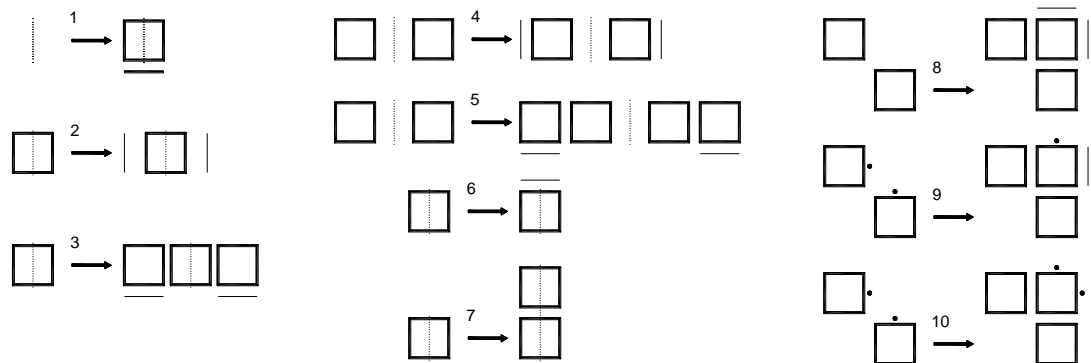


Figure 6.13: Cell rules 1-10 for the Palladian grammar adapted from (Stiny & Mitchell 1978)

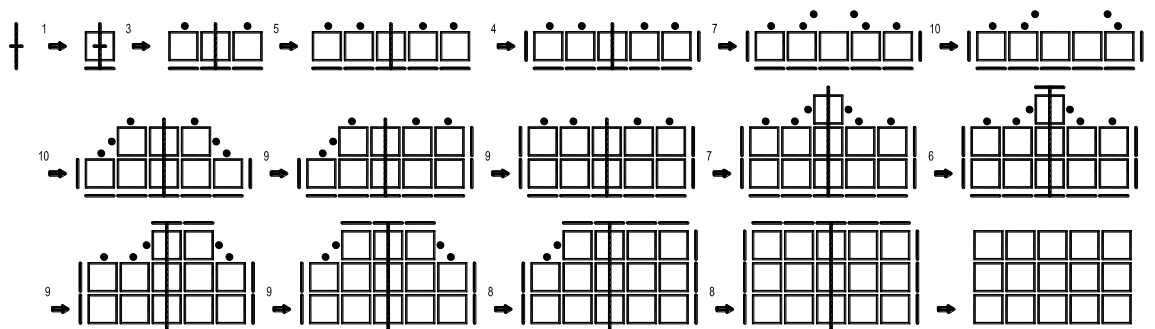


Figure 6.14: Application of the Palladian grammar rules to generate a 5x3 grid floorplan

The original grammar for the Palladian villas was based on a grid process. Understandably grid does not qualify the building system used in these villas and its structural concept, but describes the way the design emerges by a process of cell placement. This cell placement creates an abstract grid that serves as base point for the further development of the house layout.

To achieve this Stiny and Mitchell developed a series of shape rules to generate this grid. This process is also far from being conventional. To achieve the grid a set of addition rules is used and one by one cells are added until the resulting grid is accomplished as shown in figures 6.13 and 6.14.

It starts with rule one. This not only places the cell corresponding to the central bay but also the symmetry axis and the required labels.

The labels to note are the adjacency label represented by a small dot allowing for the introduction of a neighbour cell or the line representing a limit and the external wall.

On the first shape rule two labels are included, the symmetry axis and the label representing the boundary line. Similarly, the second rule also places a central cell but bound to limit labels. Sequentially rule 3 adds 2 new cells alternatively in both sides symmetrically. These cells are not perfectly aligned with the central bay cell but offset of a given gap distance. This gap distance has one purpose to create the necessary space for the wall thickness in between.

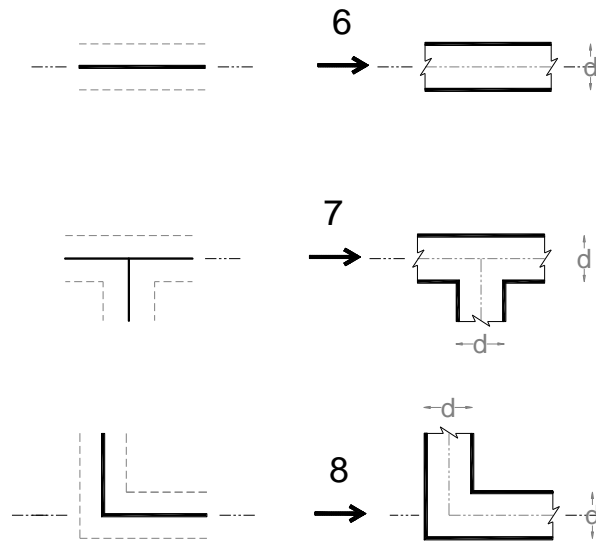


Figure 6. 15: Alternative Palladian grammar wall placement shape rules

Similarly rule 4 and 5 apply the same principles of symmetry but allow other instances to occur like the cell placement by using the axis as reference. While rule 4 prepares the grid for the design of the next row by placing limit cells at the extremes, rule 5 allows further iterations within the same row placing limit labels on the bottom edge.

These rules are followed by rules 6 to 8 which are responsible for wall placement, wall junctions and resolving corners or terminating rows. The attentive use of these referring always to the dot, symmetry and limit labels allow for the correct placement of cells and completion of grid.

The same does not apply in the alternative grammar formulation. While the original grammar automatically derives the wall thickness from the cell position, this proposed formulation is an abstract outline of the final layout. The layout to this point is schematic and one dimensional.

To achieve the level of detail required the lines will give place to double lines offset and spaced according to Palladio's real wall thicknesses. This is illustrated above in 6.16 with rules 7-9. As reported in 'il Quattro libri' wall thicknesses should be a module of the vincentine grid usually two units. To achieve this principle four rules were included that transform a single line into a wall representation.

Respectively rules 6 to 9 oversee the possible wall thickening scenarios that one could face. Rule 6 transforms a single line into 2 parallel lines that replicate a wall representation. Rule 7 a T junction or wall corner. Rule 8 a crossed junction and rule 9 a corner detail or an L shaped configuration. These rules should be replicated symmetrically taken into account the symmetry axis, however its obligatory application makes it unavoidable.

This explanation covers the positioning and determination of wall representation. However does not explain in detail how the wall confinement is settled.

The wall confinement is conferred of great importance since it is responsible for the settlement of the right proportions of rooms. The proportions are mentioned by Palladio himself and also observed in his projected and built work. It is known the classical predilection for golden ratios

(ratios derived from squared shapes or its lateral projections through an arc). These golden ratios are described by many including Vitruvius.

Within the most common rectangular ratios used by Palladio we can find the 1:1 1:2, 1:3, 2:3, 3:4, $1:\sqrt{2}$, $1:\sqrt{4}$.

Another important feature often discarded and not much discussed but observed in our research is the modular feature of these designs. It seems possible that Palladio used a unitary system to design in a modular way where the vincentine foot was the base unit. Walls observe a 2 feet measurement, doors 3 feet, the minimum sized room as mentioned in his writings is 8 feet and so on. This can be seen in the parameterization provided for each rule in table 6.1. All shape rules follow this modular notion.

The proposed alternative grammar is based on the subdivision principle. Divisions take place recursively in order to design the house. These divisions will further on define the room confinement. This means that the distances between each division have to relate to the preferred proportions in order to create acceptable rooms.

It is also referred in the section above that the shape rules used are parametric and therefore support certain parameterizations and different measurements. However they still have to abide to the modular feature described.

Therefore if we overlap a notional grid this grid would show a tartan distribution of equally spaced lines distancing one vincentine foot from each other. This grid constitutes a base point for the subdivision. The computer tool efficiently utilizes this grid to guide and offset each subdivision to the grid alignment.

6.4.2. Door placement:

Door and windows placement are important characteristic to take into account when analysing Palladian villas. These elements have an important role both in the spatial articulation allowing communication between adjacent spaces and maintaining the language perpetrated by Palladio. These constitute important compositional elements both in the villa's interior and with an important reflection into the façades.

Internal doors are usually centred within the circulation area and windows symmetrically positioned in the façade, taking a vertical centre line as axis. There are different window sizes for the different levels, the larger sizes being on the main floor, followed by the top level, whilst the smaller windows are on the lower ground floor. The design is completed with a label deletion (a technical stage), which is the eighth and final stage.

As mentioned before the notion of symmetry occupies a main role in the house composition both in floorplan (east and west wings are mirrored compositions) and in the façades. Palladio destined a chapter of the 'Quattro libri' to discuss the anthropomorphic features of the main façade comparing it to the human face. Just like the face the main façade should have two main composition elements, one pair of eyes (even numbers) materialized by the windows allowing

the viewing reference and mirrored and a mouth (in odd number) centred allowing the in and out flow materialized by the main door.

Another important concept suggested by Palladio is that 'void should overlap void and mass should overlap mass'. This means that doors and windows whenever possible should align in plan so that the front view and façade should have this composition of voids and mass.

In many villas is possible to visualize through the main building from the entrance to the back access through the interior (as examples villa Poiana and Emo). This feature allowed for easy indoor outdoor access in many of the rural villas. This is exemplified in villas such as Emo, Poiana, Pisani to name a few emphasises the principle that void should overlap void.

Like any other design, plan and elevation are constructed simultaneously. The grammar sequence allows the room planning and layout followed by the positioning of the main entrance. The main entrance constitutes a key feature in the whole construction. Often this element creates an obtrusion supported and decorated by a colonnade, or a series of columns. The columns are included centred within the façade and in even numbers. This allows the entrance to be aligned with the symmetry axis and framed by the columns. The main door occupies the centre point. This suits the internal layout allowing an easy access towards the central bay and therefore a straight communication with the core social space. Opposite to the main façade another secondary entrance with more humble composition.

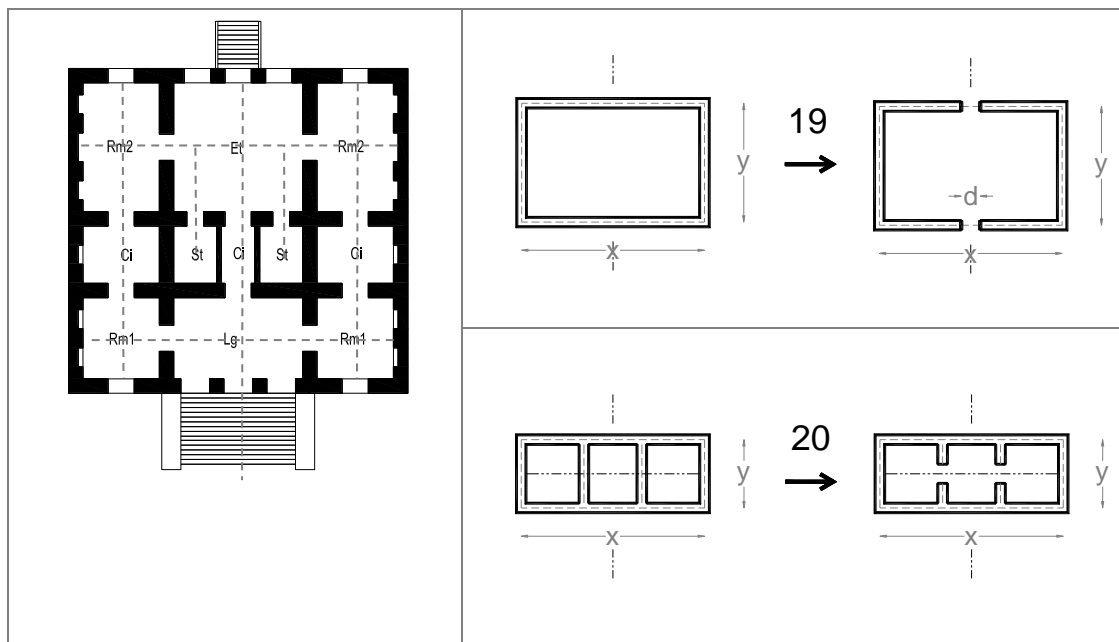


Figure 6. 16: Door openings alignment (Illustrated on the left villa Emo overlapped with the door alignment axes in dashed, on the right rules 19 and rules 20 of the alternative grammar illustrating how the entrance and rear entrance door is introduced aligned with the symmetry axis and the alignment of internal doors centred within the rooms.)

The internal doors of rooms that lie on top of the north-south axis align straight with the main entrance. This follows the rule established by Palladio of mass/void alignment. Parallel axis to the north-south can be drawn in the west or east wing by the external bay. This second axis

should be centred with the smallest room existent in the external bay. In order to keep the symmetry, the same should be replicated in the opposite wing.

Perpendicular to this north-south axis, a west to east axis should be drawn centred with the core room. Whichever walls this axis trespasses a door opening should be drawn allowing the adjacent rooms to communicate. A parallel to this west to east axis should be drawn to every horizontal bay of the villa floorplan centred with the smallest room it cuts. This exclude rooms destined for vertical circulation since they only allow access from one end usually from the central bay outward. This should conclude the internal wall design.

6.4.3. Window placement:

With the internal doors designed the next step is window inclusion. These will reflect profoundly into the elevations helping to determine the overall appearance of the house. Palladio dedicated a chapter describing how the external façades should look like in his treaty. Understandably this has a great impact into the whole house setting out including the floorplan. This way plan and elevation are in-dissociable and need to be closely articulated in a sequential manner. Palladio described the villas façades as a product based on human features. As explained above, this translates into an even number of windows. There are three ways to place windows. The first relates directly with the portico design. The main entrance is framed by a portico or a loggia. This framing is executed by the support system of the entrance element. The support is provided by an even number of columns obeying to the Doric, Ionic or Corinthian classic ornate style. These are the three great styles of columns as prescribed by Vitruvius (Vitruvius 2001).

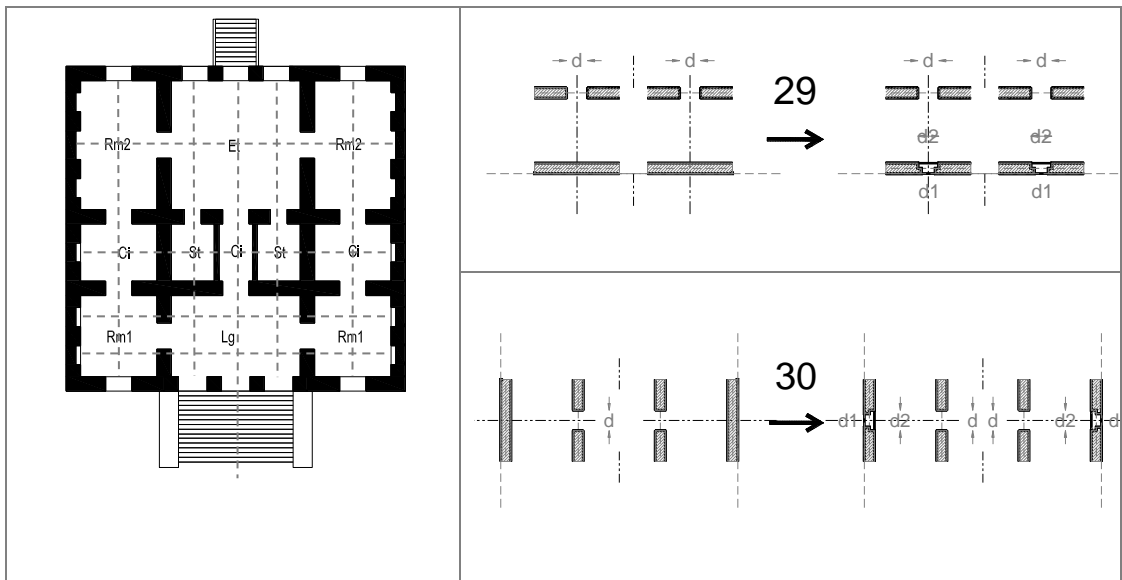


Figure 6. 17: Sequence of window openings design (Illustrated on the left villa Emo overlapped with the window alignment axes in dashed, on the right rules 29 and 30 of the alternative grammar illustrating the introduction of windows in the main façades (north and south) centrally aligned with the internal doors, and windows in the west and east façade following the same principle, respectively)

The columns are shown in even numbers because they not only frame the entrance, but also provide access through the central free bay towards the main door and also frame windows in between a set of a pair of columns as illustrated. Overall were identified a set number of columns that Palladio commonly used: either four columns (the most frequent option as seen in Ragona and Pisani), six (like villa Caldogno) or eight (like in villa Sarego) (figure 6.1). The second window construction method derives from the plan analysis.

From the door openings alignment that axis should project all the way to the façade and create a window opening. This should only happen after the windows in the central bays were created by the first method. The third and last method relies on symmetry. East and west wings should reflect the same scheme which implies that a mirroring of the window elements should take place. The window placement rules can be found in Figure 6.17.

Other three rules apply in order to design façade openings:

- Every room adjacent to a façade should have at least one window. Window numbers range from one, two, four or six as maximum.
- Most windows connect straight with the exterior, however occasionally a window can connect to the exterior covered access space when aligned in the central bay. There are also built cases where Palladio applies a walled-in decorative window just to keep the façade harmonic composition or simply create a sculpture niche as famously pictured in villas Poiana and Pisani.
- The last rule to be described with regards to the window composition is the proportion. With no exceptions the window sizes are calculated taking into account the distance between columns. The windows are always elongated rectangles placed with the height as the largest dimension. The windows should be equally distanced between the pair of columns and its width should be half the distance between columns. Its height should reflect a proportion of 2 times $\frac{1}{6}$ of the width. This applies only to the main raised ground floor. Upper level windows should reflect a proportion that varies with the floor to ceiling height which is reduced as we go up. Palladio expresses the height variation clearly 'the lower will have to be one fifth higher than the upper one' (Palladio 1997). This increased the perspective distortion perception.

6.4.4. Staircases:

The first book of Palladio's treaty devotes a section to the design of staircases. However, this section is targeted to internal staircases and no example is dedicated to external stairs. The staircases pictured in this section are Palladio's recreations and in some cases inventions that were tested in some of his work. Most of them are not typical classical elements which probably fits within the Mannerism style more than the Renaissance. There are spiral, elbow and crossing straight flights of stairs, and most surprisingly elliptical spiral stairs. Ellipses are not classical shapes and were not included in canonical classical Greek or Roman architecture. These elliptical stairs were used in villas Cornaro and Pisani Montagnana as internal vertical circulation.

In all villas observed the structure repeats itself. There is a '*piano nobile*' where the main social functions take place. This is a raised ground floor and its finished floor level varies from house to house depending on the site conditions and on the height of the lower ground level. Regardless of the difference of internal and external heights a grand staircase is always required. First to mitigate the level difference, second to create an expectation and a scenic effect. The staircase is used as integral part of the villa's functional ornamentation.

The reason why Palladio did not dedicate a section of his treaty to this element is most likely because he used classical principals inspired from Vitruvius's ten books of architecture that he was commissioned to translate in collaboration with one of his most devoted clients.

Most houses despite its Manneristic approach have a classical façade. In a sense the true innovation lies in the floorplan and internal features.

From the villas observed, most show straight stairs like villa Emo shown above. Others like Malcontenta showcase an elbow twin staircase by the river Brenta facing façade where most guests would arrive by boat from Venice.

Identified within the corpus of existing/documented villas were the following types of stairs:

- Simple straight
- Capped straight
- Twin flights
- Twin elbow flights
- Splayed straight flights
- Splayed round flights

The design staircases occupy a section in the shape grammar rule set with shape rules 31 to 36 forming part of stage 8 as pictured in Table 6.1.

6.4.5. Loggias:

Part of the detailing stage is the loggia design. The loggias are another important feature in the renaissance architecture that comprises a covered outdoor space that works as a transition between outdoor and indoor. In Palladio's work the loggias are of two kinds the recessed/inscribed within the villa bounding box and the projected outdoors.

The loggias can also show different configurations despite their relative position. The loggias in the Palladian villas vary from rectangular (with 1:1 to 3:1 ratios) to 'T' or 'I' shapes.

Most often than not they are articulated with the main event space or the core room. For that reason, they sit above the axis in floorplan and are symmetrical. Normally each villa exhibits only one loggia, often in the main façade facing south. But there are exceptions like villa Valmarana, Pisani Bonetti and Badoer that present two loggias in opposite facades. This can be explained by the relative position within the plot. Some of these villas have dual entrance, a main entrance facing the access and an 'entertainment access' that faces the river or main gardens where the guest entrance was placed. In these cases, the two loggias would sit opposite on the same axis.

The exception to this rule is once more Rotonda which allocates loggias with porticos in all four facades stressing the double symmetry nature of the floorplan. This is a particular feature since all Palladian villas show bisymmetry along the North-South axis only. 'La rotonda' shows two perpendicular axis of symmetry.

In the set of rules created the loggias related rules are 37 to 40.

6.4.6. Colonnade:

The last section of the detailing stage is the insertion of the colonnade or '*colonatta*'. This is the name given to a series of columns displayed on the edge of a loggia or simply supporting a terrace or portico.

Palladio dedicates his first book to the correct design, first of the column, and second the colonatta. The four books are arranged from the particular to the general, starting with one specific architectural element, to a set of complex elements as stairs and concluding with villas designs and other engineering and architectural constructions in the last book.

In the first book he describes the three styles of columns Ionic, Doric and Corinthic with attached dimensions and proportions and carries out describing their use in colonnades and required spacings.

Apart from classical colonnades he also uses arched colonnades of square section and 'Serlianas' (the Serliana window was developed by Serlio an early Mannerist who preceded Palladio's work in a couple of decades, the Serliana consists on a main circular arch doubled by two smaller arches in either side).

In his villas he uses series of 4, 6 and 8 columns. These always in even numbers allow for a straight entrance line which is centred within the colonnade.

There are also variations within the colonnade termination. These could be free standing or half trimmed by an incoming wall or side wall. Additional columns can create the returns along the opposite façade.

These colonnades often resemble classical architecture specially when topped by a triangular portico. The rules that enable the design the colonnades are represented in the grammar between rules 43 and 48.

These terminate stage 8 and conclude the design of the floorplan.

Not included in this study is the design of the façade.

Other elements are key in the design of these that are of significance for the design of Palladian elevations.

Other architectural elements deserve a special attention and description since they seem to follow a common canon of design both in designed and erected Palladian villas, such as:

- Porticos
- Pediment, entablure and base
- Roof

In conclusion we can combine a elementary set of rules to be foreseen, this encompasses: planar west to east symmetry and elevation vertical symmetry, predominance of rectangular floorplans and rectangular rooms of ratio not larger than 2:1, a larger room coincident with the axial line with configurations ranging from rectangular to cross, T or I shaped, door and window openings aligned and on axis parallel to the main construction walls, no rooms with dimensions smaller than 8 Vicentine feet (approximate 2.8 m), a central bay centred within the floorplan allowing a centred access.

The rules above described are illustrated as shape rules in the upcoming pages in Table. 6.1:

6.5 Palladian functional rules

This section addresses the set of rules developed to assign function to Palladian villas. As discussed previously Palladio left an elaborate and complete set of indications, suggestions, built examples and drawings that describe not only his signature style but what he thought was good practice worth sharing. He dedicated quite a big portion of his treaty to proportions, composition and to several architectural elements within the design however very little addresses function specifically.

Information was gathered by the form of images, pictures and drawings of built villas which helped identified how the space was used. Some pictures showed furniture and other elements which facilitated the analysis. Additionally, visits to villas Caldogno, Poiana, Rotonda, Valmarana, Barbaro, Emo, Cornaro, Pisani and la Malcontenta, helped visualizing the space and identifying functions and typological features that relate to uses. It was also considered the space itself and among each house the spatial features and location of each room was considered in order to 'assign' or determine which function was held in each space.

After analysing several examples, it was possible to draw some conclusions.

There is a hierarchical partition of spaces. The central space of the house has larger proportions and most of the other spaces are arranged radially towards this one. This also means that this space is highly connected and easily accessible from almost all other divisions. This space also benefits from easy access from the outside and often connects directly or in straight line via a hall or lobby to the main entrance. Taking into account the connectivity of the space, its size and proportion and one of Palladio's rare references to spatial context this room is clearly destined to entertainment with an important social role.

From a size point of view, the rooms adjacent to the main façade are the largest right after the core room and benefit of natural daylight and external views. These rooms with rectangular profile and ratios no larger than 2:1 are destined to house functions like living areas, work places such as studies and libraries, drawing and parlour rooms and eventually dining rooms when associated with other common areas. The smaller rooms occupy the lowest rank in the hierarchy or simply have a more straightforward occupation. Smaller rooms lying on top of the symmetry axis provide halls or lobbies for spatial distribution and weather buffer. Rooms of small size adjacent on a central bay perpendicular to the entrance are often corridors or small connectivity antechambers between the core space and the peripheral rooms. Rooms in central bays around the core in larger sized villas (often with an elongated, elliptical or triangular profile) are destined to accommodate vertical circulation like staircases.

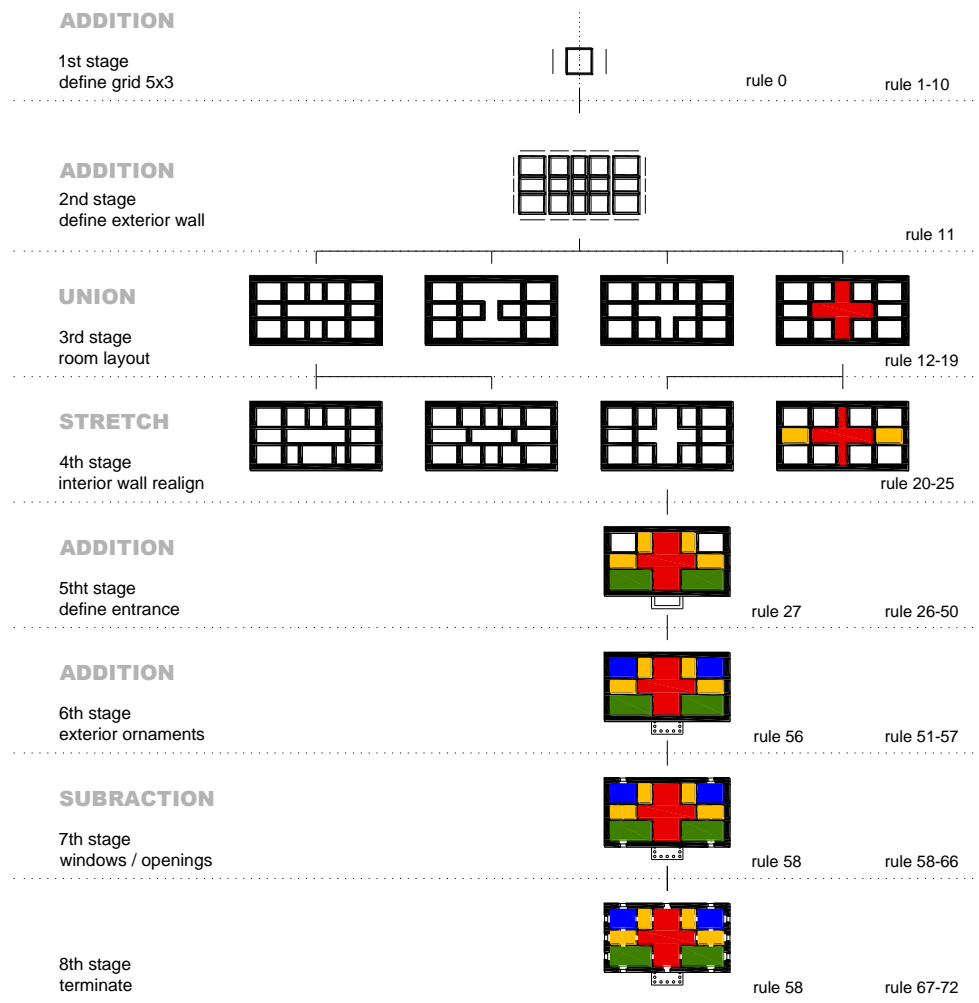


Figure 6. 18: Functional derivation of Palladian grammar

Taking these notions into account, is possible to illustrate the derivation of the functional assignment for the Palladian villas (Figure 6.18).

1. The derivation starts with the insertion of the bounding box that illustrates the villa limit. The second stage allows the application of 2 rules. These rules allow the subdivision of the footprint into two or three spaces using respectively a horizontal division or a double vertical division. With this operation two functional zones are allocated living (L) and entertainment (E). (E) occupies the most central spot as explained and/or the largest divided area. (L) occupies the most peripheral area adjacent to the façade to allow for natural daylight. (L) occupies the surround (E) for the three spit result.
2. The subdivision process will carry on for a number of steps until at least three bays orientated north to south are included and at least two bays spanning from west to east. The next stage on the derivation diagram provided proposes to the previous partial solutions another iteration of the division rules application. This step allows the creation of a number of bays into the house floorplan. The functional assignment to these

designed spaces will obey what was observed and prescribed above namely to what occurs to central east to west bays and north to south bays. During this third step circulation is introduced with the tag (C). This spatial allocation tag will be assigned to any interior bays between entertainment and living spaces tagged respectively with (E) or (L). Circulation can also be assigned to central bays adjacent to the façade if they are coincident with the symmetry axis or the mid axis perpendicular either the north to south or the west to east axis). This means that circulation can be assigned in transition spaces that connect two living spaces or the house exterior access towards the core room. As the diagram shows circulation can be assigned to create an entrance lobby or hall or to create buffer areas between the core space and living spaces.

3. The following steps allow the recursive application of subdivision up to the completion of the interior layout and spatial assignment. According to the original Palladian grammar the subdivision process was allowed and demonstrated up to the inclusion of 5 bays from west to east and 3 bays north to south. An alternative grammar encompasses a more flexible layout while also abiding to Palladio's design system. Observation showed that villas such as Cornaro and Barbaro have a grid of 4x3 and villas like Gazzotti, Mocenigo and Valmarana support a grid of 4x5. Some of these examples are illustrated in 'Il Quattro Libri '. Therefore, it seemed acceptable to allow for 4x5 as a maximum number of bays. The subdivision rules incorporate that and allow for this as a maximum. This number of bays translated into number of subdivisions results into five, 3 horizontal divisions and 2 double symmetrical vertical divisions. In other words, the development of the division process in the worst case scenario can take up to 5 stages accompanied with the spatial assignment.
4. The following step relates with spatial concatenation. This when associated with spatial function acquires special shape rules conditions. If the spatial configuration differs from a rectangle in footprint this determines only one use, entertainment. If a result of a spatial merging is an I, T or cross shaped it should have a (E) tag. Similarly, if the space has a proportion of ratio larger than 2:1 this space should be allocated for circulation. The last step from the functional assignment point of view relates with the positioning of the main access or entrance and that terminates this task.

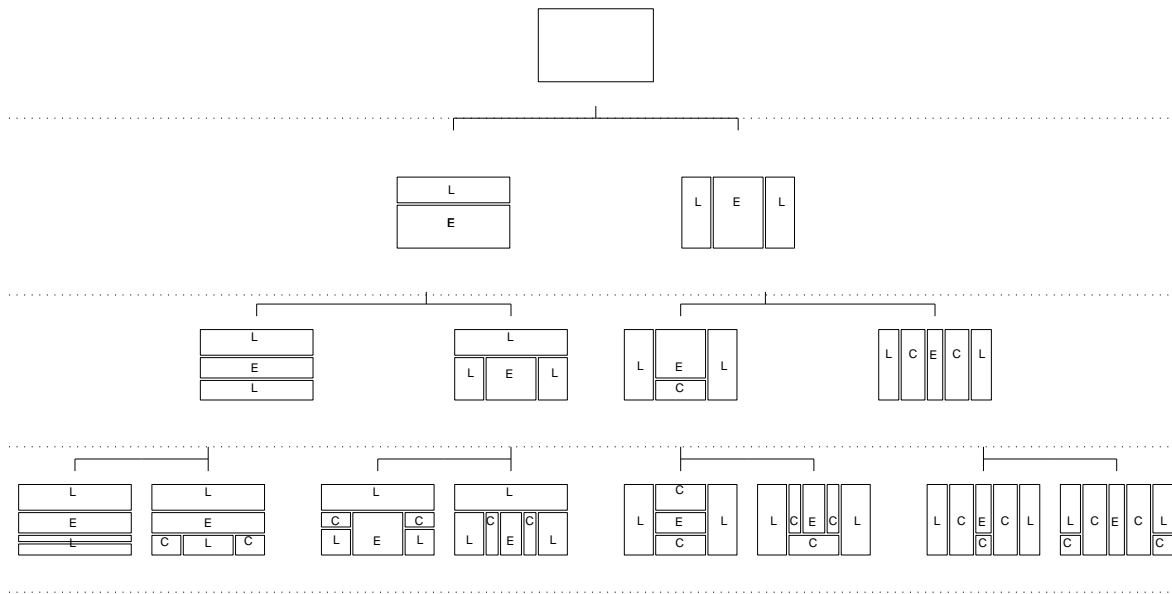


Figure 6.19: Partial functional derivation for Palladian villas

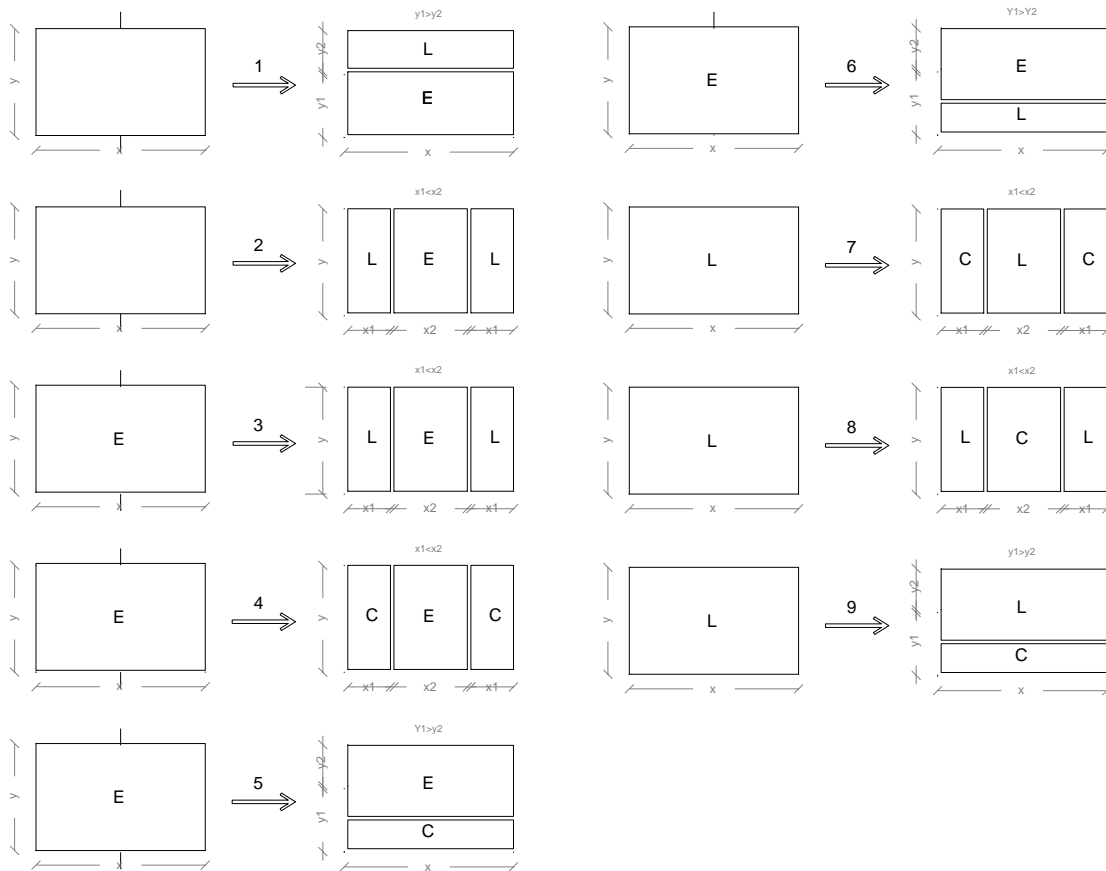


Figure 6.20: Spatial function rules for Palladian villas based on existing examples observed

The specific rules for spatial assignment are gathered in the illustrations above (6.19-20).

1. Rule 1 illustrates and details the manner in which the first spatial zoning is inserted. The first division allows for the placement of entertainment and living.
2. Rule 2 allows a similar placement but for vertical divisions.
3. Rules 3 to 5 continue the application of the living and entertainment spaces but taking into account the subdivision second iteration following what was prescribed for the centrality of the entertainment core space and the periphery of the living areas.
4. Rule 6 allows the placement of the first circulation space.
5. Rules 7 to 10 propose the second iterations of the circulation space insertion. The circulation is inserted taking into account the principles of connectivity and allocation by the central bays.
6. All of these rules are presented parametrically with variables derived from the villas boundary dimensions. The largest space in the house, the core room should be at least, a third of the villa's width up to 2:3, while its depth should also be at least a third of the overall depth. The remainder living spaces should have its minimum dimension smaller than a third of the correspondent overall width or depth and its maximum smaller than 2:3 but never to exceed the area of the core room. The circulation spaces while with large elongated proportional ratios such as 2:1 should not be larger in area than any other room in the villa. This way a clear hierarchical relationship is arranged among spaces allowing a visitor to easily identify the spaces while navigating.
7. The seventh stage starts the exterior spatial assignment by adding a porch. This porch is adjacent to the build fabric and is included as an extension. Therefore the rule type is predominantly an addition rule. This stage proposes three different configurations of porches.
8. The eighth stage creates the entrance point. To do so 10 rules are included that generate variations on accesses points. These rules propose a formulation that is in its core a subtractive method. To the external wall and perimeter of the house a penetration is introduced by a subtraction. To this shape a rectangle is subtracted and the endpoints are matched to finish either ends. Rules 31 to 41 create this set.

In most of these houses there is a smaller secondary entrance usually connected to the service area or kitchen. This service access is opposite to the main entrance and connects straight to the back yard. This stage groups rules 42 to 45 and follows the principles described for the previous stage.

The ninth stage constitutes an important stage by potentiating communications among spaces. To the functional areas designed is now left the ability to flow from one space to the adjacent space. Therefore, rules 46 to 50 have an important role creating a functional working diagram of adjacencies. As a shape rule it is mainly a subtractive rule where shapes are subtracted from others to generate an alteration. Spatially allows for navigation between two spaces as illustrated in the diagram. These rules create internal doors.

The eleventh stage creates internal details such as fireplaces, chimneys and windows. To do so this stage proposes several rules arranged from rule 51 to 68 which are a diverse set of addition and subtraction rules.

The final stage is responsible for terminating the design and deletes remaining labels.

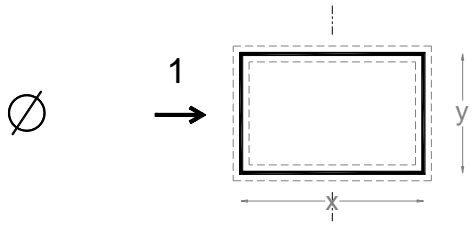
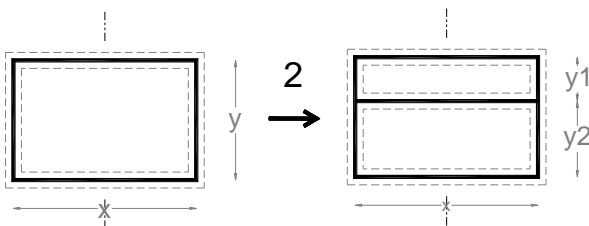
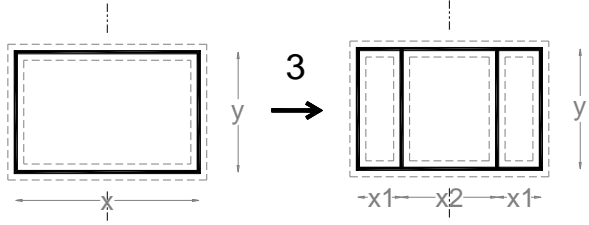
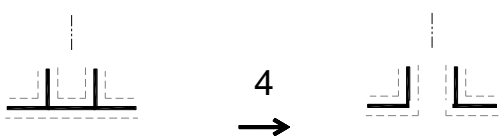
In the diagram of these orthogonal grid houses there are many assumptions that can be retained. First this grammar could have been easily achieved using a grid grammar and to explain it is only needed to observe a floorplan of one of these houses. Second the functional assignment comes hand and hand with the shape rule system and in fact they are one and only process. Third functions are assigned by a series of sequential steps where spaces are included from social to services, from general to particular and leaving to last the connection links. This seems to be a preferred method for grammarians and it is patent both in the Palladian villas creation and the Malagueira grammar.

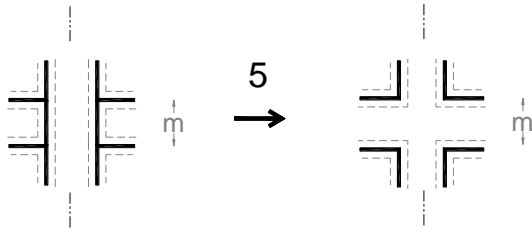
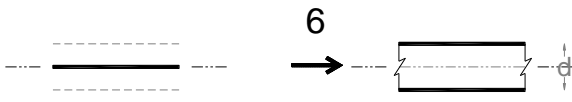
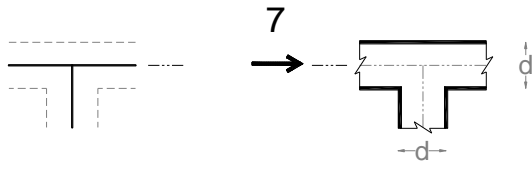
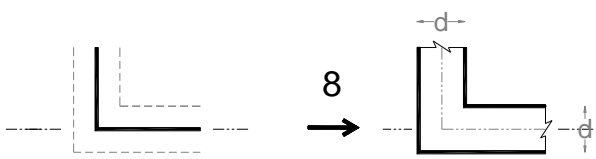
Even though a grid method would be an obvious step when considering the grammar inference, is also understandable that subdivision allows for a more elegant and efficient grammar in this case since there are a series of examples of non-alignment of walls or omissions of borders and cells. A grid method would only be feasible with a series of concatenation/merging rules and border deletion. This is certainly important knowledge to use when rethinking a housing grammar.

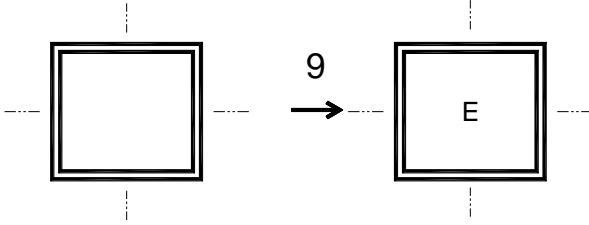
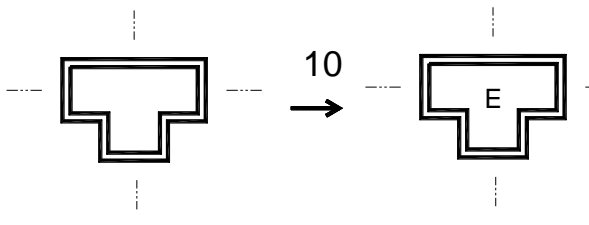
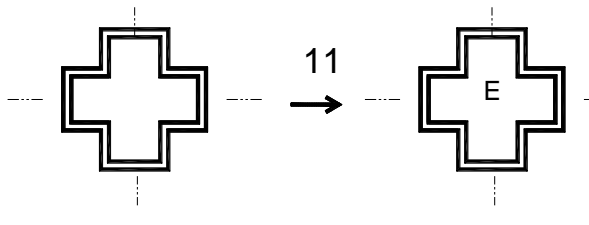
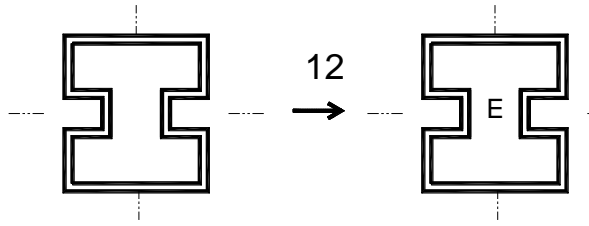
The sequence of spaces used also does not fall too far from the examples studied so far. The main spaces created are living 'l', dining 'd', kitchen 'k', sleeping/bedroom 'b', closet 'c', porch 'p' and entrance 'e'. In this single house type no singular spaces is included unlike the central core space at the Palladian villas.

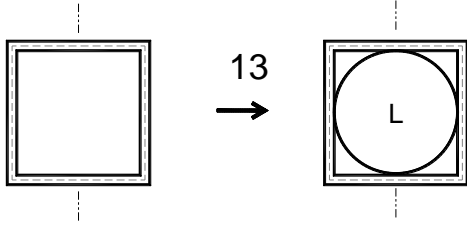
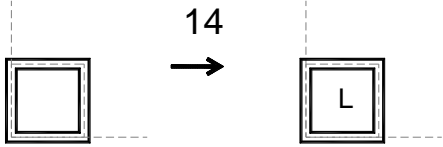
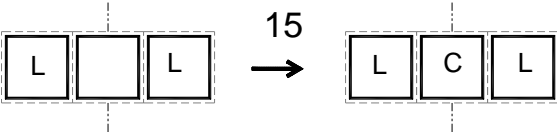
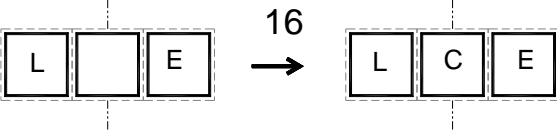
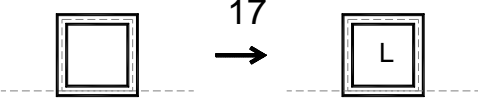
In conclusion this constitutes a good precedent study for a generic housing grammar.


The presented shape and functional rules were then assembled in a structured shape grammar as shown in table 6.1:

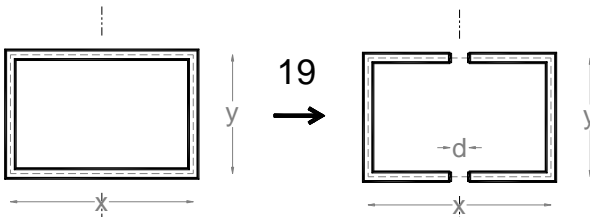
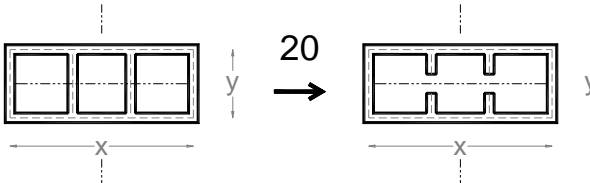
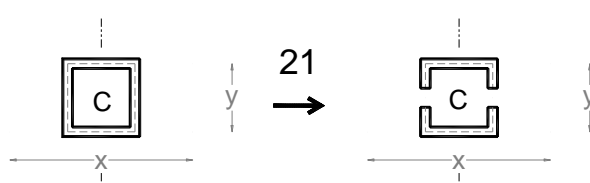
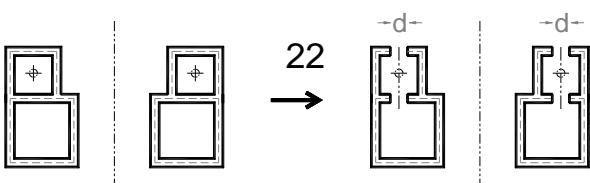
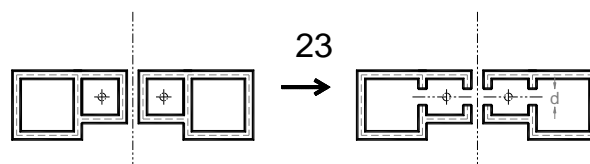
STAGE 1		
BOUNDARY DEFINITION		
RULE 1	ADDING BOUNDARY	PARAMETERIZATION
		$y/x = n/m$ $m = [3,5,7]$ $n = [2,3,4,5]$ allowed ratios: 1:1, 4:3, 3:5, 4:5, 3:7
STAGE 2		
SPATIAL SUBDIVISION		
RULE 2	HORIZONTAL SUBDIVISION	PARAMETERIZATION
		$y/x = n/m$ $m = [3,5,7]$ $n = [2,3,4,5]$ $y = y1 + y2$ $y1 = y/n \vee y1 = y/3n$ $y1 = y2 \text{ (for } n=2 \vee n=4 \text{)}$
RULE 3	VERTICAL SUBDIVISION	PARAMETERIZATION
		$y/x = n/m$ $m = [3,5,7]$ $n = [2,3,4,5]$ $X = 2.X1 + X2$ $X1 = n \vee X1 = 3.n/2 \vee X2 = 2n$ $y1 = y2 \text{ (for } n=2 \vee n=4 \text{)}$
STAGE 3		
SPACE MERGING		
RULE 4	HORIZONTAL MERGING	PARAMETERIZATION
		$X=n$

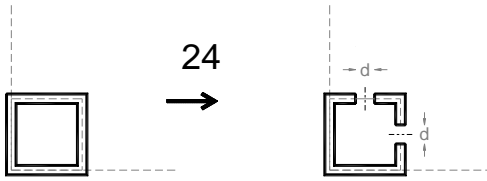
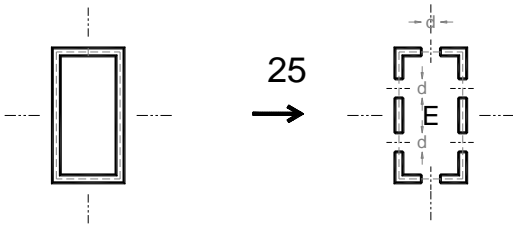
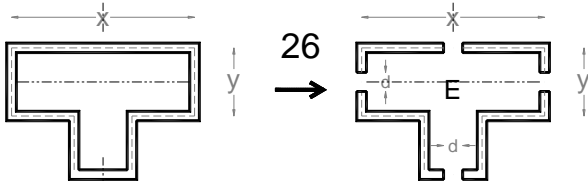
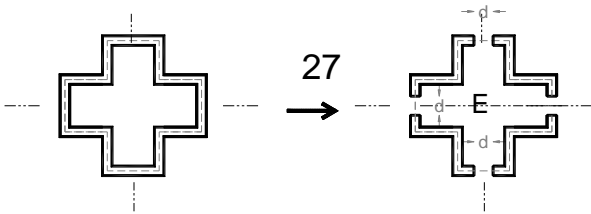
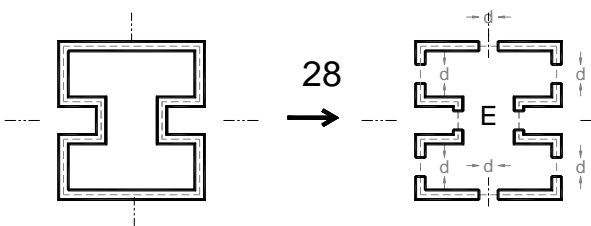
RULE 5	VERTICAL MERGING	PARAMETERIZATION
		$Y=m$
STAGE 4	WALL THICKENING	
RULE 6	SINGLE WALL	PARAMETERIZATION
		$d \approx 2$ Vicentine feet $d = 750$ mm
RULE 7	WALL 'T' JUNCTION	PARAMETERIZATION
		$d \approx 2$ Vicentine feet $d = 750$ mm
RULE 8	WALL CORNER JUNCTION	PARAMETERIZATION
		$d \approx 2$ Vicentine feet $d = 750$ mm

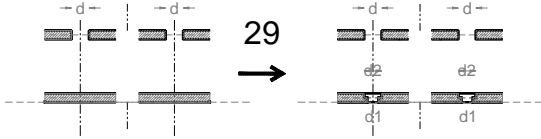
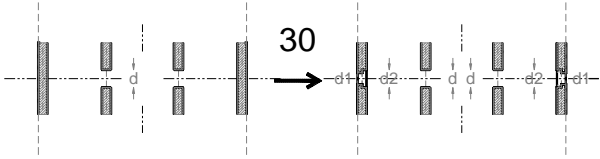
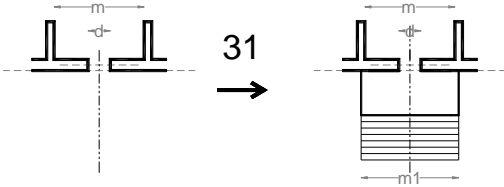
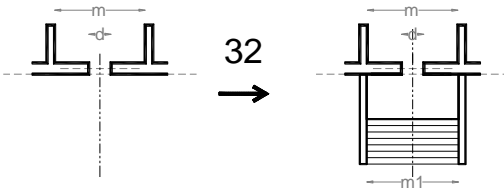
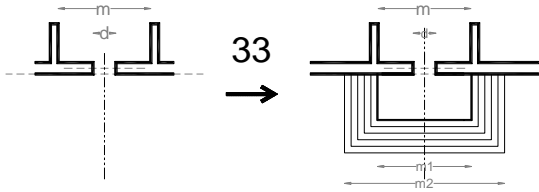
STAGE 5	FUNCTIONAL ASSIGNMENT	
RULE 9	CORE SOCIAL SPACE	PARAMETERIZATION
		E = entertainment space
RULE 10	CORE SOCIAL SPACE	PARAMETERIZATION
		E = entertainment space
RULE 11	CORE SOCIAL SPACE	PARAMETERIZATION
		E = entertainment space
RULE 12	CORE SOCIAL SPACE	PARAMETERIZATION
		E = entertainment space

RULE 13	CORE SOCIAL SPACE	PARAMETERIZATION
	 <p>13</p>	E = entertainment space
RULE 14	CORE SOCIAL SPACE	PARAMETERIZATION
	 <p>14</p>	L = living space
RULE 15	CORNER LIVING SPACE	PARAMETERIZATION
	 <p>15</p>	L = living space C = Circulation
RULE 16	LIVING SPACE	PARAMETERIZATION
	 <p>16</p>	L = living space C = Circulation E = entertainment space
RULE 17	LIVING SPACE	PARAMETERIZATION
	 <p>17</p>	L = living space

RULE 18	LIVING SPACE	PARAMETERIZATION
	 <p>The diagram illustrates Rule 18, showing a transformation of a square space labeled 'C'. On the left, a square with a double border and a dashed line through its center is shown. An arrow labeled '18' points to the right, where the same square is shown again, but with a single border and a dashed line through its center. This indicates a change in the space's configuration or parameterization.</p>	<p>C = Circulation</p>

STAGE 6	DOOR ASSIGNMENT	
RULE 19	MAIN ACCESS DOORS	PARAMETERIZATION
		$d = 4 \times vc$ $1vc = 0.354 \text{ m}$
RULE 20	INTERNAL DOORS	PARAMETERIZATION
		$d = 4 \times vc$ $1vc = 0.354 \text{ m}$
RULE 21	INTERNAL DOORS	PARAMETERIZATION
		$d = 4 \times vc$ $1vc = 0.354 \text{ m}$
RULE 22	INTERNAL DOORS CORE SPACE	PARAMETERIZATION
		$d = 4 \times vc$ $1vc = 0.354 \text{ m}$
RULE 23	INTERNAL DOORS CORE SPACE	PARAMETERIZATION
		$d = 4 \times vc$ $1vc = 0.354 \text{ m}$

RULE 24	INTERNAL DOORS CORE SPACE	PARAMETERIZATION
		$d = 4 \times vc$ $1vc = 0.354 \text{ m}$
RULE 25	INTERNAL DOORS CORE SPACE	PARAMETERIZATION
		$d = 4 \times vc$ $1vc = 0.354 \text{ m}$
RULE 26	INTERNAL DOORS CORE SPACE	PARAMETERIZATION
		$d = 4 \times vc$ $1vc = 0.354 \text{ m}$
RULE 27	INTERNAL DOORS CORE SPACE	PARAMETERIZATION
		$d = 4 \times vc$ $1vc = 0.354 \text{ m}$
RULE 28	INTERNAL DOORS CORE SPACE	PARAMETERIZATION
		$d = 4 \times vc$ $1vc = 0.354 \text{ m}$

STAGE 7		WINDOW ASSIGNMENT
RULE 29	WINDOWS	PARAMETERIZATION
		$d = 4 \times vc$ $d = d2$ $d1 = d2 - (2 \times 1vc)$ $1vc = 0.354 \text{ m}$
RULE 30	WINDOWS	PARAMETERIZATION
		$d = 4 \times vc$ $d = d2$ $d1 = d2 - (2 \times 1vc)$ $1vc = 0.354 \text{ m}$
STAGE 8		DETAILING AND ORNAMENTATION
RULE 31	STAIRS – STRAIGHT FLIGHT	PARAMETERIZATION
		$m1 = m$ v $m1 = 3xm$
RULE 32	STAIRS – STRAIGHT FLIGHT WITH CAPPED ENDS	PARAMETERIZATION
		$m1 = m$ v $m1 = 3xm$
RULE 33	STAIRS – SPLAYED FLIGHT	PARAMETERIZATION
		$m1 = m$ v $m1 = 3xm$ $m2 = m1 + (\text{num steps} \times \text{dist} \times 2)$

RULE 34	STAIRS – SPLAYED CIRCULAR FLIGHT	PARAMETERIZATION
		$m_1 = m$ v $m_1 = 3xm$ $m_2 = m_1 + (\text{num steps} \times \text{dist} \times 2)$
RULE 35	STAIRS – TWIN FLIGHTS	PARAMETERIZATION
		$m_2 = m$ v $m_2 = 3xm$ $m_1 = (\text{num steps} \times \text{dist})$
RULE 36	STAIRS – TWIN ELBOW FLIGHTS	PARAMETERIZATION
		$m_2 = m$ v $m_2 = 3xm$ $m_1 = m_3$
RULE 37	SINGLE CELL RECESSED LOGGIA	PARAMETERIZATION
		m
RULE 37	TRIPLE CELL RECESSED LOGGIA	PARAMETERIZATION
		$m \rightarrow 3 \times m$

RULE 39	TRIPLE CELL RECESSED LOGGIA	PARAMETERIZATION
		$m \rightarrow 3 \times m$
RULE 40	PROJECTED TERRACE	PARAMETERIZATION
		$m1 = m$ v $m1 = 3 \times m$ $m2 = m1$ $m3 = m2 - r$
RULE 41	COLUMNS – EXTERNAL ORNAMENTS X4	PARAMETERIZATION
		m $m2 = m1$ $m1 = m / 3$
RULE 42	COLUMNS – EXTERNAL ORNAMENTS X4	PARAMETERIZATION
		m $m2 = m1$ $m1 = m / 3$
RULE 43	COLUMNS – EXTERNAL ORNAMENTS X4	PARAMETERIZATION
		m $m2 = m1$ $m1 = m / 3$

RULE 44	COLUMNS – EXTERNAL ORNAMENTS X 6	PARAMETERIZATION
		m $m2 = m1$ $m1 = m / 5$
RULE 45	COLUMNS – EXTERNAL ORNAMENTS X 6	PARAMETERIZATION
		m $m2 = m1$ $m1 = m / 5$
RULE 46	COLUMNS – EXTERNAL ORNAMENTS X 6	PARAMETERIZATION
		m $m2 = m1$ $m1 = m / 5$
RULE 47	COLUMNS – EXTERNAL ORNAMENTS X 3	PARAMETERIZATION
		n $n1$ $n1 = m / 2$
RULE 48	SERLIANA – EXTERNAL ORNAMENTS	PARAMETERIZATION
		m $m2 = m1$ $m1 = m / 3$

Table 6. 1: Set of alternative grammar shape rules

6.6. Villa Derivation

Derivation describes the generation process in which the recursive application of shape rules leads to a final design. The original grammar showed how a successful design can be achieved, using the example of the derivation of the La Malcontenta villa (Figure 6.15). This was originally built between 1559-60 in the outskirts of Venice and is illustrated in Palladio's "Il quattro libri" (Palladio 1997). In order to fully grasp the differences between the original and the alternative grammar a derivation exercise was performed. Despite the divergences (namely a more economical use of rules and process), both grammars contain the same number of stages. The new derivation process has 14 steps, using 10 shape rules sometimes recalled recursively (Steps 2, 5 and 6 using Rule Three, and Steps 3 and 4 using Rule Two). The first step is the boundary placement, followed by division in Steps 2 to 6. The division sequence of rules 3, 2, 2, 3, 3 splits a rectangle into three, or creates a variation of a 3x5 grid in which cells occupy a larger area for future occupancy. Step 7 deletes the vertical boundaries of the central cell. Step 9 offsets the preliminary layout by duplicating lines 2 feet away from each other and creating the wall placement. Step 10 deals with the intersections between wall lines and refines corners. Step 11 includes the entrance feature. Step 12 proposes functional uses by tagging each room. Step 13 creates openings in walls with the array or for window and doors. Step 14 terminates the design by erasing the labels.

The following figures show respectively the partial derivation of the Malcontenta Villa using Stiny and Mitchell's grammar and the same derivation using the alternative version. Overall the original grammar allows for a substantially greater number of steps. The basic outline requires 15 steps to be abstractly recognisable. Up to step 15 only cells are being added to the design. From then on transformation rules take place to manipulate the proportions and concatenate spaces. These operations require in total eight additional steps. To this is added detailing with inclusion of porticos, ornamentation and external stairs.

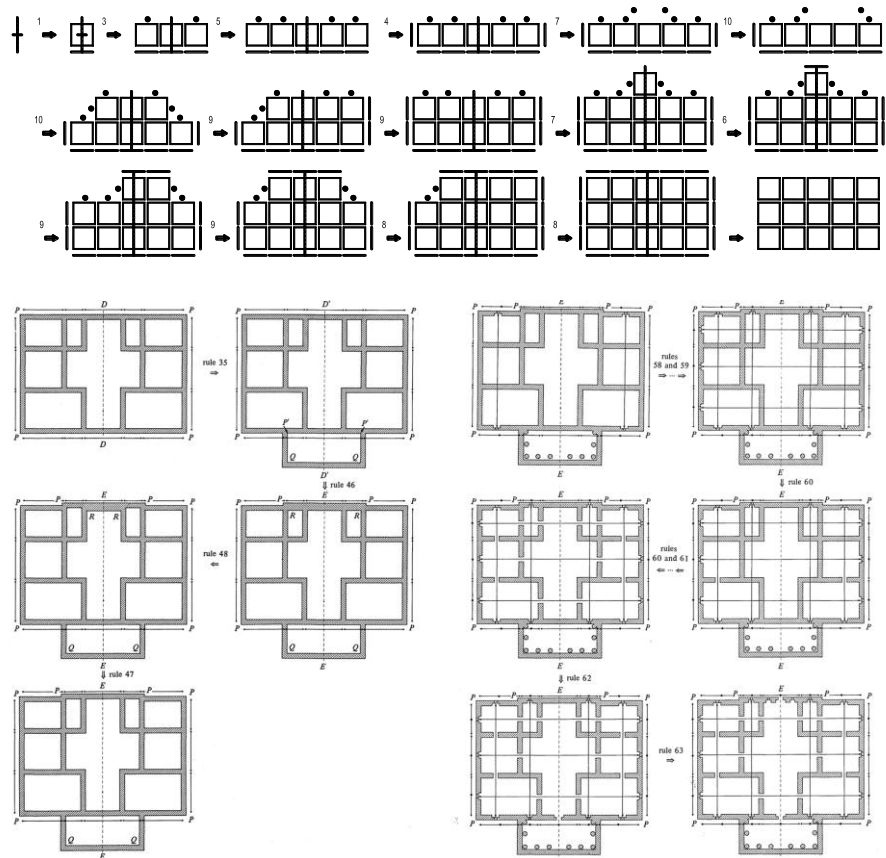


Figure 6. 21: Partial derivation of the Malcontenta Villa using Stiny and Mitchell's grammar

The alternative grammar promotes a more straight-forward and economic approach. This does not translate into a better or worse option, but in a grammar that might be easier to use since it requires less steps. The reduction of steps translates into a reduction of opportunities for errors whilst applying the rule set. With the alternative grammar the basic house layout is outlined in step 9. From this point detailing and functional assignment takes place. The full process is concluded in step 15.

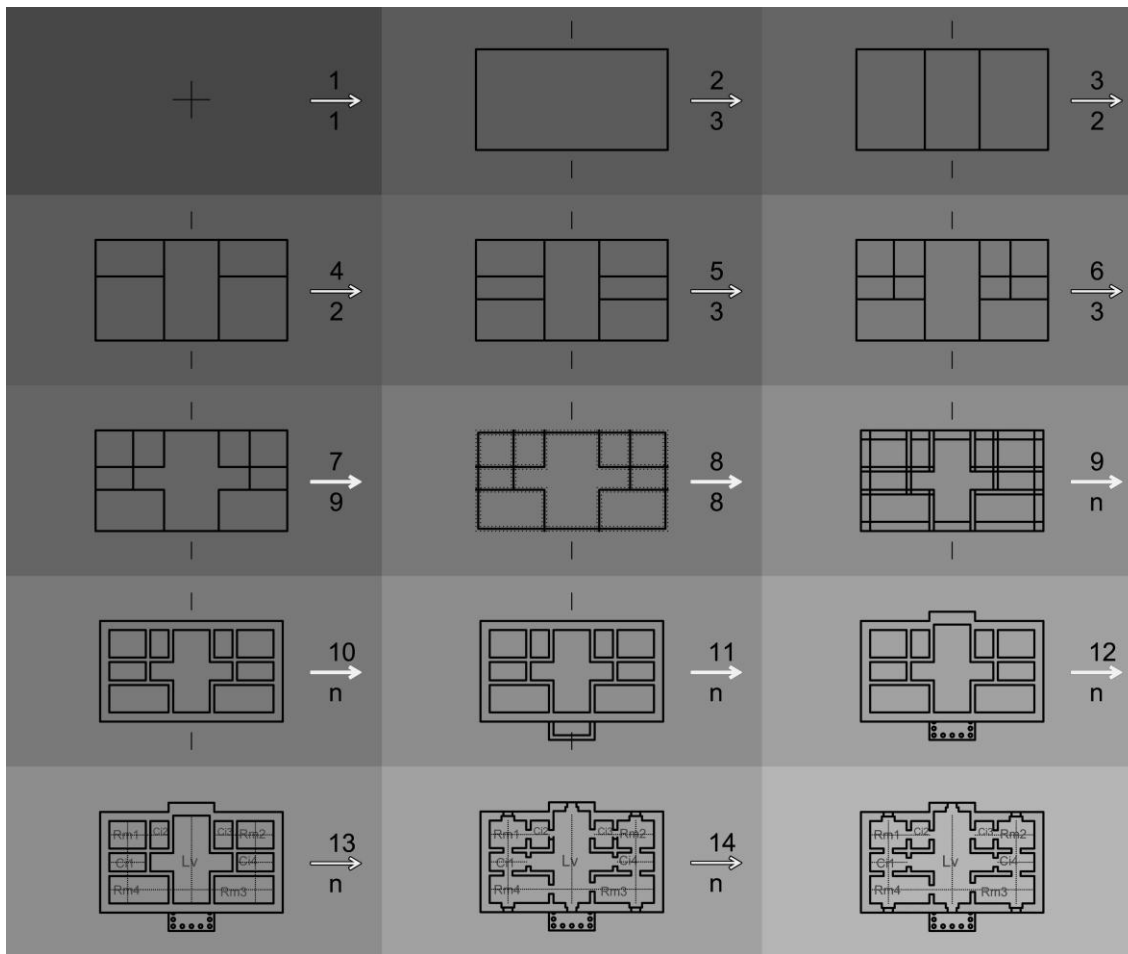


Figure 6.22: Derivation of the Malcontenta Villa using the alternative grammar

6.7. New Designs

An important feature of shape grammars is the ability to encapsulate expert knowledge from an experienced designer and recreate its design process enabling design exploration and production of a new corpus of solutions. This know-how can then be used in an aided manner by an inexperienced user whilst using the grammar.

Previous shape grammars studies, proposed new set of solutions and generated an extensive sample of novel designs. The creation of new designs using a shape grammar might be a polemic subject, especially if referring to an architectural style developed by an architect that did not supervised or authorized a study of this kind. However existing designs encode a level of knowledge and know-how that sometimes is even imperceptible to the designer but emerges intuitively when designing, fruit of experience and previous practice in problem resolution. By recreating existing corpus of designs, grammars sum much of that underlined knowledge both in geometry, space, adjacencies and spatial communication. This can then be used to generate new designs that comply with the style archetypes and rules. A shape grammar potentiates design exploration. The benefits are several. It can help historians determine within reasonable doubt if an unsigned design belongs to the believed author, it can test the generative potential of a given design system, it can test the level of creativity and diversity a design has, and it can allow the generation of unexpected and still desirable solutions.

The Palladian grammar, the first complete architectural shape grammar, proposed 230 new designs (as shown previously in Figure 6.10 combining 20 for villas with a 3x3 grid and 210 for villas with a 5x3 grid. To calculate these numbers Chris Earl and Ramesh Krishnamurti used both combinatory calculus and colour grammars to illustrate the new solutions. This was later published in a work entitled 'Counting Palladian Plans'(G Stiny & W Mitchell 1978). From all the examples described and illustrated some are clearly representative of the rule production system and the Palladian style, others appear to be a repetition of cells and fail to engage as a quality architectural design product. This is also commented by the authors and expressed in the work 'An evaluation on Palladian plans' (Stiny & Gips 1978). They establish that other values have to be considered to add aesthetic and spatial quality to some of the villas designed. Noticeably unwritten rules such as the need to include a larger room in the house centre and avoid that large proportioned rooms can occupy extreme locations. This is a post-analysis of what was rehearsed with the grammar, however these could have been easily incorporated into a rule integrated within the grammar. The inclusion of spatial quality rules such as these above mentioned would have avoided the criticism that some of the examples raised amongst art historians and architects when faced with a tartan grid that merely represented an array of equally sized rooms. Nevertheless, some Palladian designs do simulate a tartan grid. A good example of that is Villa Godi with its unusual 5x2 grid or Villa Angarano that uses a 3x3 grid resulting on a display of 8 rooms as demonstrated in Figure 6.1.

What is also not discussed is the reason why the authors focused on 3x3 and 5x3 grids and omitted other villas examples pictured in the 'Il quattro libri'. However, Palladio also designed

and built villas with a 3x2, 3x4, 5x2 and 5x4 overlying grid, like villa Barbaro, Cornaro (3x4) Valmarana, Godi (2x5) Mocenigo and Gazzotti (5x4). This means that potentially more rules and more examples can be added to the new corpus of designs.

As for the Prairie houses, Eizenberg and Konning do not produce any evidence of combinatory calculus that supports the generation of multiple novel solutions. They do show very feasible designs of what could easily be Prairie houses. As homage to the first researchers in the field they named it Stiny, Mitchell and March house. The real potential of the grammar that showcases a great number of design rules and expresses designs volumetrically would add great interest to the study.

The Malagueira grammar showcases a great amount of new solutions ranging up to one million. This is linked to several factors:

First the case study and amount of houses designed by the original architect reached almost 300. This obviously enriched the experiment with a multiplicity of existing examples.

Second, and as a result of the first point, it benefits of a large corpus of shape rules up to 120, which exceeds any of the others shape grammars. This allows a higher level of options and more precise detailing

Third and last, the grammar encapsulated rules that were vetoed directly from the original designer which allowed refining and perfecting the formalism itself. The subdivision grammar developed fits Knights characterization of descriptive grammar and as explained these have a great generative power, probably the grammar type with greater generative power (Knight 1999). Another important factor to note is that most cases within the new corpus are not only feasible but acceptable by Siza himself. (Duarte 2001).

The alternative grammar manages to propose new solutions that could be added to the corpus identified by Stiny and Mitchell. From the new solutions five new design results constituted the examples showcasing underlined grids of 2x3. Included in this set is a small villa designed by Palladio and villa Godi illustrated in the four books with a 5x2 subliminal grid. On the other hand, 60 examples were identified to illustrate the possibilities of 3x4 villas. In both cases the following rules were supervised. Examples that allowed to many concatenation processes to the point that these villas could be mistaken by smaller grid sets were scrapped. In the event of rooms with larger proportions one equally or larger proportioned room would occupy the central bay.

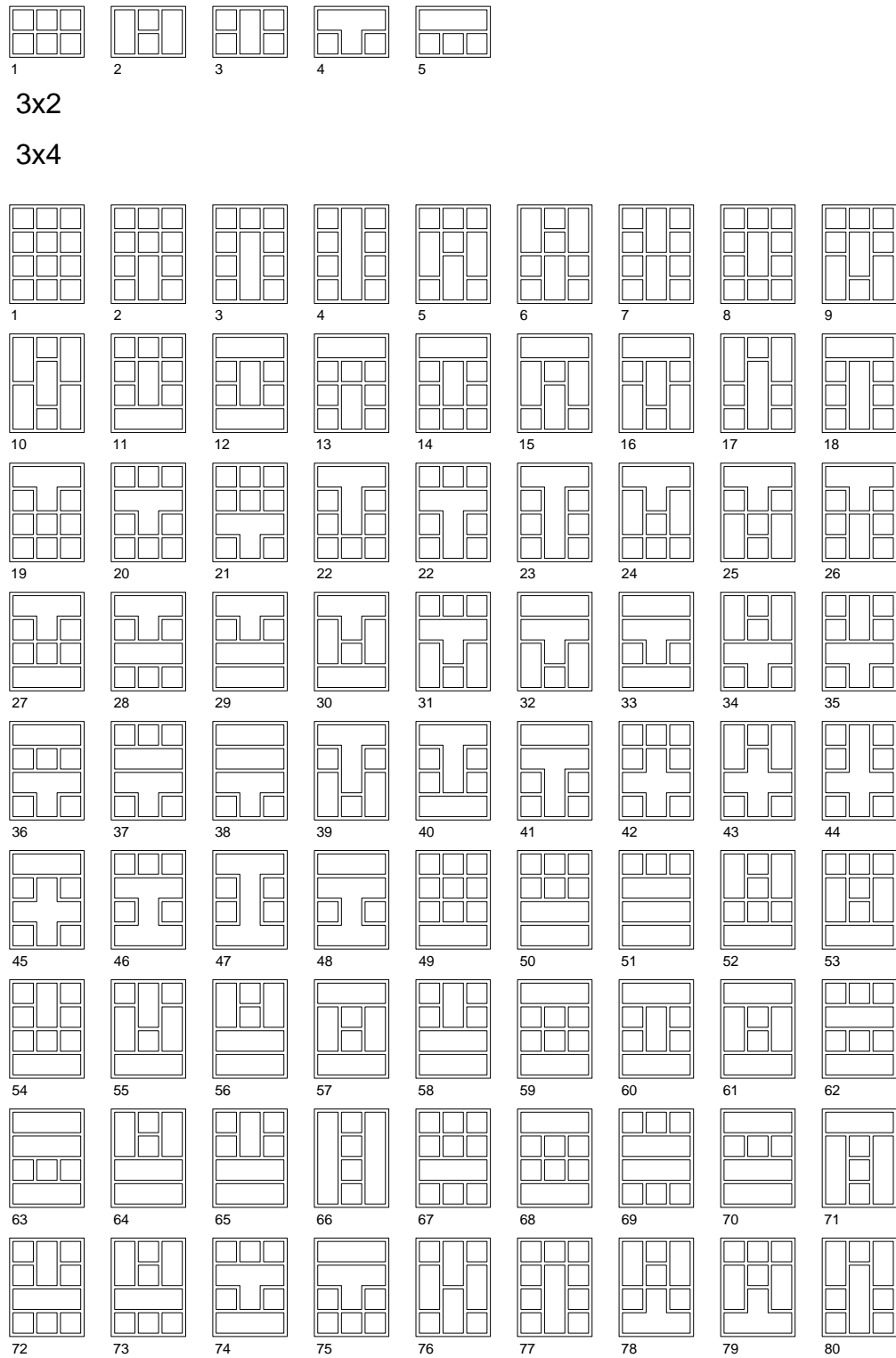


Figure 6.23: New corpus of solutions of the alternative grammar with 3x2 and 3x4 bays

All rooms had rectangular footprints, but the central room could allow also other configurations such as cross shaped, T or I. The north/south axis was the symmetry axis which meant that west and east wings had mirrored configurations. The number of bays over the west to east axis had to be always in odd numbers resulting into grids that are (2x3, 3x3, 4x3, 3x5 and 4x5), in order to accommodate a central room in the central bay. The new set illustrated below accommodated the villas also designed by Palladio with 2x3, 4x3 and 4x5 and new examples. Some could join Palladio built examples, others would struggle. An important thing to note is that this representation only focus on core base configuration, the spatial proportion still allowed a different parameterization that most likely would allow some of the examples to look more feasible. The described new corpus is illustrated above in Figure 6.23. Not surprisingly some real examples are part of the alternative grammar new corpus. Villa 44 is a good reproduction of Villa Barbaro and villa 8 an approximation of Cornaro.

As validation derivation can be used as a means of testing if the designed villas fit the style using the grammar. Famously the Palladian grammar showcases the partial derivation of villa la Malcontenta Foscari, showcasing successfully how this house can be generated using the shape rules. However as mentioned by Stiny some villas are not included in the set of design solutions and an example of that is la Rotonda. The explanation can be:

- Most of the Palladian villas within the studied universe present in plan bi-symmetry and Rotonda is very close from a perfect double symmetry North/South and East/West and a boundary ration of 1:1
- It allows a concentric design inscribed within a regular and symmetrical polygon (square)
- The centre core space is circular (there is only another villa with such a room)
- The design is singular within the work of Palladio and is illustrated as a main isolated example of domestic architecture as the first in the second book and the only that illustrates plan, section and plan
- The characterization of villa is questionable, taking into account its dimension, geographical, topographical and relative location within its plot, its building tectonic singularity with an inclusion of a dome can be considered as a Palazzo

Is not only the Palladian grammar that fails the representation of la Rotonda, Hersey design system and software plan-maker also infamously represent this floorplan with an approximation that resembles very little the original. Hernsey proposed a rule based system based on a division method inspired by musical harmonies. This method was the put into practice using a computer tool to design floorplans. This tool was named 'planmaker'. A subsequent tool was also created and named 'façade maker in order to generate facades to complement the plans generated.

The inability of generating concentric Palladian villas such as 'La rotonda' is mentioned by the published work. This is noted in the conclusion by the authors:

“The method we have described works well for most types of Palladian villas, but there are other designs by Palladio that it could never produce. First of all, Planmaker cannot produce circular rooms as in the Rotonda. To correct this fail we could of course tell the program to inscribe a circle every so often in one of the square rooms it so frequently produces. But unfortunately there is more to it. The rotundas must sit precisely in the centre of the whole. To extrapolate a rule from these two examples, we would have to allow Planmaker to inscribe a circle only when the square is central and the plan symmetrical on both the x and y axes. Theoretically Planmaker can produce such plans, but statistically the chances of doing so are nearly zero.” (Hersey 1992)

However, the inability to design a circular centre room is not the only issue with Planmaker. Figure 6.26 shows its best approximation to Villa Rotonda. The proposed design system bases its subdivisions in musical fractions and all proportions derive from this. In order to maintain the Palladian internal and external proportions the set of rules proposed by the Planmaker are too hermetic to allow most designs. The result is a very inflexible layout that hardly resembles la Rotonda. Instead of 8 rooms, shows 6, the centre room is not circular and does not allow the typical cruciform communication and the floorplan appears to be rotated, hence it is illustrated here in the most believable configuration.

Is La Rotonda one of a kind or can this be represented within the style and generated through the shape rules provided?

Regardless of the limitations and specific characteristics mentioned above the villa still resembles the main Palladian style, keeping symmetry as a main principle, being easily represented by a tartan grid (5x5) and allowing for a main centre room with larger dimensions and with a regular polygonal configuration. To test these two derivations were proposed. First a derivation using the original Palladian shape rules and second the alternative grammar rules.

The first attempt is illustrated in Figure 6.24. The derivation process allows for 31 steps were different rules are applied recursively. The first 21 steps are only applied in order to generate the main grid. This underlined grid to support the Rotonda basic layout would be a 5x5 cells. This would allow the correct internal and boundary proportions. The immediate following steps propose the space concatenation as allowed by rules 12-23. The larger rooms along the main and opposite facades are also concatenated generating the canonical 2:1 proportion, these abide to merging rule 13. In order to create the centre space three merging processes would take place, the conversion of three adjacent cells as per rule 12 and the concatenation of already merged cells. This rule is not allowed by the grammar (unless we allow for shape immergence) hence constituting a rule introduction.

The derivation illustrated shows 2 colours, black and red. The red colour illustrates a breach in the rule system in order to satisfy the layout. The merging of previously merged cells is one of the five examples of rule adjustment. The following step would allow for a third stage recursive

merging step were the already merged cells would allow for a consecutive merging. This would allow for a new central room shape resultant from a cruciform union with a square. The two additional rule breaches would allow for a grid manipulation in X and Y varying the adjacent cells from a 1:1 to a 4:3 proportion and the last rule would introduce a circular space.

The replication of Villa Rotonda by the alternative grammar would require less effort as clearly illustrated in Figure 6.25.

The derivation would comprehend 17 steps combining mainly subdivision processes and very few merging steps. The first 8 steps are recursions of subdivisions over the previously introduced square boundary. These single vertical and mirrored vertical subdivisions have to comply with the bilateral symmetry principal based on the North-South axis. Step 9 allows a particular circular division shape rule for the centre room. In step 10 merging is prompt with the concatenation of the perimeter rooms coincident with the Cartesian axis and the centre rooms. The three last steps transform the abstract underlined layout into a bi-dimensional floorplan recreating correct wall thicknesses.

The result is a floorplan that is compatible with la Rotonda's proportions and layout. This corroborates and positively evaluates the set of rules proposed by the alternative grammar.

Additionally, this alternative grammar has proven to be more economic and elegant summarizing in a more restive number of rules a similar generative power.

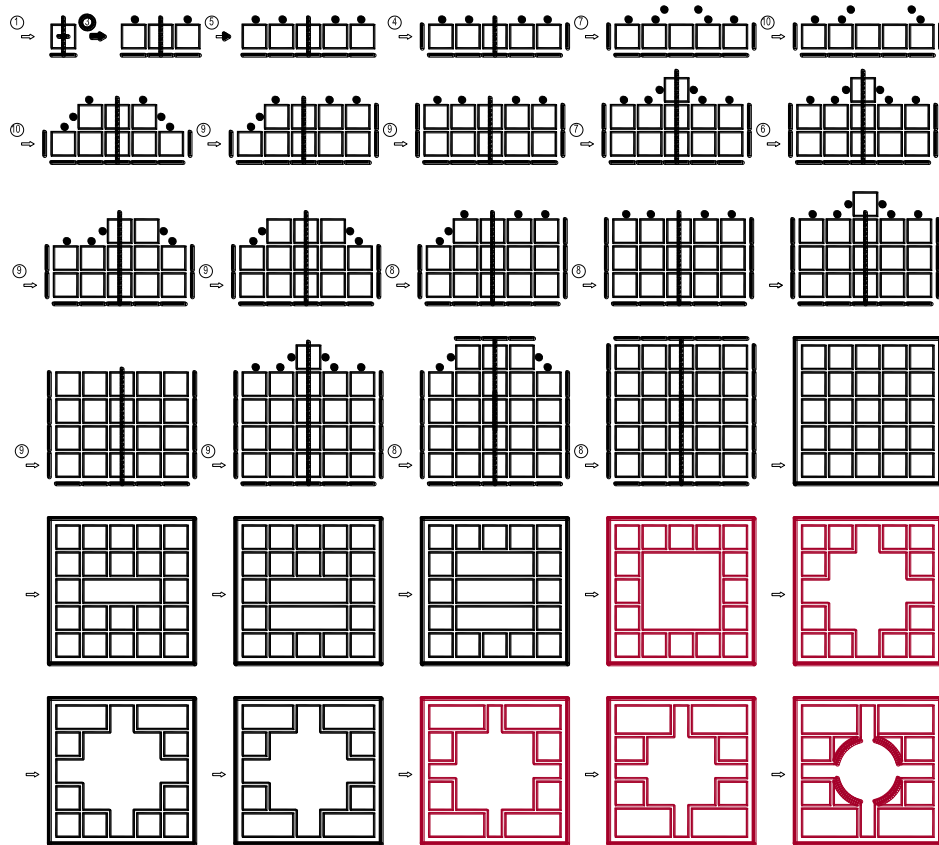


Figure 6.24: Villa Rotonda al Capri derivation using Palladian grammar

(The red floorplans indicate rule adaptations in order to achieve a design compatible to the villa floorplan, it would require at least 5 new shape rules applied over 31 steps to achieve the basic layout based on a 5x5 grid)

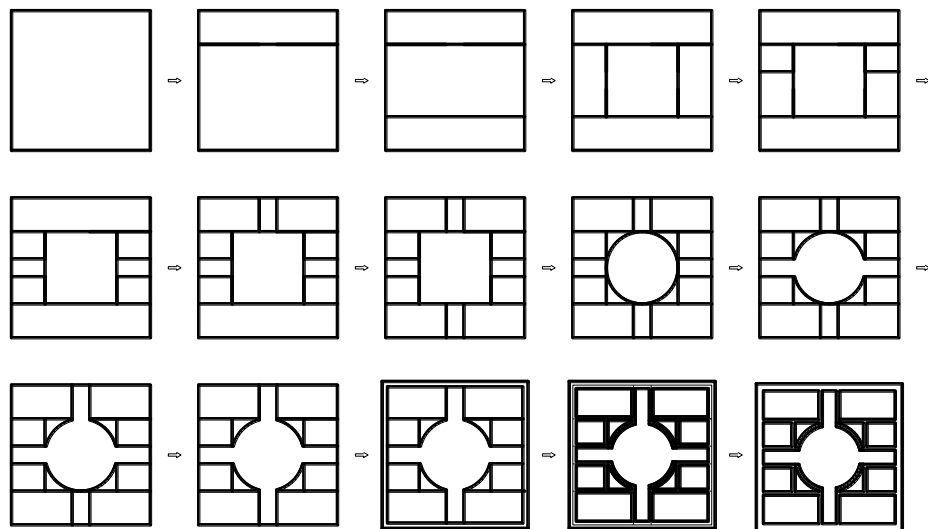


Figure 6.25: Villa Rotonda al Capri derivation using Alternative Palladian grammar in 17 steps

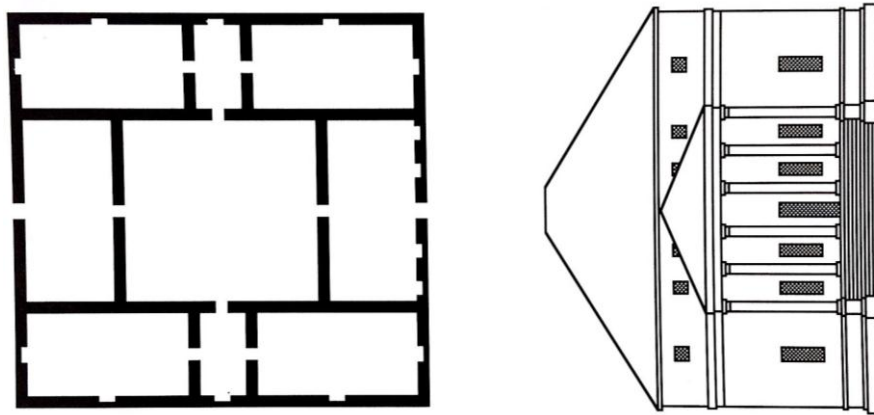


Figure 6. 26: Villa Rotonda al Capri approximation generated by Plan-maker design system developed by Hersey and Freedman in (Hersey 1992)

(The floorplan circular room cannot be reproduced and only 6 of the 8 perimeter rooms can be recreated. The façade allows only a pitched roof that does not replicate the typical dome)

The grammar has been demonstrated to be effective in terms of recreating the villa. Previous attempts by either Stiny and Mitchell original shape grammar and Hersey and Freedman's replicated the design system of Andrea Palladio, but failed with particular examples. Additionally, it was illustrated in Figure 6.23 a set of potential solutions the alternative grammar can generate using subdivisions that split the boundary into spaces of underlined grid of 3x2 and 3x4. This illustrates a range of possible designs.

This new set of design solutions is useful to attest the potential of the grammar. Almost all shape grammars in the past exemplified how a new design solution could be presented. To better illustrate this a new design is illustrated and its derivation process is shown in Figure 6.27.

This new villa was illustrated previously as number 25 and according to most principles in Palladio design seems to represent the style correctly. This constitutes an example of a novel design not recreated by previous systems or grammars.

The villa is symmetrical with its main symmetry axis splitting the villa East and West. The main room with larger proportions sit on top of the symmetry axis and allows communication to all its adjacent rooms. This core room shows a T shape and is the only space that ranges from one façade to its opposite. Two other larger rooms are arranged symmetrically and propose a 2:1 ratio which contrasts in hierarch with the circulation spaces of square shape.

Up to step 5 the derivation process allows only subdivision, followed by stage 6 that allows one simple merging to create a special central geometry. This is followed by the wall thickening process. The following process is the introduction of entrance by creating in the central bay a recessed loggia. This loggia is then ornamented with columns in even number and a staircase of the same width of the bay. This step is then followed by the inclusion of entrance and opposite access. The internal doors will align firstly with the main access, secondly in parallel aligned with the middle line of the smaller rooms and thirdly in perpendicular. The last derivation

step allows for window openings which follow the principle of void alignment, hence these are placed in line with the internal doors.

The result is a villa consistent with the Palladian style with many similarities with villas Cornaro, Sarego and Emo, both in structure internal distribution and in basic layout.

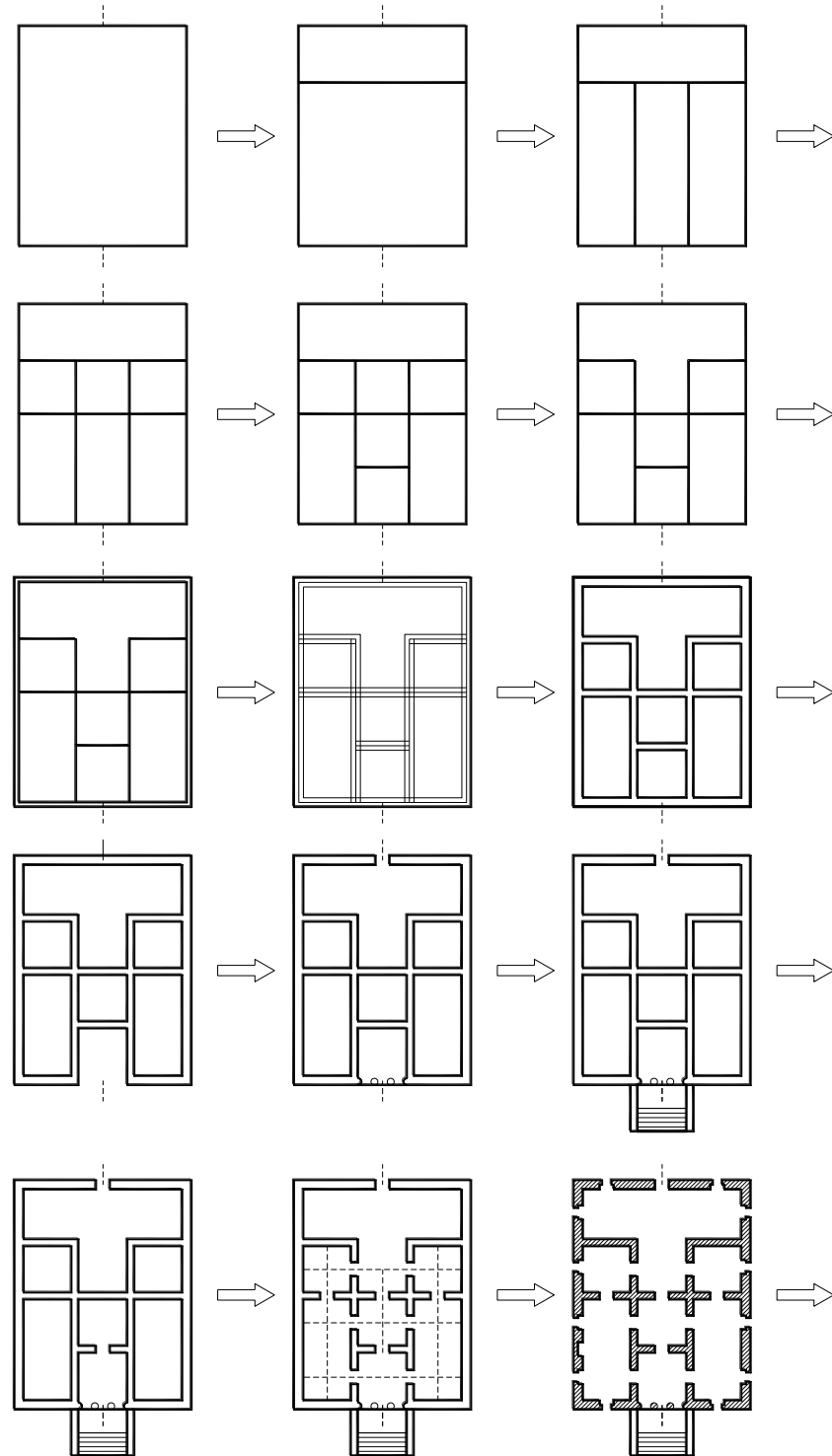


Figure 6. 27: New villa solution generated with the alternative Palladian grammar

6.8 Computer implementation

The computer implementation combines the shape rules, parametric formulation and conditions and actively assists in the complex application of the shape grammar. In addition, it reduces errors and secures the generation of solutions. Many languages were considered before AutoLISP, edited in Visual Lisp, was selected as a dialect due to its object-based nature which facilitates implementation, using AutoCAD 2012 as a platform. The script was developed in a similar way to the grammar described in this section (Figure 6.21). It focuses first on the plan view in 2D and then evolves to encompass the overall modelling of the villa, allowing for immediate visualisation of the design results in real time. The rules are implemented using different functions that can be called up recursively if desired and form and function are addressed sequentially:

1. Stage 1 consists of the overall definition. The user is asked to choose the insertion point and area and the boundary is set using Palladio's proportion ratios, either a square, a $\frac{1}{2}$ rectangle or other appropriate ratios. The minimum and maximum areas are given as output and the user's input is optimised to the proportional ratios. Once the boundary is drawn, an indicative matrix is inserted using Palladio's Vicentine measurements (1ft \approx 0.354m). The dimensions are rounded off to this system and the proportions conform to symmetrical classical principles.
2. Stage 2 allows the user to intervene in the design of the interior layout. Subdivision rules allow for horizontal and vertical divisions and the mouse input is registered on screen, with location justified to the nearest possible position in the Vicentine matrix. Both rules can be recalled a number of times, restricted only by the remaining free space and maximum grid size.
3. Stage 3 involves spatial concatenation using a process of cell border deletion. The user can erase lines to generate a larger room and the input is recorded and repeated when necessary to maintain symmetrical layouts.
4. Stage 4 transforms indicative lines into masonry wall dimensions. The exterior walls will be 2 Vincentine feet deep and the interior walls 1 foot deep. The offset process takes the centre line drawn and reproduces two parallel lines in separate layers.
5. Stage 5 reads the design accomplished so far and positions and sizes the entrance and rear porticos, emphasising the north/south symmetry axis with a similarly sized central bay.
6. Stage 6 creates openings in walls for doors and windows. Windows are added symmetrically to the façades in an arrangement corresponding to Palladio's specification for even numbers. Every room should have an even number of windows. Exterior doors are arranged along the symmetry axis of each façade in uneven numbers.
7. Stage 7 involves the assignment of functions, accessing the data input and addressing functions in terms of spatial situation (central or peripheral), connectivity (first level

connection), adjacency and type (living, social or distribution areas). Highly integrated spaces such as the central area connect with other rooms to create social areas. Rooms positioned in corners and represented as squares or rectangles with ratios of preferably $2/3$, $3/4$, $4/5$, $3/5$, $1/2$ are living spaces (studies or living rooms). Rooms between other densely connected spaces (i.e. with two or more connections) that are smaller and oblong (with proportions of $1/2$, $1/3$, or $1/4$) are distribution spaces (circulation areas, corridors, staircases and ante-chambers). This function labels each space and creates the adjacency diagram by connecting the adjacent circulation areas.

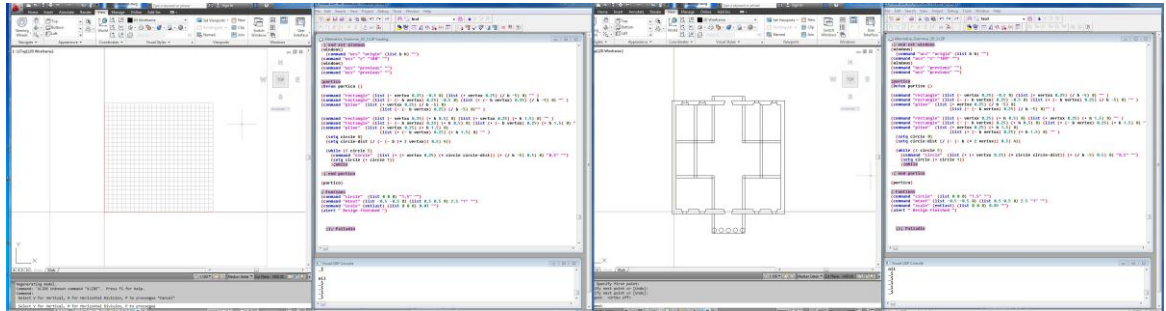


Figure 6.28: Computer implementation

6.9 Chapter results and conclusions

The main breakthrough for this Chapter is the delineation of an operative alternative grammar for the Palladia villas. To the extent of our knowledge no two grammars have illustrated the same subject whilst using different formulation. Despite the different processes and even approach both grammars proved to attain:

1. existing corpus of solutions
2. new corpus of solutions that are share from both grammars
3. independent corpus of new solutions for each grammar

This is significant as it points towards two ways of achieving similar results whilst using different processes. One grammar proposes a methodology based on specific additive shape rules, the alternative a formulation based on the subdivision rule. (Benros, Duarte, Hanna; 2012 and 2014).

It was observed that the latter produces results in a more economic manner and uses a method more intuitive to designers.

The alternative grammar also allowed:

1. the design of existing examples that the original grammars did not
2. the design of existing examples that previously constructed rule based systems did not allow
3. a novel set of design solutions not allowed by previous studies

However, if one or the other is more efficient that is not necessarily worth discussing. The novelty is that this proves that two independent grammars can produce overlapping results and here lies the contribution to shape grammar theory.

The univocal relationship between the principle 'To one corpus corresponds one grammar' is now put into question. Likewise, the principle 'To one grammar corresponds one corpus' is also up for discussion. In previous chapters we have proposed generic shape rules. These rules were presented parametrically and using a rule schema. If the validity of the first principle was proved wrong by the present chapter is now worth checking the validity of the second principle too.

Another conclusion drawn is that the most prolific grammar is also the most flexible. This flexibility does not correlate directly with the number of shape rules available in the set. The flexibility does not correlate either with the extent of the grammar formulation. As observed the original Palladian grammar proposed an extensive set of rules divided within 8 stages.

Grammar flexibility can be achieved by a set of specific shape rules that graphically showcase spatial relations and parametrically allow for a wide range of parameters whilst maintain the desired relationships.

Flexibility can be 'quantified' by the number of novel solutions the grammar can generate, and (perhaps more importantly) the full inclusion of all available existing solutions within a language,

7.	Generic Grammar	239
7.1	Generic shape grammar methodology	241
7.2	Shape Grammar formulation	247
7.3	Generation of solutions and the derivation process	253
7.3.1	Derivation of Palladian villas	255
7.3.2	Prairie House derivation	256
7.3.3	Malagueira single-family house derivations	265
7.4	Generic grammar syntax	267
7.5	Generic Grammar structure	270
7.6	Generic grammar validation	290

7.1 Generic shape grammar methodology

The methodology used to construct a new generic grammar with a process of analysing grammar types. This generic grammar targets different architectural contexts and styles started. The hypothesis determined that the three different branches of grammars (grid, additive and subdivision) could be derived from a common branch named generic grammar.

This branch would contain the grammars and many others. On a low level, the three-diagram hierarchy hosts grammars for specific architectural styles. Apart from the common ground derived from the generic grammar, it contains the specific variations and shape rule parameterisations responsible for the typical style features of each language. They may be as general as envelope ratios and shapes for decorative elements such as columns and niches, but characteristic enough to personify style.

Each of these grammar types is characterised by the recursive and consistent use of the shape rule types, namely the use of grid, addition rules and subdivision rules respectively. This does not mean that each grammar uses one type of rule exclusively, but that the most common sets of rules conform to one of these types. Grids can be produced by the use of addition rules (as in the Palladian grammar, which used simple cell addition rules to create the main grid and still comply with the symmetry criteria) or subdivision (the Malagueira house types may resemble grid floor plans, even though this is achieved by a succession of subdivisions of an overall shape). Another important distinction that can be made between the three grammar types is self-inscription within the envelope boundary. Some housing floor plans are immediately contained within a bounding shape (usually a closed polygon). Other envelope shapes are obtained by addition and subtraction operations which result in complex polygons or, to be more precise, irregular polygons. These shapes can also be inscribed in regular polygons or rectangles when their extreme edges coincide with the bounding shape limits. However, in the last scenario this justification for the bounding shape is to some extent forced and usually only possible once the overall design is concluded. It occurs when top-down design approaches are used (such as Wright's Prairie House scheme), whereas contained shapes usually emerge from bottom-up approaches. The method which unifies the approaches always uses a rectangular bounding box that surrounds the edges of the design.

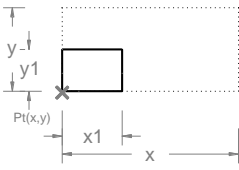
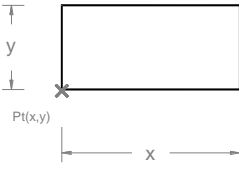
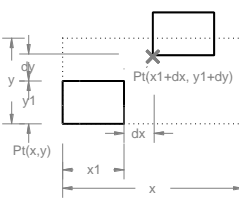
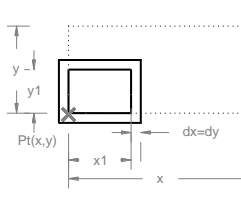
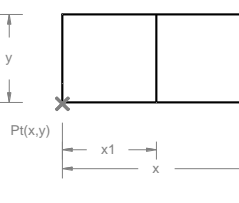
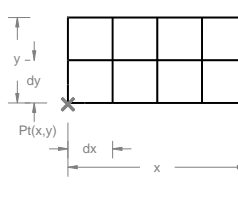
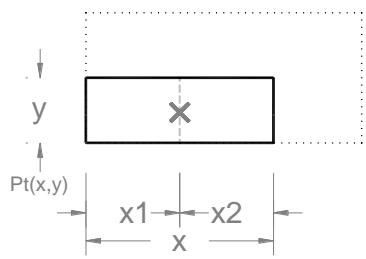
Top-down or bottom-up approach:			
Self-contained within boundary?			
No			Yes
Bottom-up		Top-down	
Not-contained			Contained
	Pt(X,Y)		Pt(X,Y)
	A= X x Y A1 =(X1xY1)		A= X x Y
Operation?			
Addition		Subdivision	
Simple	Offset	Subdivision	Grid
			
Following steps			
Detailing:			
Concatenation			
1.			
			
2.	Subtraction		
3.	Addition		

Figure 7. 1: Self-contained shape/bounding box diagram

With self-contained shapes this bounding box coincides with the limits of the envelope (as in the Palladian villas) or the limits of the plot (as in the Malagueira houses). For designs derived from shape addition contained by irregular polygons, a bounding box is applied surrounding the limits of the overall geometry. This is illustrated in the diagram above in Figure 7.1.

Once the boundary is defined and set, the detailing takes place. From this it may be concluded that the generic grammar uses a top-down approach.

The next step, true to the nature of subdivision grammars, is a subdivision process in which a series of divisions take place within the bounding box. This process will take the specific features of each style into account, with the shape rules customised to reflect each style in the first stage. One example of this is the repetition of vertical divisions in the Palladian villas to guarantee symmetry, or the angular divisions to allow 1/3 ratio triangles for the Malagueira houses. Simplifying a generic division rule therefore involves taking the bounding polygon shape and splitting it into two sub-shapes. The specific or particular rule splits the same bounding shape into two shapes with a certain shape rule parameterisation in which different conditions compete to ensure design quality and maintain consistency in terms of the permitted shapes, proportions, angles, maximum and minimum dimensions and supported areas.

All generic shape rules described in previous chapters (addition, subtraction, offset and concatenation) can be applied in the generic shape grammar. Obviously the subtracting rule comes across as a fast track to subdivision. In fact the subtracting proposes the geometric difference between two dissimilar polygons that coincide in one or more points. This process is illustrated by the following picture in which a smaller rectangular shape is subtracted from a larger rectangle with one coinciding vertex and two sides. The result is a six-sided enclosed polygon with a greater level of complexity and more vertexes than the original shape.

The same result could be achieved with two successive shape subdivisions, one vertical and one horizontal division, each coinciding with at least one side of the shape, followed by a deletion process. This proves that the subtraction and the subdivision rule can be condensed or reduced to two, even though the latter requires a greater number of steps to obtain the same result. However, elegance, economy and speed do not necessarily translate into simplicity or logic. For simplicity of use, subdivision remains the most useful rule for now and the one responsible for naming and typifying the shape grammar.

The subdivision stage uses the subdivision rule recursively a number of times. The number of iterations is limited by the grammar itself and the expressions of the architectural language.

Once the basic layout has been obtained, other stages follow. One important stage following subdivision is concatenation which, in fact, constitutes the first stage in which modification occurs. The designed borders are manipulated within the previously designed layout to accommodate the required spaces. Concatenation/cell merging constitutes the deletion shape rule type $X \rightarrow X - \text{part}(b(X))$ in which X symbolises the original shape and 'b' its border, whereas 'part' is a portion. In other words, part of the boundary of the original shape is removed or deleted. The resulting solution ' $X - \text{part}(b(X))$ ' has a larger total area than the original X area, since it will include the adjacent cell. The rule comprises a deletion and an addition.

The assignment of functions is directly linked to each of the styles pictured in the three original grammars. It is often associated with the shape rules and cannot be separated from them, although in the case of the Palladian grammar this is open to discussion. As explained in previous chapters, the original grammar did not reflect on functions and Palladio's "Il quattro libri" did not discuss the issue. Extrapolations were made, however, to incorporate this housing dimension into the grammar. The grammar also aimed to address functions in a unified manner. Regardless of the housing type to be generated, functions occur parallel to shape generation and are introduced at the time of the first shape split (sub-division). Like the progression and definition of shape, which is successively detailed, function is first drafted generically (i.e. interior vs. exterior, public vs private) when some first level zoning is introduced, then progressively detailed and separated into individual rooms with particular uses (i.e. living room, study, entertainment space, kitchen, dining, bedroom, etc.).

Function assignment rules are usually additive rule types which can be expressed by $\emptyset \rightarrow X$, or even $X \rightarrow X + y$. They can also be alteration rules type $X \rightarrow Y$ when a function is being refined or detailed, having been changed from a zone type to a functional room (i.e. the interior is replaced at some stage by a living area and later by a study). These particular rules vary according to house type and address specific stylistic features.

Adjacencies play an important role in defining space, as can be seen in the figure 7.2 provided. The rooms that are immediately adjacent to a certain space help define each room and therefore the style itself, as can be seen in the diagrams illustrated in Figure 7.2. Palladian adjacency graphs are usually radial and this is explained by the nature of the house and its social role of providing an entertainment scenario. The main and central room in the plan is also the focal point of the design and in some way all the rooms are connected to it, either directly or via an intermediate room. Prairie houses present a diagram that resembles a tree structure. The focal point is, by definition, the fireplace, as the centre of the home and family life. However, this focal point occupies the geometric but not the functional centre. This is because it is centred within the social/living quarters of the house but, due to the way in which the circulation, services and other areas are linked, breaks with centrality. The Malagueira grammar presents an almost grid-like or spider-web spatial distribution. Spaces are linked on the basis of direct adjacency connectivity and therefore one space connects to the space immediately adjacent to it and the next one to the following space, thus creating a path from the public and more exterior layers towards the most intimate and interior spaces. The methodology is based on three consecutive tasks: the development of generic shape rules, the creation of a generic grammar formalism and structure and lastly the development of specific parameterization to represent different languages. The strategy used to create the generic grammar followed this principles:

1. Top-down approach
2. Self-contained strategy based on a polygonal boundary
3. Subdivision as a method to provide detail
4. Common shape rules to address the generation process
5. Variation and detail conferred by the parameterization in each shape rule

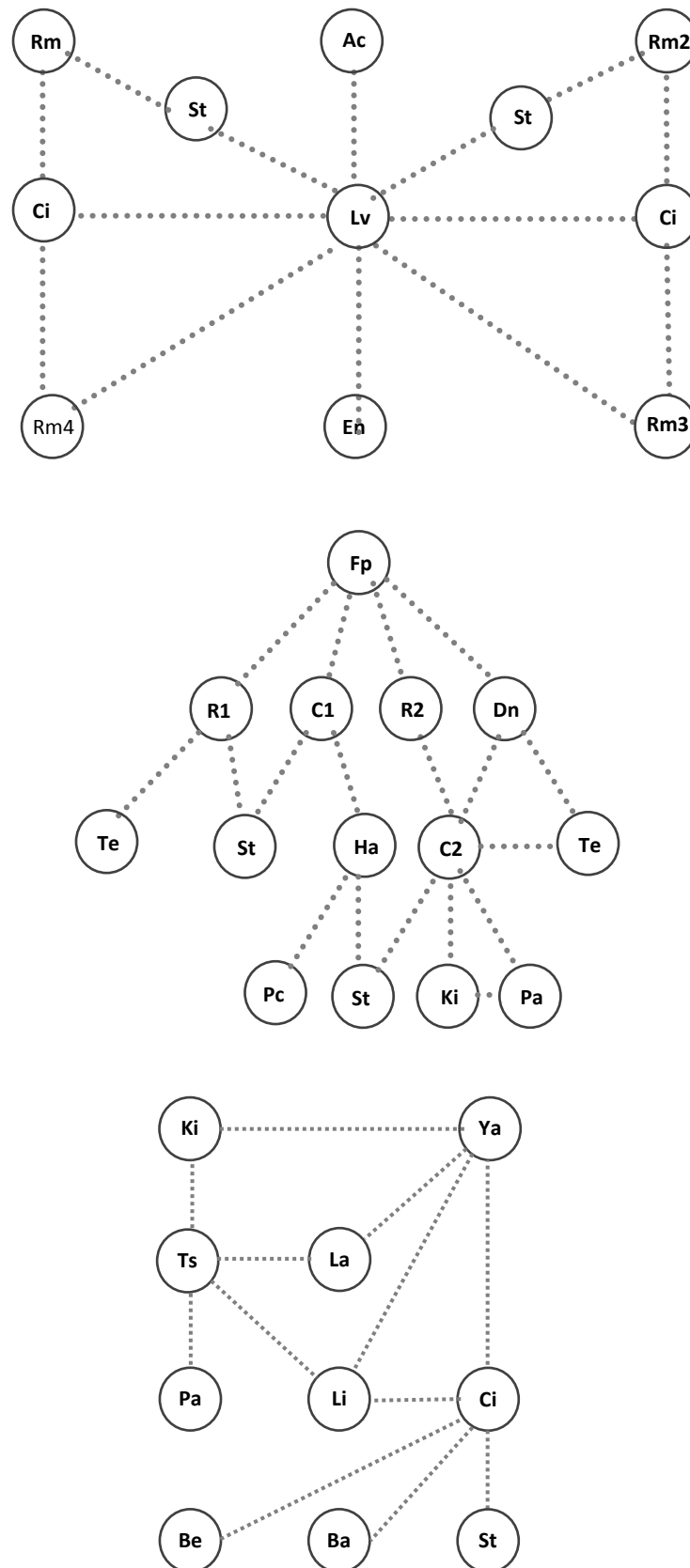


Figure 7. 2: Adjacency diagrams for the three grammars (Palladian, Prairie and Malagueira)

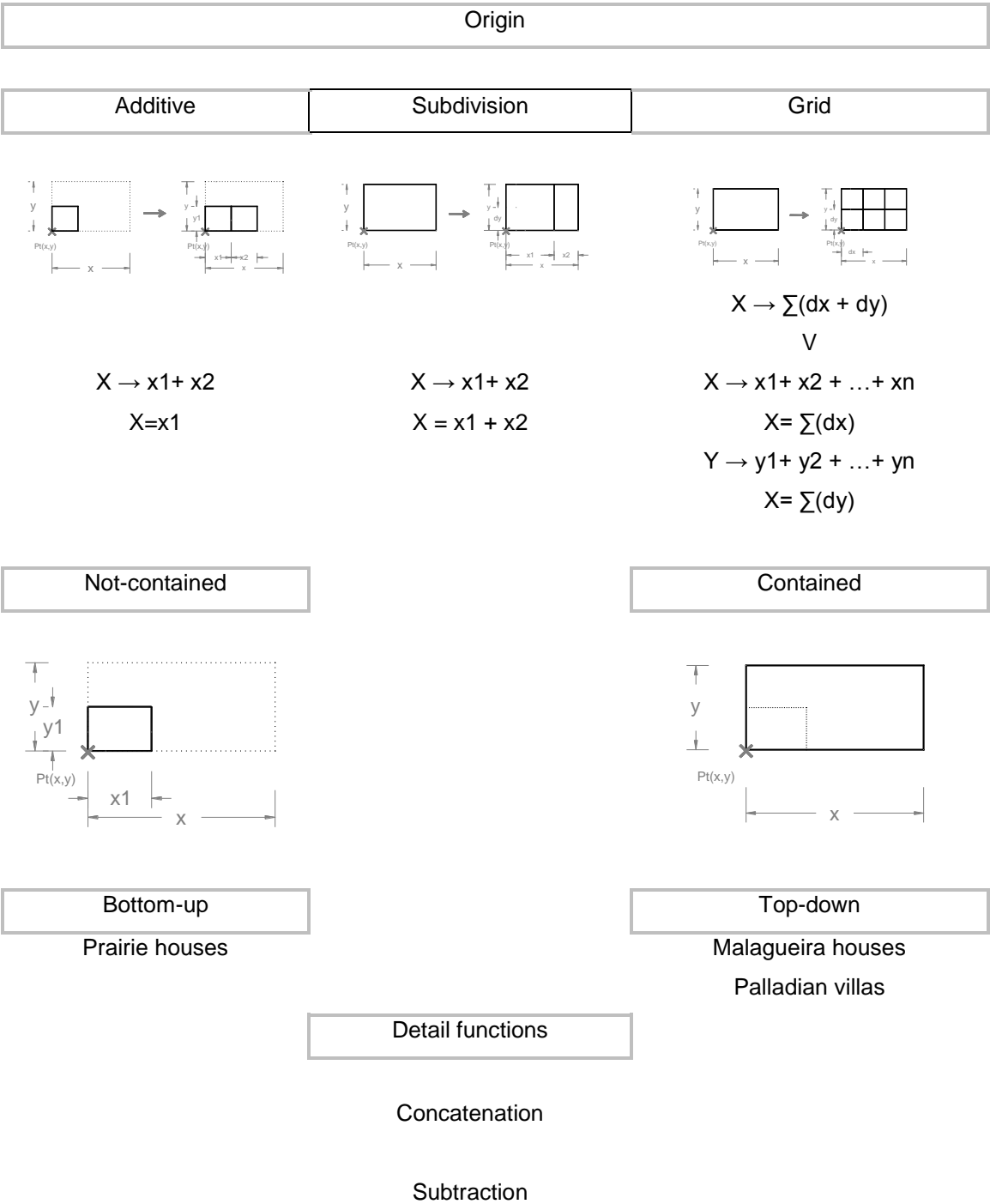


Figure 7. 3: Top-down and bottom-up approach diagram

7.2 Shape Grammar formulation

A generic shape grammar was developed for the creation of Palladian villas, Prairie houses and Malagueira houses, supported by the choice of a top-down approach using a subdivision grammar. A new grammar structure and a new set of parametric shape rules were developed.

The alternative grammar for the Palladian villa uses a methodology developed by Duarte (Duarte 2001) for the Malagueira house and later adapted to an urban context for the Marrakech grammar (Duarte et al. 2006). The subdivision grammar famously implemented by Stiny in the Ice-Ray grammar (Stiny 1977) encompasses a very simple shape rule set of sequential subdivisions of polygons. The grammar showed how a polygon could be divided in two, generating complexity just by introducing diagonal cuts into a rectangle and sequential cuts to triangles and pentagons. The end result was a complex ice-ray window frame that resembled the traditional Chinese ice-ray lattices. This proved how a simple five-rule grammar could lead to an almost endless set of new solutions. The Malagueira grammar used a similar process, starting with the boundaries of the plot and evolving through subdivision into smaller inner spaces.

Palladio's villas always propose a rectangular envelope geometry with fixed proportional ratios. This rectangular outline can be further divided to originate each room. Likewise the use of subdivision is a common design choice for architects, this was the process selected to accomplish the task.

A new set of rules was developed to address all three grammars. The selected grammar does not try to replicate the process of the designers' work or methodology but instead aims for a generic formulation that can accommodate three instances of design that are not only independent but also dissimilar.

It attempts to achieve a consistent solution as efficiently as possible, whilst also creating a system that is easy to use. The grammar adopts a top-down approach and uses subdivision.

The new generic grammar allows for eight stages from start to finish and the design of three types of houses, namely Palladian, Prairie and Malagueira.

Any grammar or rule manipulation will generate a hybrid version of the houses. This is avoided by parameterization and labelling. Several conditions apply to design a solution within a specific language.

As shown in the tree diagram 7.4 the sequence of stages in the grammar are:

1. The first stage designs the envelope shape or boundary. In all cases it is a 4-sided polygon or rectangle. For non-rectangular envelope geometries such as those for the Prairie houses, further geometric transformations are required to detail the envelope
2. The second stage introduces the subdivision. Two rules can be applied at this stage, namely horizontal divisions or vertical divisions. Vertical divisions provide double divisions in cases like the Palladian villas, where bi-lateral symmetry has to be observed. At this stage, both rules can be applied recursively a number of times to the point where the maximum grid with the maximum number of cells is reached (a condition for the Palladian villas) or the divisions fill the minimum space required for a living or circulation area. Other proportional aspects are monitored to ensure that design consistency is maintained.

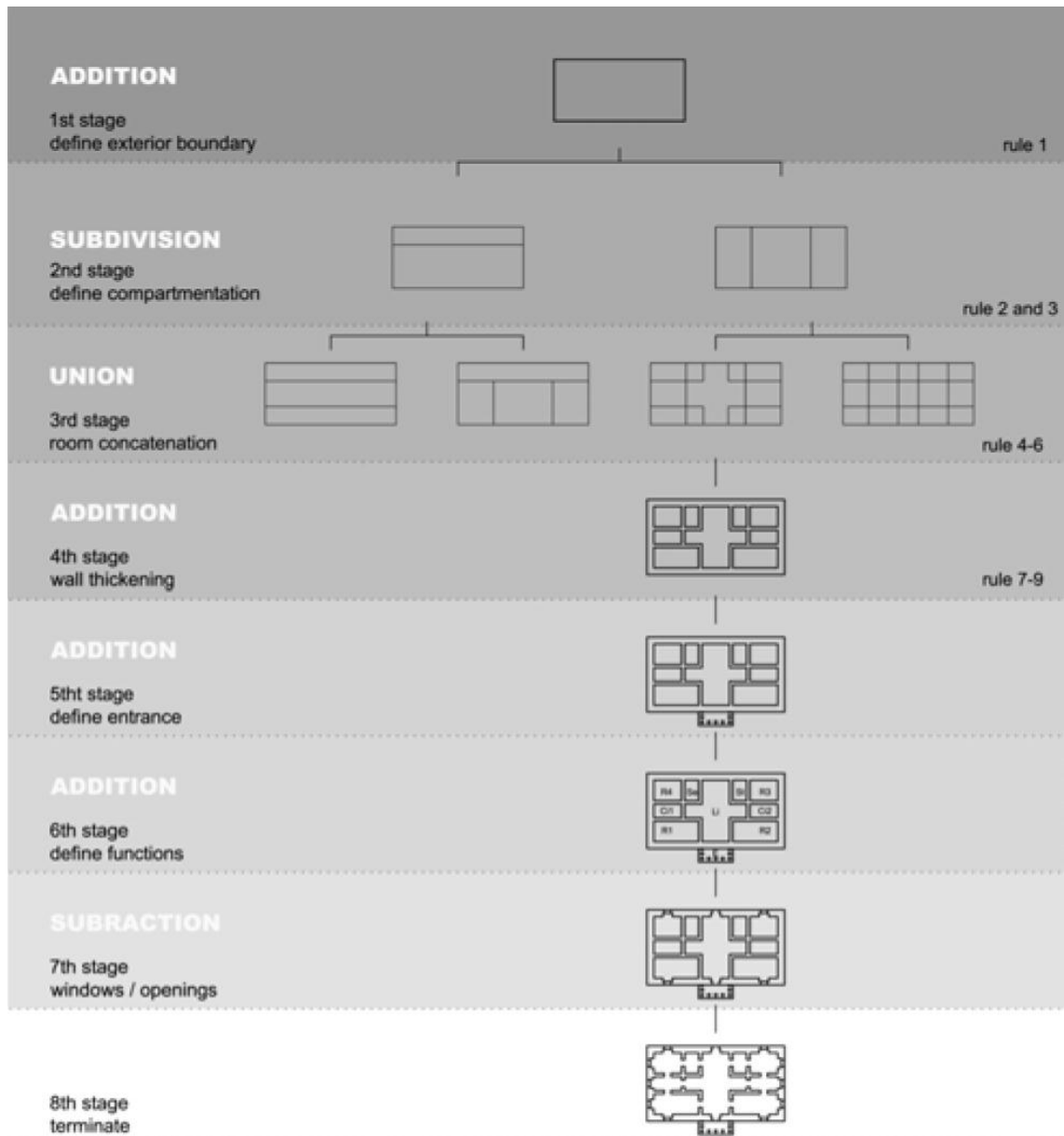


Figure 7. 4: Generic grammar tree diagram illustrating the derivation of Palladio's Villa La Malcontenta

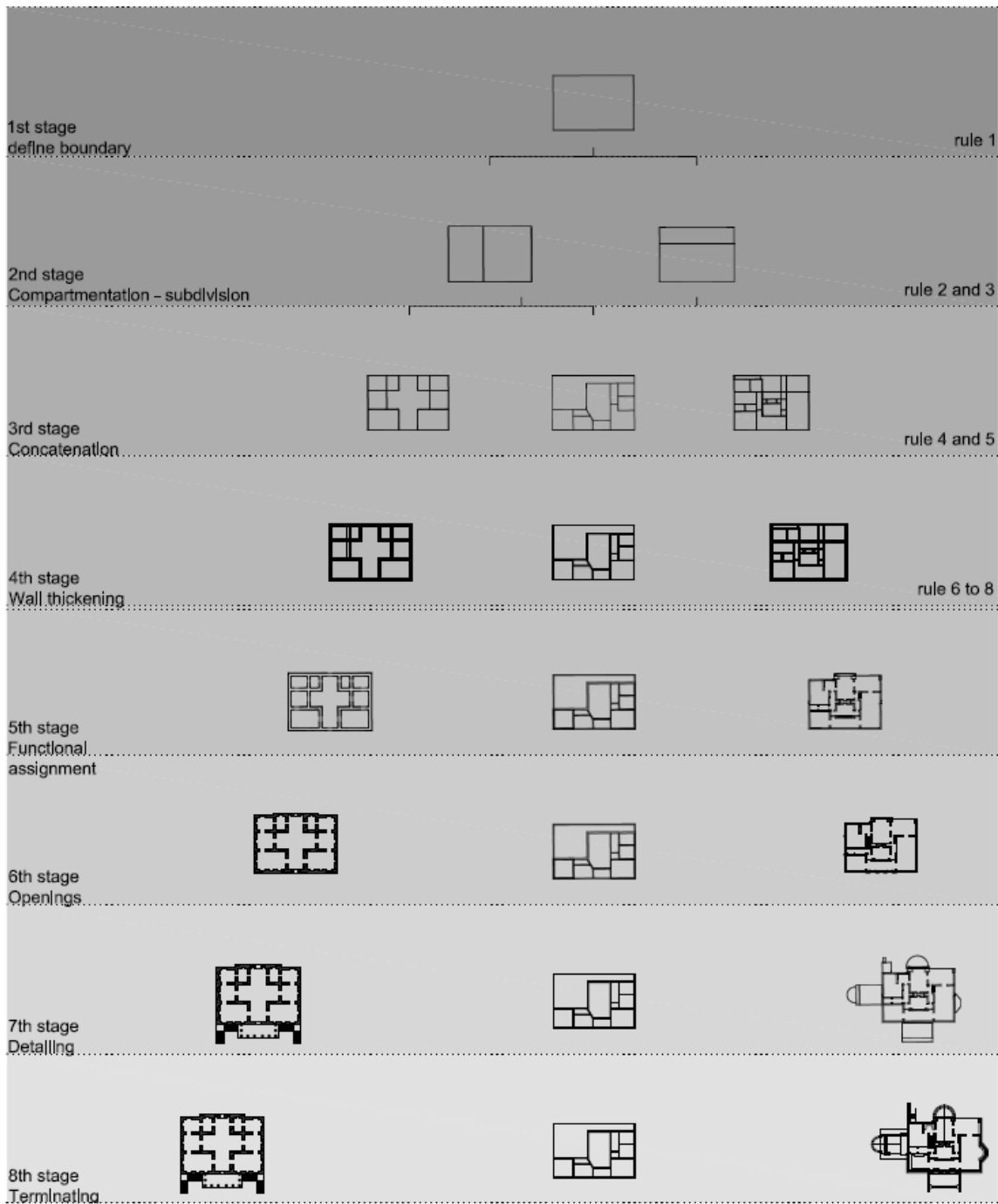


Figure 7. 5: Generic grammar tree diagram

At the end of this stage the outline of the interior layout should be visible or at least intelligible. This outline will be made clearer in the next stage.

3. Stage three encompasses two basic design rules for concatenation and cell merging or, to be more accurate, cell border deletion. Both rules follow the symmetry conditions for the Palladian villas and involve a simple formulation for the Prairie and Malagueira houses. Rules 4 and 5 present two simple border deletion methods for vertical and horizontal situations, as shown in Figure 7.4. The final interior layout is now complete and the design progresses to other aspects of construction.
4. The fourth stage proposes wall thickening. Limits and spaces were previously represented by bi-dimensional lines but at this point, Rules 6 to 9 are responsible for converting either lines or corners into proper wall representations with a double line to represent a specific thickness. In the case of the Palladian villas, the thickness of the interior walls reflects the masonry technology used and they are relatively thick, whilst the exterior walls are even thicker. The rules address these differences and convert corners and intersections from 2D abstractions to standard thicknesses.

Figure 7.5 illustrates the generic formulation and generation of the three style branches consisting of the Palladian, Prairie and Malagueira houses. As shown after the fourth stage, (highlighted by a double dotted line) the particularities of each style are added.

This stage marks the conclusion of the common branch of the generic grammar. Further customised steps will be exclusive to each specific style.

It is important to note that the main house layout is now set and defined and it is the detailing stages that will determine the particular features of the house style. This does not mean that the main style features have not been included prior to this point. On the contrary, in each of the generic rules for the common branch the style is controlled through the exclusive parameterisations of the shape rules. One good example is the first rule that determines the external boundary. The Malagueira houses have fixed dimensions predetermined by the plot. Prairie houses propose specific rectangular ratios, as described in the villas designed by Palladio and even the Prairie houses abide by minimum and maximum bounding box ratios.

The generation process in the steps which follow is dedicated to detailing.

5. The fifth stage of the generic grammar proposes an addition step, enabling certain features of each style to be added to the exterior, complementing the envelope with entrances, porticos, porches, exterior spaces and/or ornamentation. In the case of the Palladian villas, this is the stage when the entrance is defined and porticos and ornamentation, such as decorative classic columns or loggias, are added. In the case of the Prairie houses, elements such as the corner volumes occasionally fitted to concave corners of verandas or terraces are incorporated. In the Malagueira houses, the small divisions or service spaces added to concave corners can be introduced at this stage. This constitutes the second additive process in the procedure.

6. The sixth stage includes internal functions. The introduction of functions represents an innovation in terms of the original Palladian grammar. Space and function is the issue covered least in Palladio's Four Books (Palladio 1997) and never addressed or labelled in plans or other drawings. It relates to the social nature of the 16th century rural Italian aristocratic villas built for the purpose of entertaining, whose main floors contained a series of rooms ranging from ante-chambers to ballrooms, drawing rooms, libraries and studies which are not always easy to identify. It was therefore necessary to research this area and through observation and comparison some assumptions could be made about space which were introduced into the generic grammar. Both the Malagueira and the Prairie house grammars addressed functions and originally proposed shape rules associated with special meaning.
7. The seventh stage constitutes a subtractive stage for introducing a greater level of refinement. At this stage openings are made in walls and interior walls can be removed to create internal circulation and incorporate doors. The exterior walls and facades can accommodate entrances and windows. Geometry, proportion and window positioning vary greatly from one house type to another. The rules were written to accommodate these differences and take into account the symmetrical features needed for the Palladian villa. One specific function is associated with each space, created in accordance with the shape rules in order to maintain spatial flow and coherence and avoid the overlapping of functions, awkward adjacencies or spatial relations not envisaged as part of the original style
8. The eighth and final stage completes the design by deleting the construction labels.

The rule structure was based on a set of rules that can be reduced to a number of shape rules that obey either addition, subdivision, concatenation, subtraction or replacement. This can be represented by schemas. Schemas try to transpose the graphic description into a simple algebraic expression. This allows for a certain level of abstraction while applying a rule. Schemas represent the shape rule without using graphical symbols. Often grammar users are limited by a graphical representation and restricted by the use of a particular rule but its abstract notion can avoid these misconceptions.

The four generic rules, as identified in chapter 4 with a specific schema are:

1) Addition:

$$\begin{aligned}\emptyset &\rightarrow X \\ X &\rightarrow X + t(X)\end{aligned}$$

2) Subdivision:

$$\begin{aligned}X &\rightarrow \text{div}(x) \\ X &\rightarrow \text{prt}'(x) + \text{prt}''(x) \\ (X' + X'') &\rightarrow \text{prt}(b(X' + X''))\end{aligned}$$

3) Concatenation:

$$\begin{aligned}(X' + X'') &\rightarrow X \\ \text{prt}'(x) + \text{prt}''(x) &\rightarrow X\end{aligned}$$

4) Subtraction:

$$\begin{aligned}X &\rightarrow X - \text{prt}'(x) \\ (\text{prt}'(x) + \text{prt}''(x)) &\rightarrow \text{prt}'(x)\end{aligned}$$

In addition to the identified generic rules specific parameterization was developed. Along with the graphic representation and each schema the parameterization for each rule was specifically developed to cater to each language.

7.3 Generation of solutions and the derivation process

The recreation of original designs using the method described above is illustrated in Figures 7.6 to 7.11.

Derivation is the exemplification, from start to finish, of the phased application of the shape grammar rules. Often, the faster the derivation, the more efficient, elegant and easy the grammar is to use.

In this experiment three existing houses designed by the original architects were selected to illustrate the generic grammar. Villa Malcontenta is an example of a typical Palladian villa, the Robie House, one of Wright's most famous creations, illustrates the existing corpus of Prairie houses, and the Malagueira two-bedroom Ab house type (according to Duarte's labelling) exemplifies a typical Malagueira family housing unit.

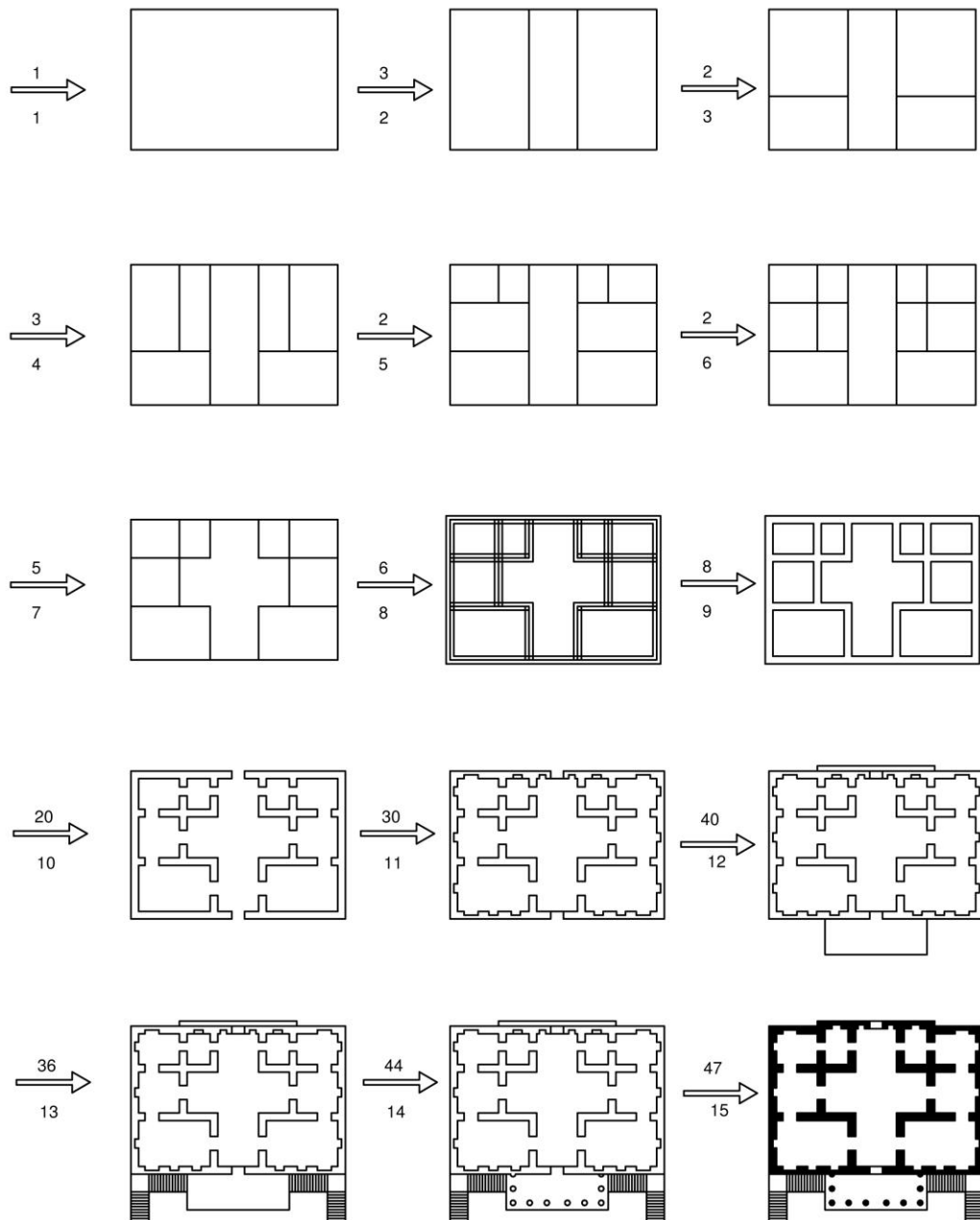


Figure 7. 6: Generic grammar derivation – Palladio's Villa La Malcontenta

7.3.1 Derivation of Palladian villas

La Malcontenta was originally designed, built and completed in Venice between 1559 and 1560 and is pictured in the 'Il quattro libri' [10]. Its orthogonal features and grid-like floor plan features a matrix that resembles a 5x3 grid. Whereas the original grammar used a grid process, producing the same design with subdivision allows us to economise on certain steps (namely extensive concatenation). The new tree diagram featuring the generic grammar is shown in Figure 7.5 and the Malcontenta derivation in Figure 7.6.

The envelope is therefore designed and established from the start. This results can be described as:

1. in the first derivation step with the application of Rule 1 (adding a four sided polygon)
2. Step 2 addresses the first main stage of the subdivision process by applying Rule 3 for vertical subdivision. As shown, this subdivision is doubled to address the symmetrical nature of the design
3. Steps 3 to 6 use the division rules 2 and 3 recursively (in the case of Rule 3, repeated again and again).
4. Steps 7 and 8 start the space merging or concatenation process. This is a fundamental step for spatial configuration in a Palladian villa. The core space or social area is often the, largest, most central and geometrically complex area in the villa. This complexity is only achieved by combining adjacent cells to form a broad regular polygon. Other larger rectangular spaces are positioned at the edges of the construction facing the facades
5. With the layout set, the 9th derivation step continues with the wall thickening, applying Rules 6 to 9.
6. The 12th step assigns functions to the previously designed spaces, and the next step creates detail and prepares for spatial articulation with the insertion of openings such as doors and windows
7. Step 13 adds new elements attached to the exterior of the envelope, namely the entrance portico
8. The villa is finished in Step 15 with the deletion of labels.

In comparative terms, the derivation of the Villa Malcontenta using this alternative method is faster than the derivation using the original grammar which is shown previously in Figure 4.1.

7.3.2 Prairie House derivation

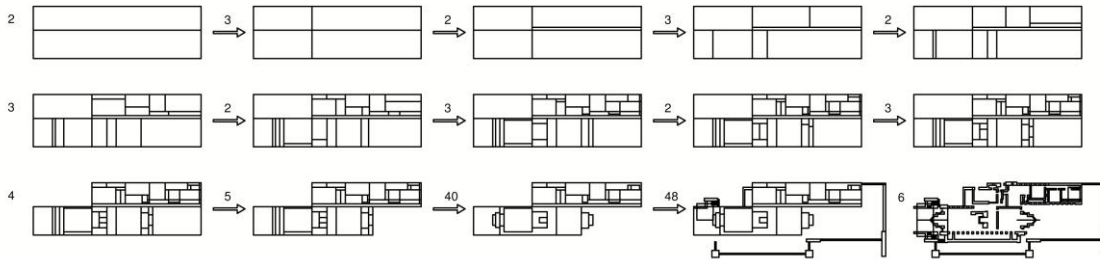


Figure 7. 7: Generic grammar derivation – Prairie Robie house

The derivation of Prairie houses using the generic grammar takes a certain level of abstraction into account. Most of the existing Prairie houses follow orthogonal principles and use polygonal shapes. With the exception of the rectangular, self-contained Prairie Winslow and Cheney houses that can easily accommodate the rules and produce an operative alternative derivation in a few steps, most have a butterfly/cross-shaped envelope. Prairie houses are created by the generic grammar using a boundary that extends to its extreme edges.

A new rule was introduced to erase the container edges after the core house was completed.

For this reason, the example chosen to illustrate the Prairie house is the Robie house, which is a unique, characteristic and well-known example.

1. As in the Palladian example, the containing envelope shape is inserted in the first step using Rule 1. This rectangular shape does not contain the construction boundary, but illustrates a containing shape that encompasses its maximum limits
2. The subdivision process is prompted straight after. The horizontal and vertical division rules 2 and 3 respectively, are used recursively in several stages until all the floor plan lines have been replicated. Step 2, which extends to the next design stage, resumes the lengthy process of subdivision and is not the most intuitive or efficient of processes at times. The process combines analysis of the maximum lines (the lines that connect opposite sides) and their replication using subdivision. The procedure starts by establishing the butterfly cross placing by first using the double division, then continuing to the finer details. Once all the lines are illustrated, the design is close to completion
3. At this point some adjacent cells need to be concatenated to remain true to the style, creating a flow of spaces that are oblong with large areas. This constitutes Steps 9-10
4. In Step 10 all the basic interior design features are represented, including the container envelope shape. This shape is then modelled to fit the design criteria for the style. The unwanted edges are erased in accordance with Rule 4 and 5 the lines from the external vertex to the connecting indoors line is erased, leaving the end results as expected
5. Steps 11, 12, 13 and 14 resemble much of what has been described previously and involve wall thickening, the detailing and creation of openings, and the completion of the design, respectively.

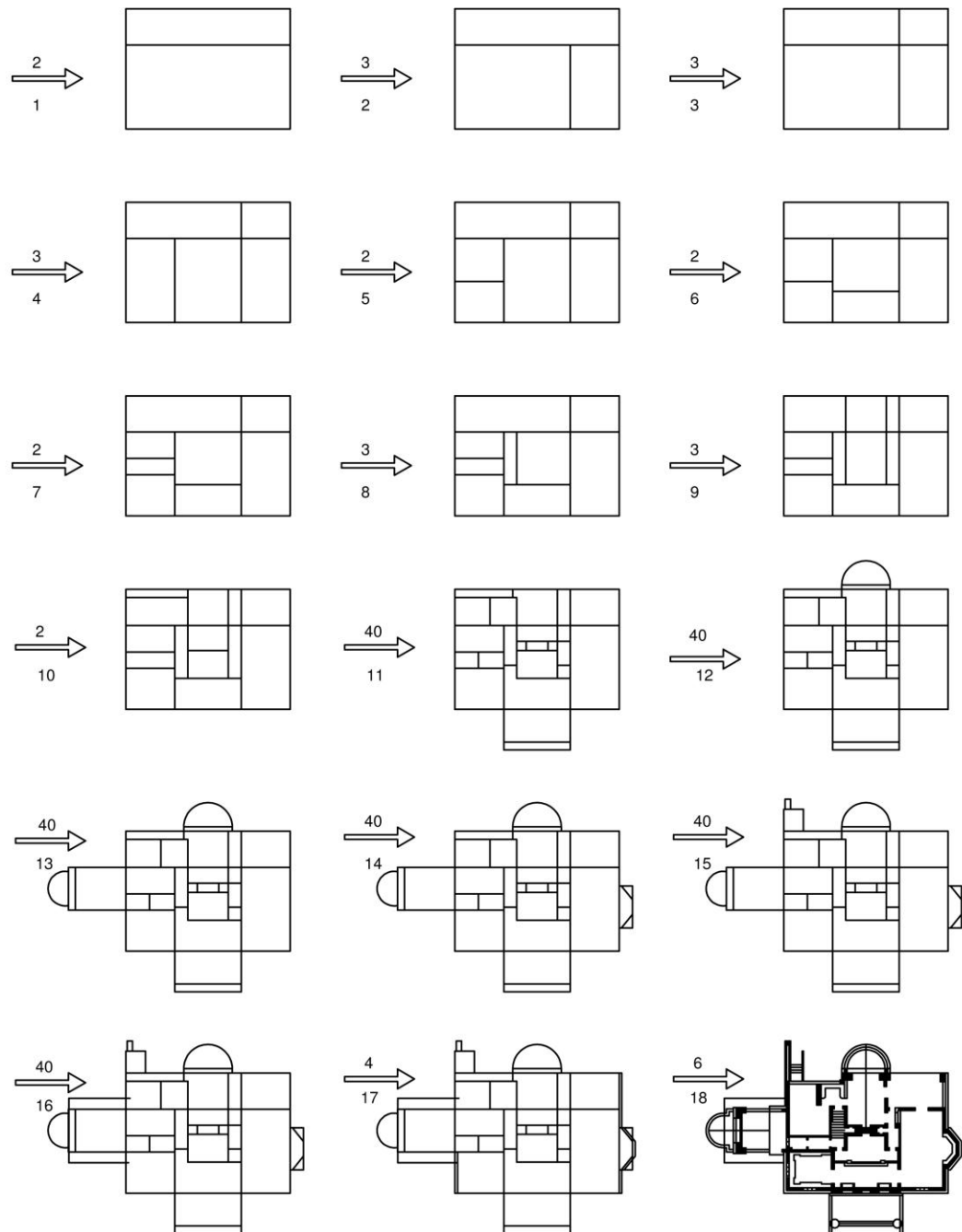


Figure 7. 8: Generic grammar derivation – Prairie Winslow House

The Winslow and Cheney houses are paradigmatic examples of Prairie houses because of their self-contained aspect. Unlike most of the typical Prairie house layouts where a cross (or butterfly-shaped) array is normally created, Winslow can be based on a rectangular bounding shape. The inside layout could easily be described by a series of orthogonal axes creating an effect very similar to a grid or matrix. This may also lead to the conclusion that with further work it would be possible to convert the Prairie House grammar into a grid grammar. The question is the relevance, effectiveness or, even more importantly, the feasibility of transforming this grammar into a more elegant one.

Again, unlike most Prairie Houses where the methodology for producing a consistent design using this generic grammar consists of implementing an abstract bounding shape as detailed in the first step and subtracting further on, in this case the first step is decisive to the delineation of this house. Moreover, the initial shape (derived from the existing design) is very similar to the overall layout of the final solution. This constitutes an exception in which new principles are explored and reflects a paradigmatic example within Frank Lloyd Wright's body of work, a transitional phase in the author's style or even the influence of the client's taste, needs and opinions.

It was also one of the first houses in the style used in this study as a case for derivation and possible conversion from an additive to a subdivision grammar. There are certain resemblances to the Palladian typology, namely the rectangular outline, grid-like interior, use of orthogonal elements, emphasis on the social area of the house which occupies the core of the dwelling and extends through communicating rooms towards the outside, progressively creating areas of privacy versus exposure, the careful and strategic addition of external elements accommodating entrance points and terraces occupying the main symmetry axis, the considered use of symmetry (an uncommon strategy in Wright's body of work), and the hierarchy between entertainment areas of generous proportions, large rooms with external views and the narrow circulation interlayer. In fact, it could be considered a modernisation of the Palladian villa concept, perhaps as a kind of tribute and certainly inspired by some of the principles discussed and tested by Palladio himself.

The derivation of Winslow House can be summarised in 18 steps from start to finish:

1. The first step is the boundary settlement, using a bounding rectangular shape that already describes the final outcome in abstract.
2. The second step involves a typical rule used in the Palladian alternative grammar to ensure symmetry between the east and west wings. This subdivision rule proposes vertical divisions by placing two vertical cuts through the outline created. This divides the space into 3 areas, a central entertainment zone and two peripheral spaces in the east and west. This is the first draft for the social area. Steps 2 to 10 provide a series of divisions that allow for further detailing of the space. Step 1 is the first horizontal division imposed, dividing the west wing into north and south.

3. Step 3 also proposes a horizontal division, this time creating a narrow space destined for circulation.
4. Similarly, step 5 creates a narrow south division which separates the large entertainment zone in the north quarter from a circulation space.
5. Steps 5 to 8 also follow these guidelines and create divisions to accommodate circulation. This is done in order to create transitions between two areas of accommodation and corridors for access. The design is now an overlay of lines that create rectangular spaces in the intersections and a succession of different cells with difficult access.
6. Step 11 to 16 introduce the prairie houses typical external elements, such as porches, balconies and porte-coucherres
7. Step 18 concludes the design process by adding some external areas to the design. In a very similar way to the process used for the Palladian villas, porticos for entrances, verandas and terraces communicating with the entertainment zones are added to the main design to create transitional spaces between the interior and exterior.

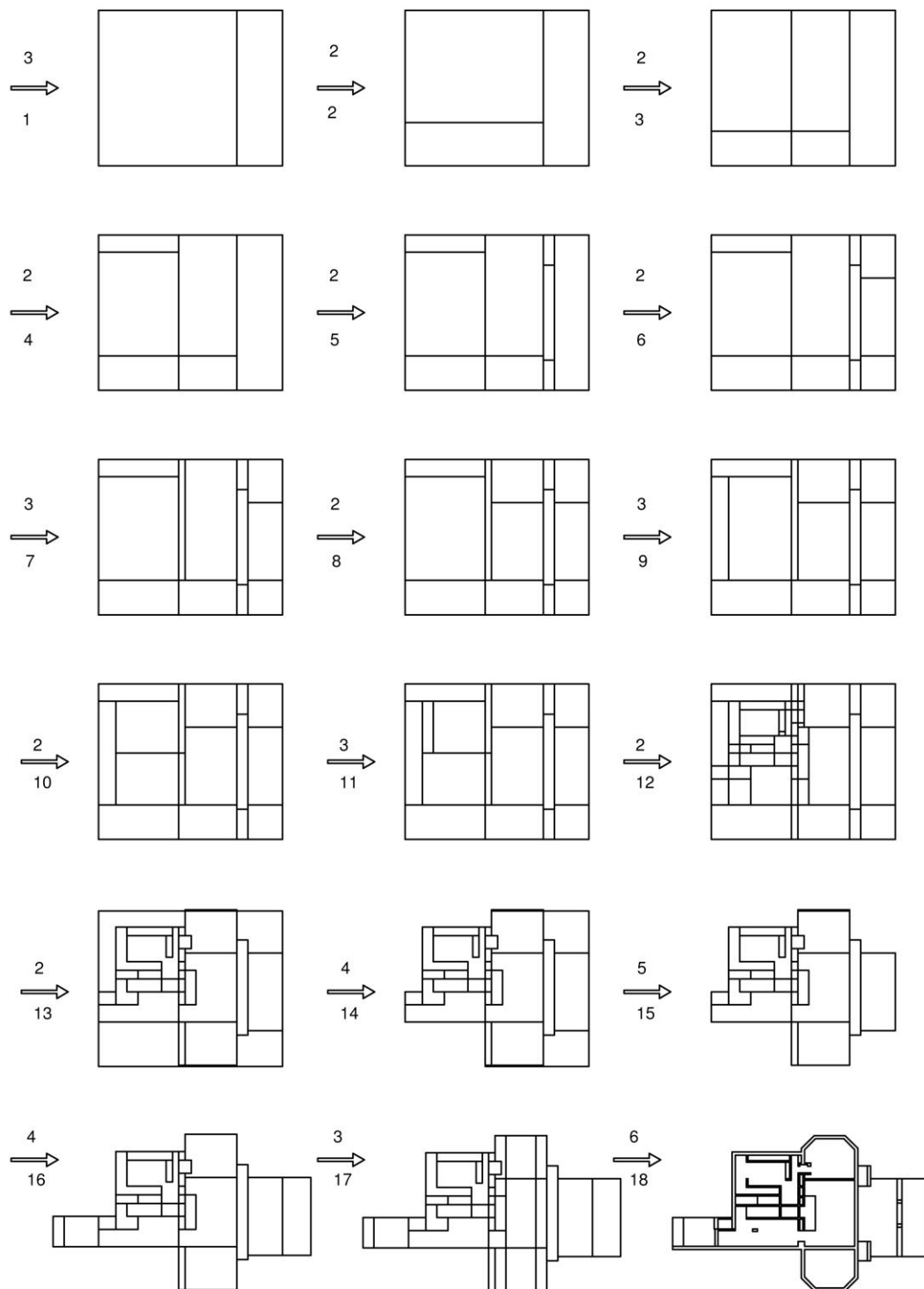


Figure 7. 9: Generic grammar derivation – Prairie Henderson House

Figure 7.9 illustrates the Henderson House derivation. Unlike Winslow House, the Henderson House fits the typical criteria for the so-called Prairie House. It has a cross-shaped layout, is composed of different volumes that combine to create the main house volumes and, in volumetric terms, is mainly horizontally driven with terraces, slabs and canopies projecting over the exterior spaces. It has typically sober decorative features, such as chamfer corners and detailing in concave corners and, last but not least, the functional spatial distribution is based on the focal point of family life, which is the fireplace.

The house floor plan does not suggest subdivision as an intuitive process, mainly because it resembles an arrangement of tree-shaped items that are combined to generate a dwelling. It therefore immediately suggests an additive design process.

Close observation also suggests features such as the orthogonal lines, intersection of different alignments, and the suggestion, albeit subtle, of a bounding box that combines and surrounds all the shapes used and the overall envelope. Experimentation proved what observation had hinted. In fact, when the generic rules were applied to generate this house in a manner similar to previous derivations, it became clear that there was more than one way to generate the Henderson House using different iterations.

The derivation illustrated above in Figure 7.9 takes 12 steps to generate the basic house layout using the subdivision process. As always, the first step is additive and the first and only rule allowed for any example is the addition of the bounding shape. This rectangular boundary lies on the extreme limits of the building plan.

Stages 2 up to 11 describe subdivision processes. In fact, due to the complexity of the house several divisions and space merging operations are performed in order to obtain the final result:

1. Step 1 proposes a vertical division, creating a narrow distribution space in the east and an enlarged area in the west.
2. The next step proposes a horizontal division that creates a clear boundary between north and south and, in this case, a boundary between the social and service areas.
3. Step 3 duplicates the horizontal divisions and creates a room definition. Step 3 creates another narrow horizontal division for a circulation area in the north end of the house.
4. Step 4 also proposes another circulation space characterised by an elongated space, this time by creating a vertical division in the east of the house.
5. Step 5 establishes the limits of another corner space and the neighbouring circulation, allocating a horizontal division in the southwest quadrant.
6. Step 14 and 15 are attempts to break through the division process. It deletes the bounding shape as required to attain the desired crossed shaped floorplan
7. Step 16 proposes external elements to the overall dwelling. It proposes the detailing of small portions of the solution and, in this case, allocates volumes and decoration to the concave corners (very much like the original Prairie House grammar (Koning & Eizenberg 1981) proposed by Koning and Eizenberg). This could also be achieved by using subdivision methods.

However, to replicate these additions two subdivisions would need to be included (one vertical and one horizontal), followed by a deletion process. For the sake of economy, this seemed the most efficient way of achieving the expected result.

8. Step 18 constitutes a different approach to the design and one of the most important processes described so far. It consists of the selective boundary deletion process, expressed in the generic grammar by Rule 6 and wall thickening.

The layout of the floor plan is achieved in abstract, but lines overlap and continue throughout the limits of the house. A process of cell border deletion is carried out to achieve space merging, which allows for the creation of larger spaces or the design of shapes that were originally just rectangles. Space merging or concatenation enables polygonal spaces to be created with a greater number of sides, proportions and configurations.

The final stage, which proposes the selective boundary or bounding box limit deletion, is a unique process and a key aspect of this generic grammar for Prairie houses. .

It allows the overall shape, derived from a rectangular container, to evolve towards a conglomerate configuration. Interior and exterior spaces are demarcated and segregated. The new house boundary is determined by this border between inside and outside space, where functions have the important task of defining these limits. All the border segments that lie between exterior spaces are rejected and erased until the design process is concluded.

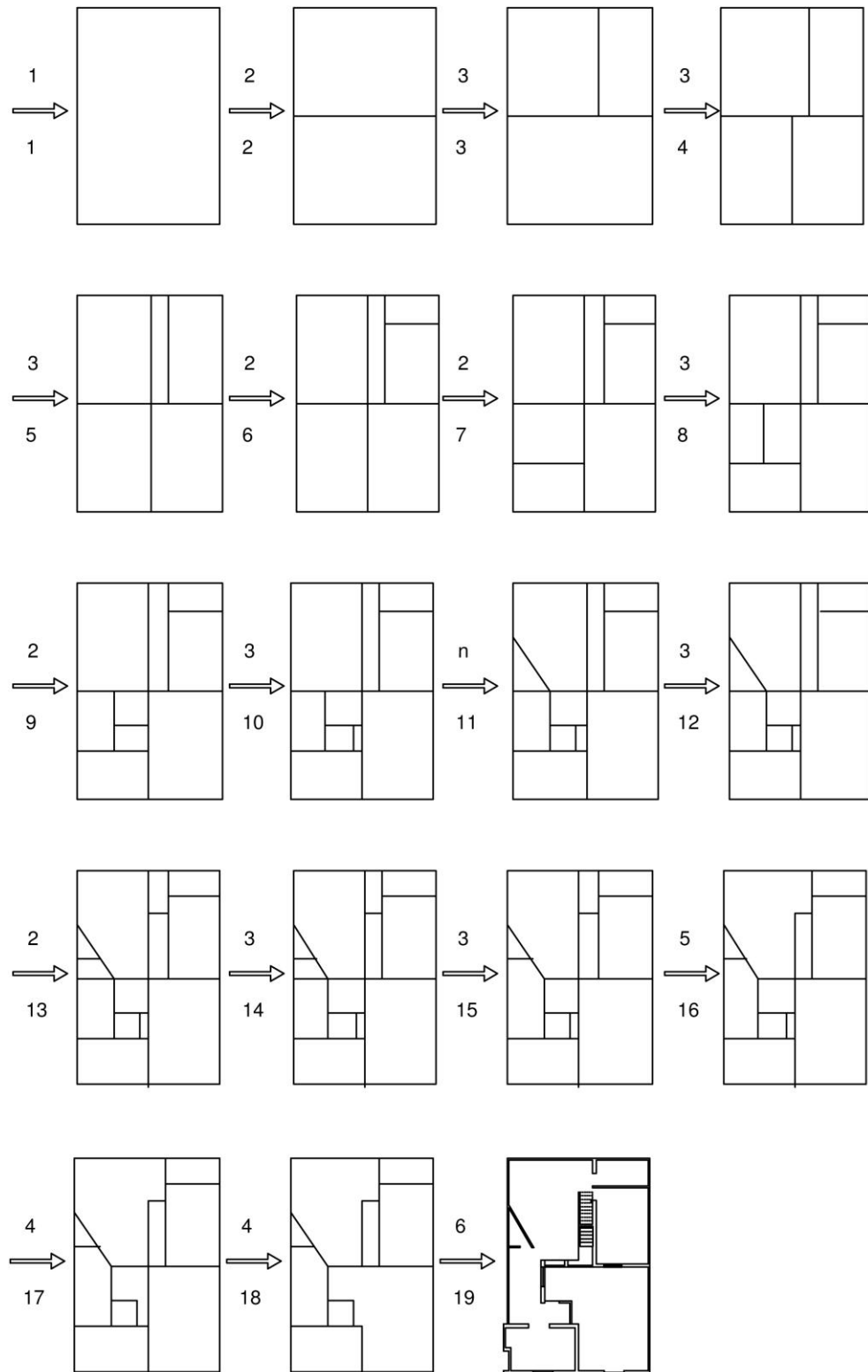


Figure 7. 10: Generic grammar derivation – Malagueira house type Ab

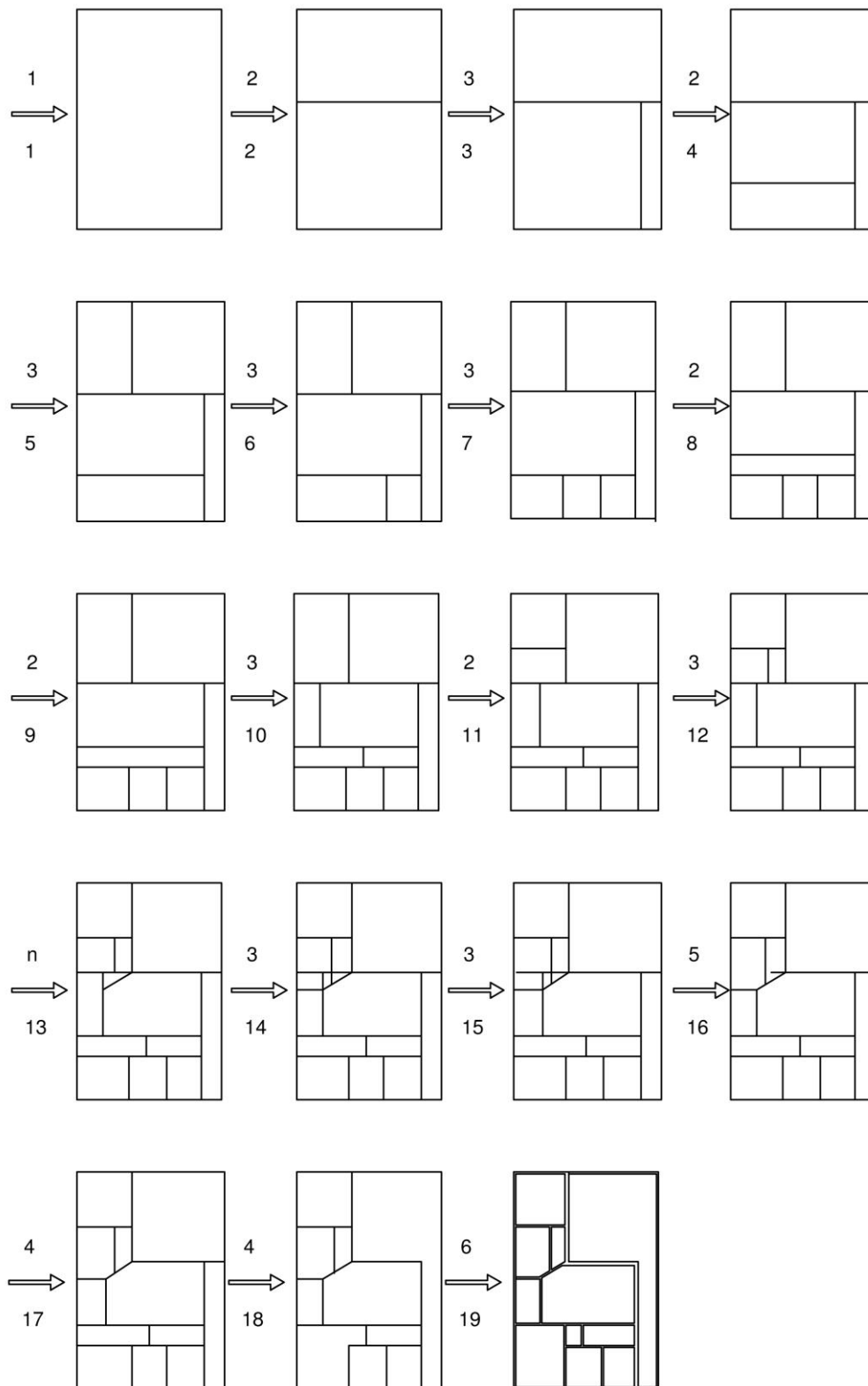


Figure 7. 11: Generic grammar derivation – Malagueira house type Bb

7.3.3 Malagueira single-family house derivations

The derivation of Malagueira houses using the generic grammar involves an adaptation of the original Malagueira grammar rules (Duarte 2001). The original grammar is a typical subdivision grammar and, as explained, is the driving force behind the design of this generic grammar.

The example in 7.10 illustrates a typical two-bedroom, two-storey, terraced, semi-detached house, type Ab using the classification system devised by Duarte. The proposed derivation uses the subdivision rules previously explained, plus particular shape rules that address Siza's spatial configurations. After the subdivision is performed, the steps that follow diverge from the original grammar and are closer to those tested in the previous derivations:

1. Step 1 is the plot insertion, which involves applying a self-contained rectangular shape. In the case of the Malagueira houses the envelope shape is not parametric, but has a fixed size that reflects the available plot space, which has the same dimensions and area for each house. This is determined by the plot dimensions of 12x8m, a perfect 3:2 proportional ratio providing a resultant plot area of 96 sqm.
2. Step 2 applies Rule 2 for horizontal subdivision, separating interior from exterior space. At this stage the yard/exterior space is allocated.
3. Step 3 applies the vertical division, creating a division between the interior functional areas. The house layout now begins with the allocation of (service versus living) zoning. Due to the true nature of this subdivision, recursive vertical and horizontal divisions are performed to carry out the zoning and spacing.
4. Steps 2 to 15 continue the recursive application of the division rules. Step 11 includes specific rules for the Malagueira design which replicate Siza's intention to create oblique cuts to produce smaller spaces and generate some spatial complexity. These rules are no more than parameterisations or generalisations of the division rules exemplified. The subdivision continues until Step 15.
5. In the 16th step concatenation is performed for the first time, offering the designer the flexibility to generate complex polygons with an incremental number of sides. This is replicated up to step 18, where the design progresses as usual with wall thickening, detailing the creation of openings and the completion of the design.

Similarly, the example shown in 7.11 illustrates another type of two-bedroom, two-storey, semi-detached terraced house, type Bb following Duarte's classification system. This house derivation involves up to 19 steps to produce the basic internal layout. Sixteen of these steps are pure subdivision processes in which the basic 12x8 m outline is successively divided to recreate the basic house functions. The steps which follow are dedicated to concatenation and wall thickening.

Only the basic layout up to stage five is illustrated in this example. The further detailing in the house is extensively described in Duarte's work, from the arrangement of doors and windows to the detailing of the façade and the inclusion of various elements such as terraces, chimneys and risers.

In comparison to the other grammars in which only vertical and horizontal divisions are performed, the Malagueira houses allow for diagonal divisions, as showcased in this house type.

7.4 Generic grammar syntax

The previous comparative and analysis work was fundamental to setting the standards and foundations for the production of this generic grammar.

A generic grammar is a grammar that allows for designs or sets of designs that are multiple and do not respect any particular feature or style. This '*per se*' is contradictory to the principle of a shape grammar, which is a formulation composed of a set of design rules that condition the design process in order to recreate a set of solutions that are stylistically related. The concept is therefore paradoxical but not entirely unexplainable. If different shape grammars deploy and recreate a particular design, all of them are independent. A similar analogy can be used to describe languages, the analogy that inspired the idea of shape formulations in the first place. If several languages have separate and distinct grammars but in many cases share common ground and if some distinct languages also share simple phrase constructions, then a higher level grammar can be elaborated to describe this branch of languages.

In this instance an attempt was made to recreate a shape grammar that would encode the parameters required to design three archetypes: the Palladian villas, the Prairie houses and the Malagueira houses. It could be argued that in order to achieve the same result, this grammar would be a combination of all the shape rules for all three grammars. However, this would result in a house type that would fit none of the above and would simply be a hybrid version. Obviously, a controlled version of the rules may be used, restricting their application to a specific stage, although this would also raise issues in terms of ordering and grammar elegance/economy.

Instead a new grammar was proposed that equated the different types to be recreated, reconsidered the grammar sequence and formulation and tried to incorporate functions. This grammar does not fit the criteria for an unrestricted type of grammar, since it implies an ordered structure and rule ordering process and clearly presents restrictions (Knight 1999). Knight classified three types of grammars (Additive, Grid and Subdivision), as cited by Prats (Prats 2007). This grammar should be used intelligently, if not assisted by a computer program to reproduce language related solutions. The rules have to be applied in a conscious way, respecting the conditions and restrictions in place. It can, however, be used freely. Hybrids have not been discussed to this point, but will be subject to reflection further on. Ideally they are not desirable and not accepted in the application of the shape rule set. The role of the computer tool is to guide and assist the rule application process and it will, to some extent, reduce the risk of undesirable solutions that do not fit any of the three styles.

Another issue emerged concerning functions. The question of typology and use is a key issue in the design, simply because different uses (e.g. housing and retail) cannot be meaningfully compared. Therefore, taking the shape grammars that were previously inferred and available into account, together with the relevance, program and type of use, housing seemed to be a good candidate.

An important analysis of the three shape grammars selected was carried out, focussing on the bottom-up approach, containment of the external fabric and grammar type.

In any grammar, the first check involves type, i.e. whether it is grid, additive or subdivision. Grammars are classified not only by the number of shape rule occurrences but also the rules that help design the basic features of the house. In the Palladian grammar the first stage and first nine rules determine the basic house layout. These rules help design a grid that will set the standard for the spaces that are later detailed. As in the Prairie houses, the first third of the grammar allows for additive rules and elements are added one by one to help design the house. This is generated progressively and the house grows in detail and in size. The Malagueira house type features a radically different approach. The first steps are additive but the true nature of the rule formulation soon becomes clear. All the rules relating to spatial creation feature subdivision and space is designed by subdividing a generic space and detailing a portion by assigning function.

Once the grammar type is determined, the type of house inscription is considered. Most of the house types selected conform to the notion of single detached housing isolated within the plot (although Malagueira is mostly semi-detached). Nevertheless, regardless of the detachment condition, some of these houses are self-contained and set within the external boundary, whilst others are not inscribed or not contained. Compact or contained houses feature in all the house types studied. Examples include the Emo, Zeno and Angarano villas and, in fact, almost all the other Palladian villas, as well as the Winslow, and Cheney Prairie houses and most of the Malagueira house types.

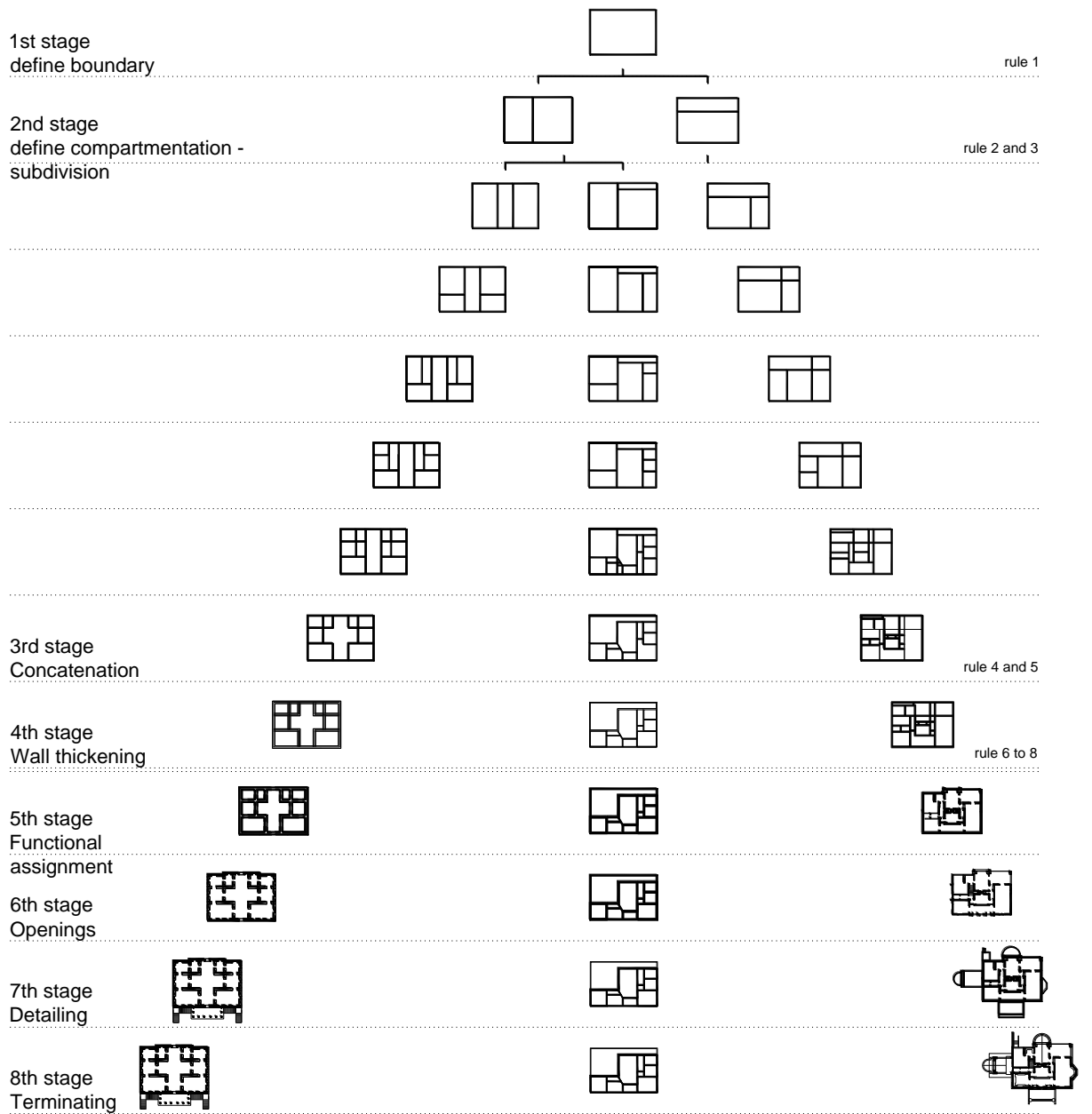


Figure 7. 12: Generic grammar structure

7.5 Generic Grammar structure

The generic shape grammar developed to replicate the three house types is based on a subdivision process similar to the original process used by Duarte for the Malagueira grammar (Duarte 2001).

This grammar contains eight stages:

1. the first stage arranges the boundary
2. the second is responsible for spatial subdivision
3. the third stage wall thickening
4. the fourth stage functional assignment
5. the fifth stage the creation of adjacencies by creating connecting doors
6. the sixth stage window design
7. the seventh stage the boundary
8. the eighth stage terminates the process by deleting the labels.

This is clearly illustrated in Figure 7.12.

These stages function similarly in the creation of the Palladio, Prairie and Malagueira houses, regardless of the house type. Moreover, the basic formulation responsible for the first step in the design concept is carried out using the same nine initial rules for all the house types. There are two differences in the house design typology route selection: firstly, the selection of appropriate labels which confer specific configurations and, secondly, the detail rules in stage seven which are responsible for providing the specific style and archetype features.

The generic grammar uses a top-down formulation. This means that the design solution is developed by progressing from the general to the particular. The design is developed from the overall outline to the room, and from this to the detailing of the openings and other specific elements.

In the fifth stage, which is responsible for functions, something specific emerges. Since functions and their corresponding adjacency diagrams are derived from style and genre specifications, this is, in fact, the first stage in which each sub-grammar denotes and branches out into a particular style. Even though most of these houses denote and propose similar living functions, their true distinction relies on the spatial organisation and articulation of different spaces. Shape and function work hand in hand in this distinction and it is clear from this stage onwards that each house type emerges as a distinct type.

The first stage is composed of the first shape rule. This additive rule allows for the design of the boundary that will contain the overall design. This rectangular shape will be applied and adjusted either to the outline of the intended building (as in the Palladio or Malagueira grammars) or to the extreme edges of the house type in question, as in the Prairie houses. This mandatory rule will be used by all types and therefore the labels PAL, PRA, and MAL are applied, standing for Palladian villa, Prairie or Malagueira house respectively. The parametric

rule designs an X x Y rectangle with specific formulations and ratios for each type, as described by the rule schema:

$$\emptyset \rightarrow X$$

A new shape is introduced to an existing shape X that translates into a transformation in the initial design stage:

$$X \rightarrow X + t(X)$$

Respectively:

PAL: $y/x = n/m \rightarrow m = [3,5,7] \rightarrow n = [2,3,4,5] \rightarrow$ allowed ratios: 1:1, 2:3, 4:3, 3:5, 4:5, 3:7

PRA: [x, y]

MAL: X= 8 m, Y=12m \rightarrow allowed ratio: 2:3

The second stage is responsible for the basic layout of the house floor plan. This stage proposes four shape rules, half of which are essentially subdivision rules responsible for the generation of most of the fabric of the house. The reminder are merging rules that deal with particular conditions and help design spaces with more complex geometries by using spatial concatenation. Shape rule 2 is responsible for horizontal subdivision. This type of subdivision can be placed in any of the three house types and allows two separate spatial/functional zones to be created by splitting the space horizontally. Regardless of the house type, the rule schema can be represented by the following expressions:

$$X \rightarrow \text{div}(x)$$

$$X \rightarrow \text{prt}'(x) + \text{prt}''(x)$$

$$(X' + X'') \rightarrow \text{prt}(b(X' + X''))$$

Therefore the resulting rule parameterisation can find intervals in: N: [x, (y' + y'')]

Similarly, Rule 3 is responsible for creating two separate spaces using subdivision. The difference lies in the direction of the split, which in this case is placed vertically. In normal circumstances the rule could be applied equally to all three case scenarios, but the Palladian villas pose some issues which must be addressed individually. The issue of symmetry patent in the Palladio language requires a vertical split to be performed and copied symmetrically across the floor plan using north-south as an axis. Therefore the rule has to allow for the proper parameterisation of this case.

N: [(x' + x''), y]

or, in the Palladian grammar:

N: [2.(x' + x''), y] ...

To clarify this, the design result of this rule would have to be clearly labelled to indicate the type of house already detailed.

Both rules described are mandatory in order to satisfy the minimum established spatial requirements. Applying both rules at least once would guarantee that a minimum number of four rooms would be created for each house, thus enabling the minimum basic house functions to be allocated, namely living, dining, sleeping and food preparation. It also allows the stylistic features of the house to be considered, since once an imaginary grid has been applied to any of

the houses, a minimum 2x2 grid will be required. There are at least two façade-facing divisions in each house and at least two bays deep in the household, justifying the use of horizontal and vertical divisions. Needless to say, the vertical split is applied to different languages with different parameterisations, resulting in a double simultaneous split for the Palladian villas where the spacing between the splits and the edges is observed and symmetry is maintained. In houses similar to those in the Malagueira grammar, proportions of 2:3, 1:2, and 3:4 can be observed, since they are the most frequent proportions for spatial occupancy.

The rules which follow are responsible for spatial modifications to the shapes created so far. They should be applied sequentially after the sub-division rules. The two rules that follow are space merging shape rules and allow for the concatenation of adjacent spaces. As discussed in previous chapters, these rules erase the cell boundaries to create a larger irregular polygonal space that would otherwise require a specific geometric rule.

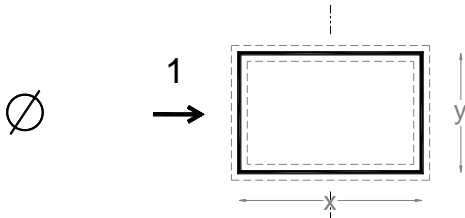
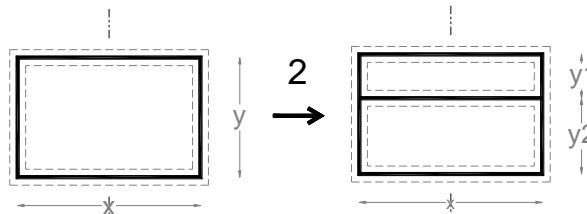
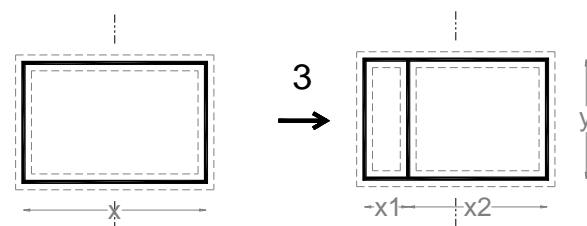
Rule 4 is applied for space concatenation. This rule can be graphically explained by the deletion of one of the boundaries, in this case a horizontal boundary.

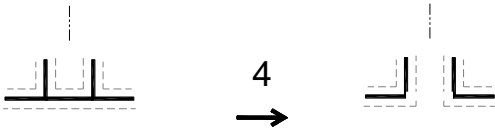
Rule 5 echoes Rule 4 but is applied to vertical conditions. This rule is also applied twice to cater for Palladian symmetry requirements.

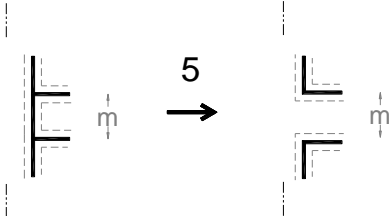
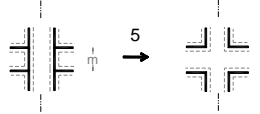
Rule 5 is the last rule in the second stage and concludes the spatial delineation.

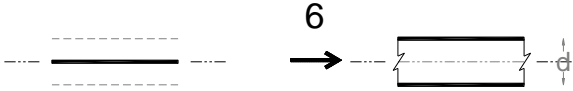
The third stage is responsible for extra dimensionality. The previous stage allowed for a basic spatial setting with single lines to denote the space. The set of rules which follow replace this graphical representation with a proper wall thickness. This is parameterised to respond to 2 Vicentine feet in the Palladian villas, the 200mm in the Malagueira houses and the 100' inches in the Prairie houses.

These generic shape rules are further described in Table 7.1.

STAGE 1		
BOUNDARY DEFINITION		
RULE 1	ADDING BOUNDARY	PARAMETERISATION
	$\emptyset \rightarrow X$	
Palladian villas		$y/x = n/m$ $m = [3,5,7]$ $n = [2,3,4,5]$ permitted ratios: 1:1, 2:3, 4:3, 3:5, 4:5, 3:7
Prairie houses		$[x, y]$
Malagueira houses		$X=8\text{ m}$ $Y=12\text{m}$ permitted ratio: 2:3
STAGE 2		
SPATIAL SUBDIVISION		
RULE 2	HORIZONTAL SUBDIVISION	PARAMETERISATION
	$X \rightarrow \text{div}(x)$ $X \rightarrow \text{prt}'(x) + \text{prt}''(x)$	
Palladian villas		$y/x = n/m$ $m = [3,5,7]$ $n = [2,3,4,5]$ $y = y1+y2$ $y1 = y/n \vee y1 = y/3n$ $y1 = y2$ (for $n=2 \vee n=4$)
Prairie houses		$[x, y]$ $Y= y1 +y2$
Malagueira houses		$Y1 = y2 = y/2$ $y1= n \times y/4 \text{ } n = [1,2,3,4]$
RULE 3	VERTICAL SUBDIVISION	PARAMETERISATION
	$X \rightarrow \text{prt}'(x) + \text{prt}''(x)$	
Palladian villas		$y/x = n/m$ $m = [3,5,7]$ $n = [2,3,4,5]$ $X = 2 \cdot X1+X2$ $X1 = n \vee X1 = 3 \cdot n/2 \vee X2 = 2n$ $y1 = y2$ (for $n=2 \vee n=4$)
Prairie houses		$[x, y]$ $x= x1 +x2$
Malagueira houses		$x1 = x2 = x/2$

STAGE 3		
SPACE MERGING		
RULE 4	HORIZONTAL MERGING	PARAMETERISATION
	$(X' + X'') \rightarrow X$ $\text{prt}'(x) + \text{prt}''(x) \rightarrow X$	
Palladian villas		$X=n$
Prairie houses		
Malagueira houses		

RULE 5		
VERTICAL MERGING		
	$(X' + X'') \rightarrow X$ $\text{prt}'(x) + \text{prt}''(x) \rightarrow X$	
Palladian villas		
Prairie houses		$Y=m$
Malagueira houses		$Y=m$

STAGE 4		
WALL THICKENING		
RULE 6	SINGLE WALL	PARAMETERISATION
	$X \rightarrow X + t(X)$	
Palladian villas		$d \approx 2$ Vicentine feet $d = 750$ mm
Prairie houses		$d \text{ ext} \approx 100 + 1/4$ " $d \text{ int} = 100$ "
Malagueira houses		$d \text{ ext} \approx 250$ mm $d \text{ int} = 200$ mm

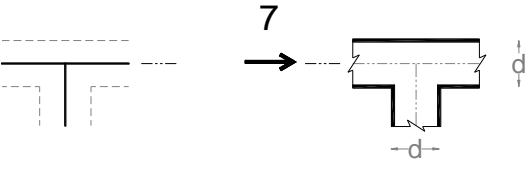
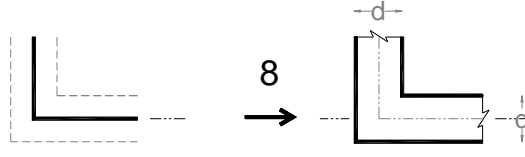
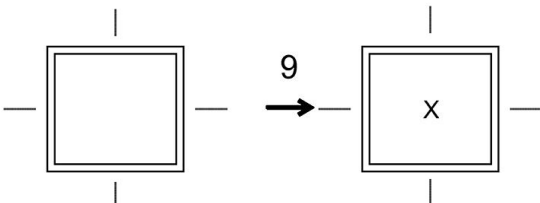
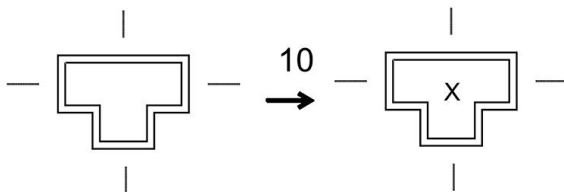
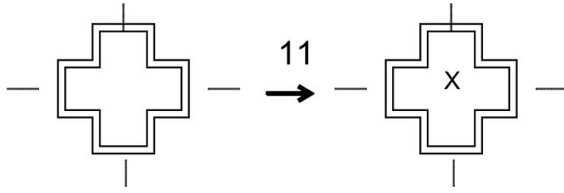
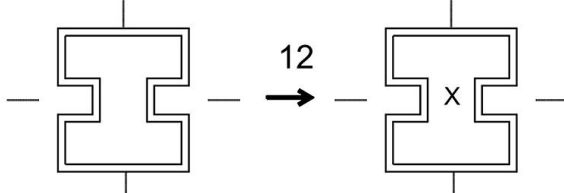
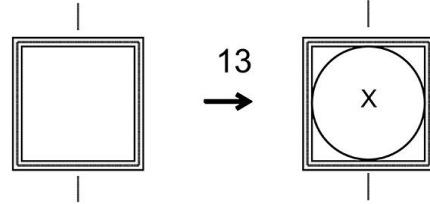
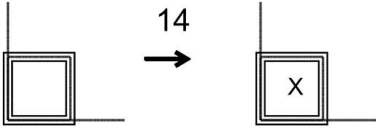
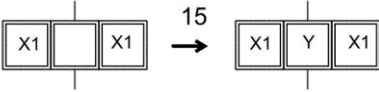
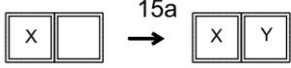
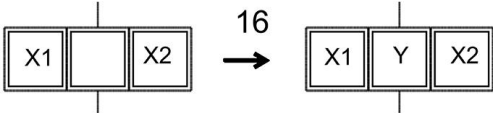

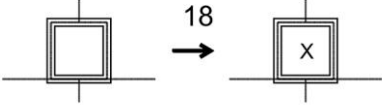
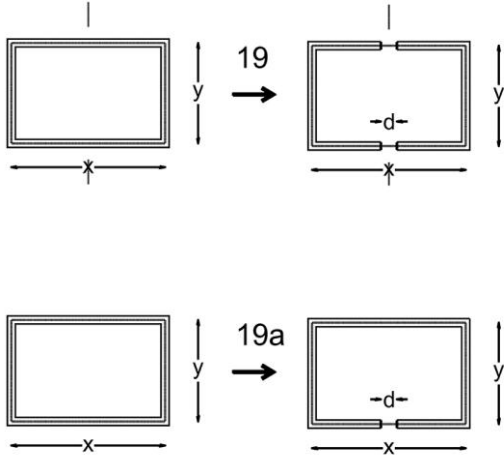
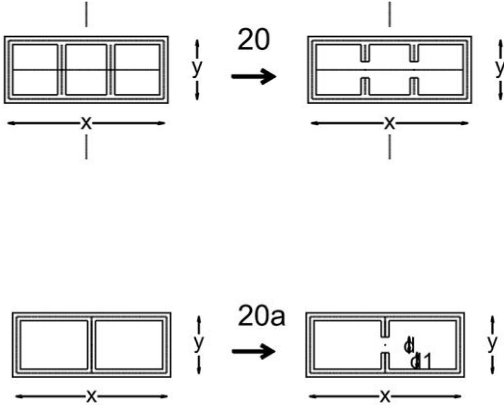
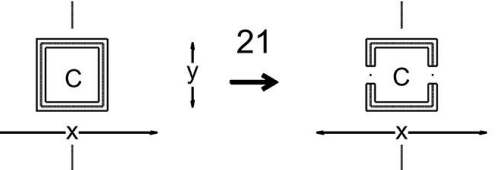
RULE 7	WALL 'T' JUNCTION	PARAMETERISATION
	$X \rightarrow X + t(X)$	
Palladian villas		$d \approx 2$ Vicentine feet $d = 750$ mm
Prairie houses		$d_{ext} \approx 100 + 1/4$ " $d_{int} = 100$ "
Malagueira houses		$d_{ext} \approx 250$ mm $d_{int} = 200$ mm
RULE 8	WALL CORNER JUNCTION	PARAMETERISATION
	$X \rightarrow X + t(X)$	
Palladian villas		$d \approx 2$ Vicentine feet $d = 750$ mm
Prairie houses		$d_{ext} \approx 100 + 1/4$ " $d_{int} = 100$ "
Malagueira houses		$d_{ext} \approx 250$ mm $d_{int} = 200$ mm
STAGE 5-8	LANGUAGE SPECIFIC STAGES	
RULE N TO M	Language Specific Shape Rules	PARAMETERISATION

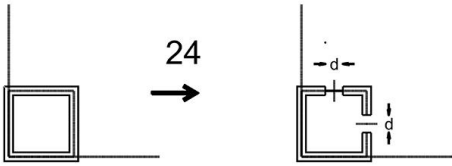
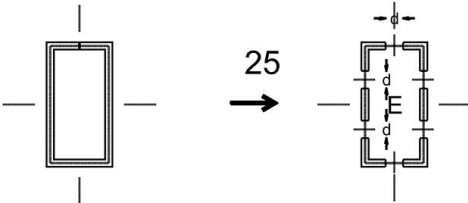
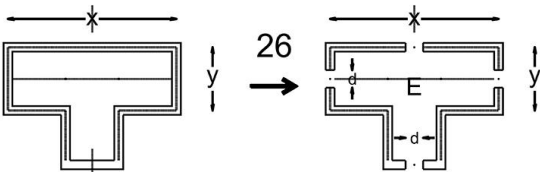
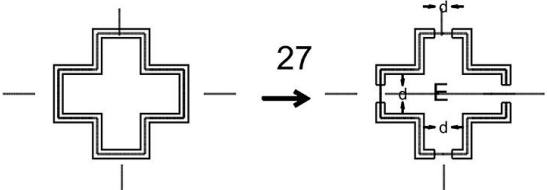
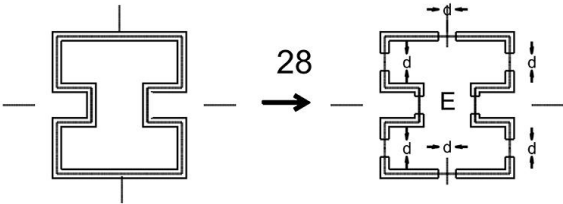
Table 7. 1: Generic grammar shape rules

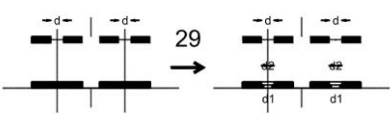
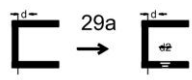
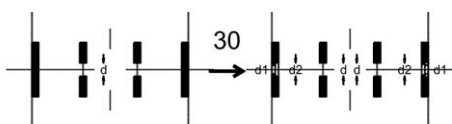
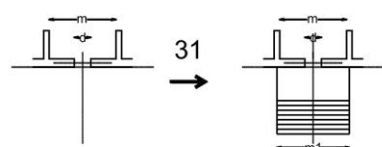
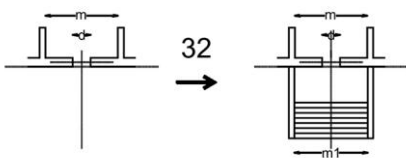
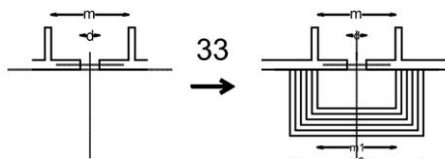
STAGE 5	FUNCTIONAL ASSIGNMENT	
RULE 9	CORE SOCIAL SPACE	PARAMETERIZATION
Palladian villas		$X = E = \text{entertainment space}$
Prairie houses		$X = Y; Y = [Lv, Ki, Be, St]$
Malagueira houses		$X = Y; Y = [Lv, Ki, Be, St]$
RULE 10	CORE SOCIAL SPACE	PARAMETERIZATION
Palladian villas		$X = \text{entertainment space}$
Prairie houses		$X = \emptyset$
Malagueira houses		$X = \emptyset$
RULE 11	CORE SOCIAL SPACE	PARAMETERIZATION
Palladian villas		$X = \text{entertainment space}$
Prairie houses		$X = \emptyset$
Malagueira houses		$X = \emptyset$
RULE 12	CORE SOCIAL SPACE	PARAMETERIZATION
Palladian villas		$X = \text{entertainment space}$
Prairie houses		$X = \emptyset$
Malagueira houses		$X = \emptyset$
RULE 13	CORE SOCIAL SPACE	PARAMETERIZATION
Palladian villas		$X = \text{entertainment space}$
Prairie houses		$X = \emptyset$
Malagueira houses		$X = \emptyset$

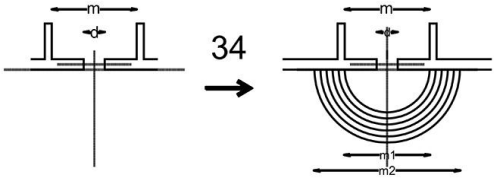
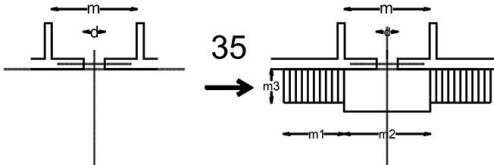
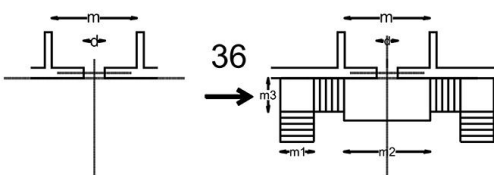
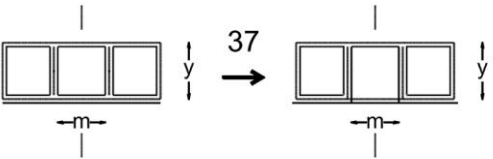
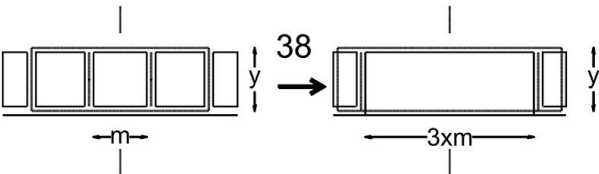
RULE 14	CORE SOCIAL SPACE	PARAMETERIZATION
Palladian villas		X = living space
Prairie houses		X= Y; Y=[Lv, Ba, Be, St]
Malagueira houses		X= Y; Y=[Lv, Ki, Be, Ba, St]
RULE 15	CORNER LIVING SPACE	PARAMETERIZATION
Palladian villas	 	X1 living space Y = Circulation
Prairie houses		X=[Lv, Ki, Be, Ba, St] Y= Ci
Malagueira houses		X=[Lv, Ki, Be, Ba, St] Y= Ci
RULE 16	LIVING SPACE	PARAMETERIZATION
Palladian villas		X1 = living space Y = Circulation X2 = entertainment space
Prairie houses		X1 = Y2 = Y = ∅
Malagueira houses		X1 = Y2 = Y = ∅
RULE 17	LIVING SPACE	PARAMETERIZATION
Palladian villas		X = living space
Prairie houses		X=[Lv, Ki, Be, Ba, St, Ci]
Malagueira houses		X=[Lv, Ki, Be, Ba, St, Ci]
RULE 18	CIRCULATION SPACE	PARAMETERIZATION
Palladian villas		X = Circulation
Prairie houses		X = Circulation
Malagueira houses		X = Circulation

STAGE 6	DOOR ASSIGNMENT	
RULE 19	MAIN ACCESS DOORS	PARAMETERIZATION
Palladian villas		$d = 4 \times vc$ $1vc = 0.354 \text{ m}$
Prairie houses		$d = 3'0''$
Malagueira houses		$d = 900\text{mm}$
RULE 20	INTERNAL DOORS	PARAMETERIZATION
Palladian villas		$d = 4 \times vc$ $1vc = 0.354 \text{ m}$
Prairie houses		$d = 2'8'' \quad d1 = 12''$
Malagueira houses		$d = 800\text{mm} \quad d1 = 200\text{mm}$
RULE 21	INTERNAL DOORS	PARAMETERIZATION
Palladian villas		$d = 4 \times vc$ $1vc = 0.354 \text{ m}$
Prairie houses		$d = 2'8'' \quad d1 = 12''$
Malagueira houses		$d = 800\text{mm} \quad d1 = 200\text{mm}$

RULE 22	INTERNAL DOORS CORE SPACE	PARAMETERIZATION
Palladian villas		$d = 4 \times vc$ $1vc = 0.354 \text{ m}$
Prairie houses		$d = 2'8'' \quad d1 = 12''$
Malagueira houses		$d = 800\text{mm} \quad d1 = 200\text{mm}$
RULE 23	INTERNAL DOORS CORE SPACE	PARAMETERIZATION
Palladian villas		$d = 4 \times vc$ $1vc = 0.354 \text{ m}$
Prairie houses		$d = 2'8'' \quad d1 = 12''$
Malagueira houses		$d = 800\text{mm} \quad d1 = 200\text{mm}$

RULE 24	INTERNAL DOORS CORE SPACE	PARAMETERIZATION
Palladian villas		$d = 4 \times vc$ $1vc = 0.354 \text{ m}$
Prairie houses		$d = 2'8'' \text{ } d1=12''$
Malagueira houses		$d=800\text{mm } d1=200\text{mm}$
RULE 25	INTERNAL DOORS CORE SPACE	PARAMETERIZATION
Palladian villas		$d = 4 \times vc$ $1vc = 0.354 \text{ m}$
Prairie houses		$X=\emptyset$
Malagueira houses		$X=\emptyset$
RULE 26	INTERNAL DOORS CORE SPACE	PARAMETERIZATION
Palladian villas		$d = 4 \times vc$ $1vc = 0.354 \text{ m}$
Prairie houses		$X=\emptyset$
Malagueira houses		$X=\emptyset$
RULE 27	INTERNAL DOORS CORE SPACE	PARAMETERIZATION
Palladian villas		$d = 4 \times vc$ $1vc = 0.354 \text{ m}$
Prairie houses		$X=\emptyset$
Malagueira houses		$X=\emptyset$
RULE 28	INTERNAL DOORS CORE SPACE	PARAMETERIZATION
Palladian villas		$d = 4 \times vc$ $1vc = 0.354 \text{ m}$
Prairie houses		$X=\emptyset$
Malagueira houses		$X=\emptyset$

STAGE 7		
WINDOW ASSIGNMENT		
RULE 29	WINDOWS	PARAMETERIZATION
Palladian villas		$d = 4 \times vc$ $d = d2$ $d1 = d2 - (2 \times 1vc)$ $1vc = 0.354 \text{ m}$
Prairie houses		$d = \{3'0'', 6'0''\}$
Malagueira houses		$d = 1200\text{mm}$ $d1 \leq d$
		
RULE 30	WINDOWS	PARAMETERIZATION
Palladian villas		$d = 4 \times vc$ $d = d2$ $d1 = d2 - (2 \times 1vc)$ $1vc = 0.354 \text{ m}$
Prairie houses		$X = \emptyset$
Malagueira houses		$X = \emptyset$
STAGE 8		
DETAILING AND ORNAMENTATION		
RULE 31	STAIRS – STRAIGHT FLIGHT	PARAMETERIZATION
Palladian villas		$m1 = m$ v $m1 = 3xm$
Prairie houses		$m1 = m$
Malagueira houses		$X = \emptyset$
RULE 32	STAIRS – STRAIGHT FLIGHT WITH CAPPED ENDS	PARAMETERIZATION
Palladian villas		$m1 = m$ v $m1 = 3xm$
Prairie houses		$X = \emptyset$
Malagueira houses		$X = \emptyset$
RULE 33	STAIRS – SPLAYED FLIGHT	PARAMETERIZATION
Palladian villas		$m1 = m$ v $m1 = 3xm$ $m2 = m1 + (\text{num steps} \times \text{dist} \times 2)$
Prairie houses		$m1 = m$
Malagueira houses		$X = \emptyset$

RULE 34		STAIRS – SPLAYED CIRCULAR FLIGHT	PARAMETERIZATION
			$m1 = m$ v $m1 = 3xm$ $m2 = m1 + (\text{num steps} \times \text{dist} \times 2)$
			$m1 = m$
			$X = \emptyset$
RULE 35		STAIRS – TWIN FLIGHTS	PARAMETERIZATION
Palladian villas			$m2 = m$ v $m2 = 3xm$ $m1 = (\text{num steps} \times \text{dist})$
Prairie houses			$X = \emptyset$
Malagueira houses			$X = \emptyset$
RULE 36		STAIRS – TWIN ELBOW FLIGHTS	PARAMETERIZATION
Palladian villas			$m2 = m$ v $m2 = 3xm$ $m1 = m3$
Prairie houses			$X = \emptyset$
Malagueira houses			$X = \emptyset$
RULE 37		SINGLE CELL RECESSED LOGGIA	PARAMETERIZATION
Palladian villas			m
Prairie houses			$X = \emptyset$
Malagueira houses			
RULE 38		TRIPLE CELL RECESSED LOGGIA	PARAMETERIZATION
Palladian villas			$m \rightarrow 3 \times m$
Prairie houses			$X = \emptyset$
Malagueira houses			

RULE 39	TRIPLE CELL RECESSED LOGGIA	PARAMETERIZATION
Palladian villas		$m \rightarrow 3 \times m$
Prairie houses		$X = \emptyset$
Malagueira houses		$X = \emptyset$
RULE 40	PROJECTED TERRACE	PARAMETERIZATION
Palladian villas		$m1 = m$ v $m1 = 3 \times m$ $m2 = m1$ $m3 = m2 - r$
Prairie houses		$m1 = m$ $m2 = m1$ $m3 = m2 - r$
Malagueira houses		$X = \emptyset$
RULE 41	COLUMNS – EXTERNAL ORNAMENTS X4	PARAMETERIZATION
Palladian villas		m $m2 = m1$ $m1 = m / 3$
Prairie houses		$X = \emptyset$
Malagueira houses		$X = \emptyset$
RULE 42	COLUMNS – EXTERNAL ORNAMENTS X4	PARAMETERIZATION
Palladian villas		m $m2 = m1$ $m1 = m / 3$
Prairie houses		$X = \emptyset$
Malagueira houses		$X = \emptyset$
RULE 43	COLUMNS – EXTERNAL ORNAMENTS X4	PARAMETERIZATION
Palladian villas		m $m2 = m1$ $m1 = m / 3$
Prairie houses		$X = \emptyset$
Malagueira houses		$X = \emptyset$

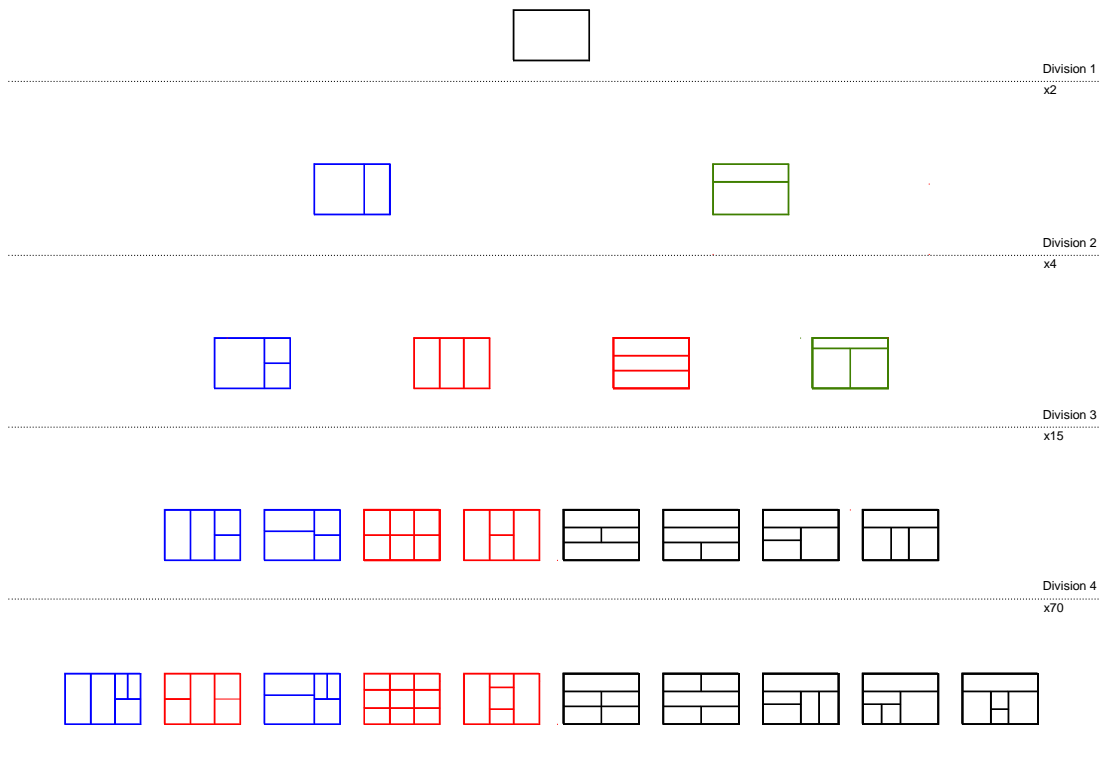


Figure 7. 13: Generic grammar diagram

The diagram aims to replicate the possible divisions that occur in this subdivision grammar by mapping the horizontal and vertical divisions.

It shows potential solutions in red for the Palladian villa, in green for the Prairie houses and in blue for the Malagueira houses. This corpus only focuses on configuration and the shapes and proportions vary parametrically.

It is possible to predict the zones of the diagram in which the different types fit. The Malagueira houses occupy the left branch, while the Palladian villas occupy the centre and the Prairie houses the right. The Palladian villas seem to share some common features with the two other types.

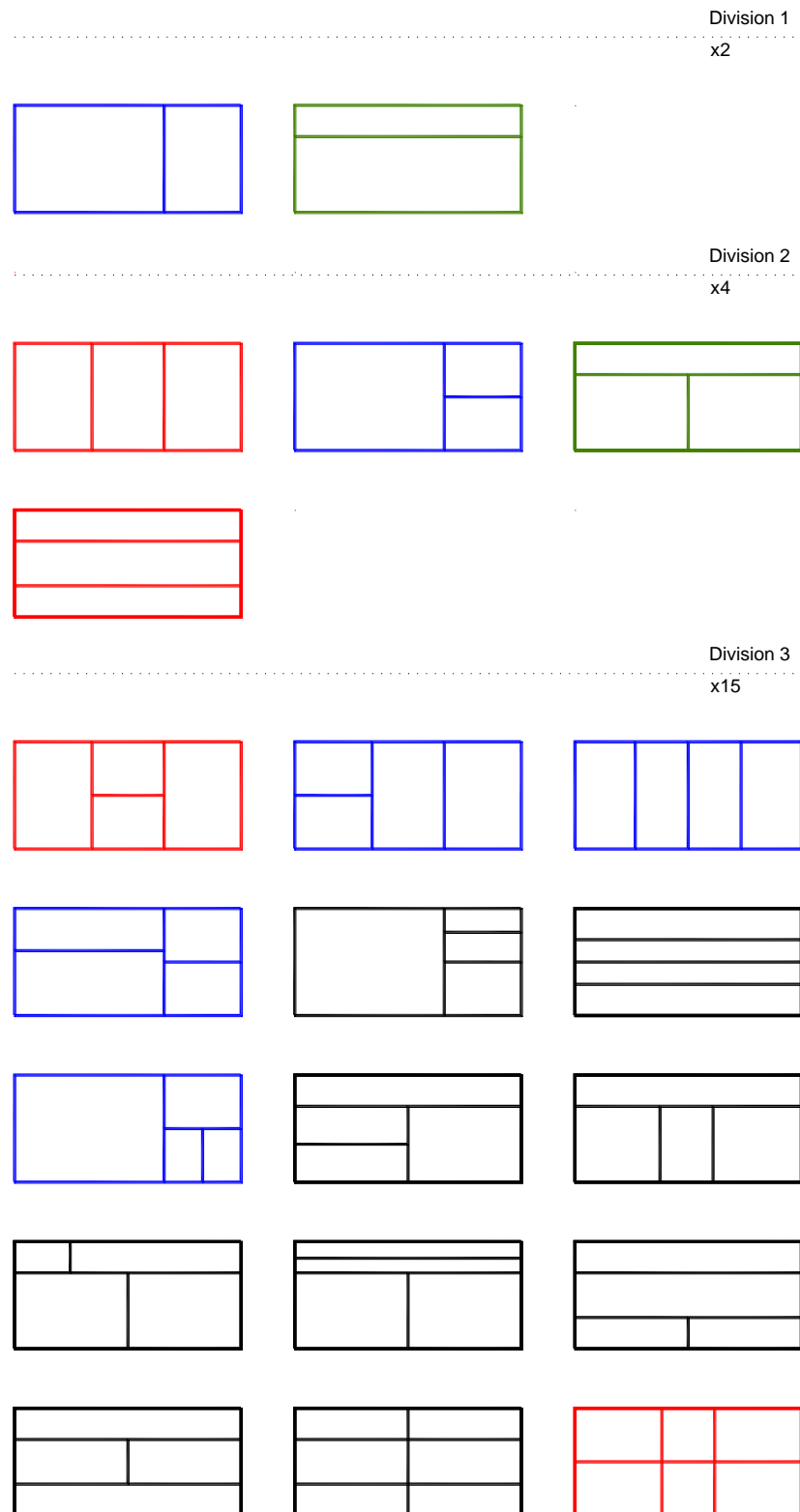


Figure 7. 14: Generic grammar corpus of solutions for 1, 2 and 3 divisions.

There are 2 possible solutions for the 1 division corpus, namely the vertical and the horizontal, 4 for 2 divisions and 16 for 3 divisions. The solutions in red represent potential Palladian villa solutions, in green Prairie house solutions and in blue Malagueira solutions. This corpus only focuses on configuration, and the shapes and proportions vary parametrically.

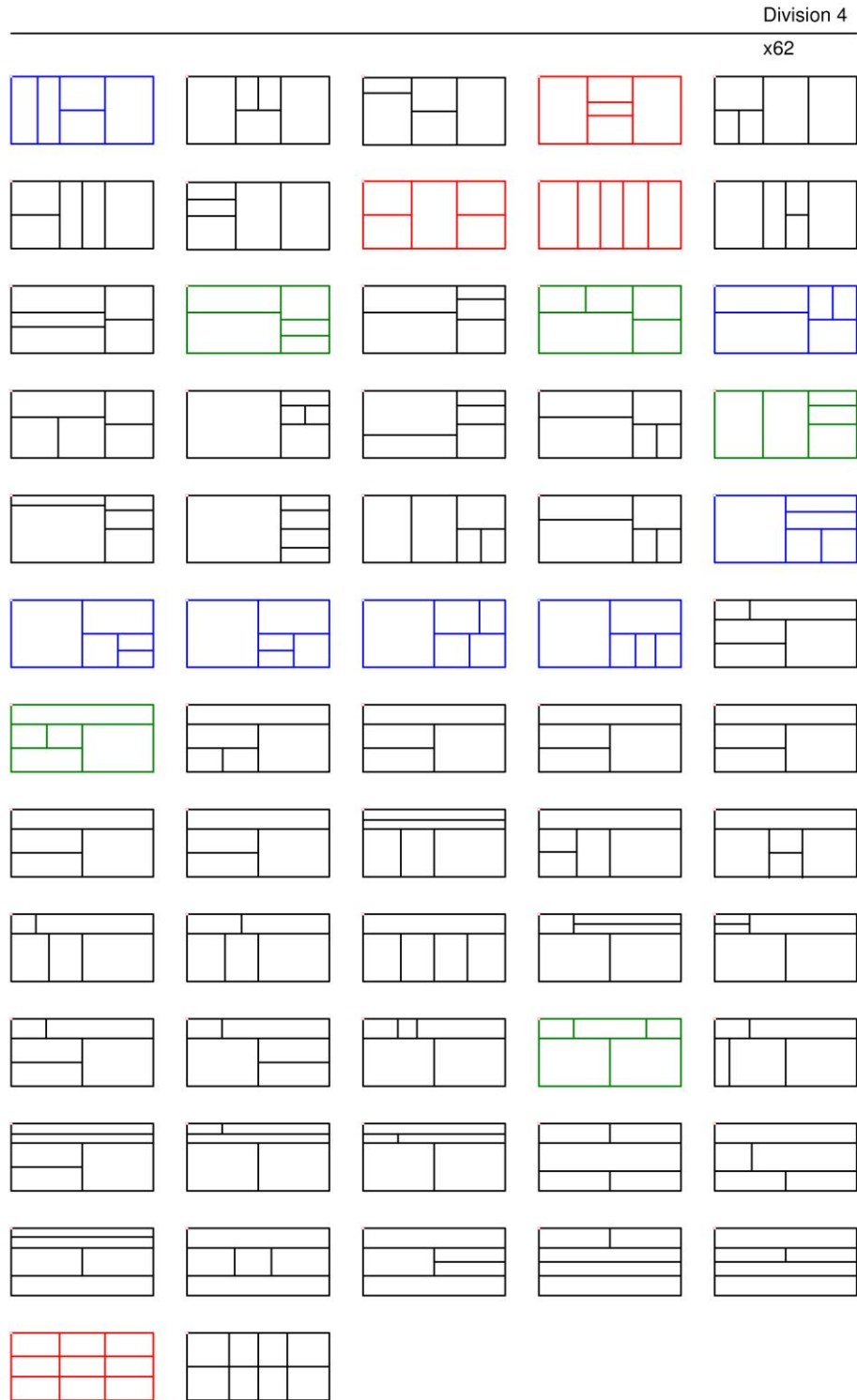


Figure 7. 15: Generic grammar corpus of solutions for 4 divisions.

There are 62 possible solutions for the 4 division corpus, combining vertical and horizontal.

These reflect configurations that are unique and can be further mirrored or replicated through rotation.

The solutions in red represent potential Palladian villa solutions, in green Prairie house solutions and in blue Malagueira solutions. This corpus only focuses on configuration, and the shapes and proportions vary parametrically.



Figure 7. 16: Generic grammar corpus of solutions for 5 divisions (part 1).

These reflect configurations that are unique and can be further mirrored or replicated through rotation.

There are 408 possible solutions for the 5 division corpus, combining vertical and horizontal.

The solutions in red represent potential Palladian villa solutions, in green Prairie house solutions and in blue Malagueira solutions. This corpus only focuses on configuration, and the shapes and proportions vary parametrically.

The bottom example in red resembles the configuration of the Palladian Villa Ragona and the one immediately above it resembles Villa Angarano, designed and built by Andrea Palladio with underlying 3x3 grids.



Figure 7. 17: Generic grammar corpus of solutions for 5 divisions (continued part 2).

These reflect configurations that are unique and can be further mirrored or replicated through rotation.

There are 408 possible solutions for the 5 division corpus, combining vertical and horizontal.

The solutions in red represent potential Palladian villa solutions, in green Prairie house solutions and in blue Malagueira. This corpus only focuses on configuration, and the shapes and proportions vary parametrically.

The bottom example in red resembles the configuration of the Palladian Villa Godi designed and built by Andrea Palladio with an underlying 5x2 grid. Additionally, there are examples in blue which fit the Malagueira criteria and are good starting points for house types Ab and Bb.

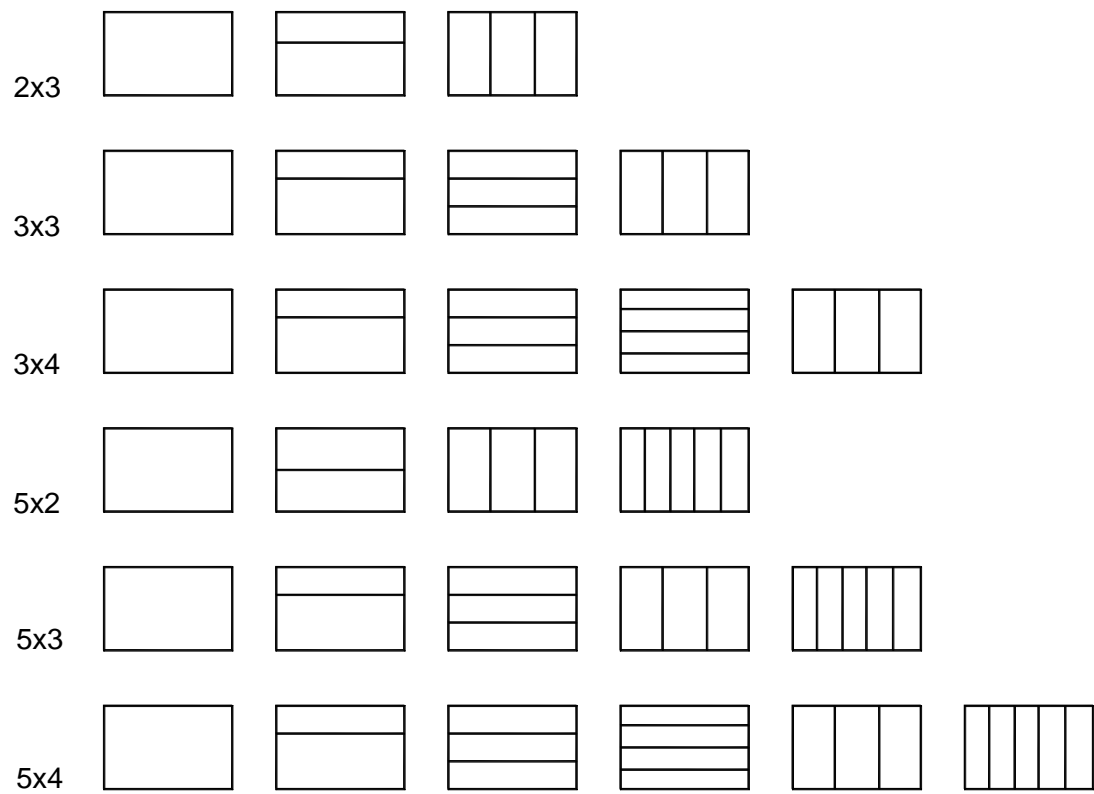


Figure 7. 18: Acceptable Palladian villa patterns

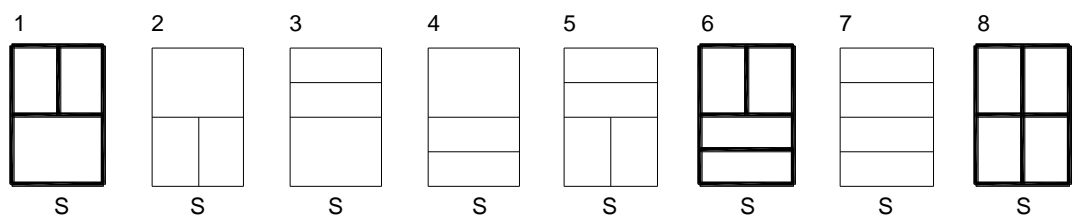


Figure 7. 19: Acceptable Malagueira house patterns

7.6 Generic grammar validation

The work described above presents a generic shape grammar that allows for the generation of not one, but several signature styles. Unlike previous work, this is not a typical shape grammar but a generic formulation that allows for the replication of more than one design style, which is believed to be a contribution to shape grammar research. To this end, the generic grammar uses the same grammar structure and the same shape rules. The shape rules are formulated as parametric and can be manipulated to generate a particular design. A case study composed of three types of grammars, namely the Palladian Villa, Prairie House and Malagueira House grammars, was selected to illustrate the scope of this generic grammar. The aim was twofold:

- firstly, to produce an alternative grammar that allowed for the alternative generation of a previously developed grammar
- secondly, to use this new grammar as a generic grammar capable of producing more than one design style.

The methodology started with a cross comparison of the grammars previously inferred and a study of their underlying styles. Each grammar was decomposed and its structure analysed. The complex sets of rules for each grammar were also analysed and similar rule formulations were pinpointed. The grammar comparison and knowledge acquired led to the idea of using subdivision grammars to construct the new generic grammar. This choice reflects the ease of use and intuitive nature of this grammar type and the adaptable nature of the subdivision process in comparison with other creative concepts. A new set of rules was developed for this new generic grammar in order to produce stylistically consistent designs. The set of rules incorporated important subdivision rules for the required conditions, such as minimum spacing and bilateral symmetry.

Figures 7.14 to 7.17 represent the corpus for solutions generated from recursive subdivisions of the outline with 1 to 5 subdivisions. The result is an extended corpus of 2 possible solutions for the 1 division corpus, (vertical and horizontal), 4 for 2 divisions, 16 for 3 divisions, 70 possible solutions for the corpus of 4 divisions, and 477 possible solutions for 5 divisions. This makes a total of 569 solutions if the generation process only applies the division rules 5 times. Experience has shown that the generation of Palladian villas, Malagueira houses and, in particular, Prairie houses takes several recursions (Palladian villas require a minimum of 8 divisions in average) to be successful. As previously seen, the Malagueira Ab house allows for 16 division steps (Figure 7.10) and the Henderson Prairie house 14 steps (Figure 7.9). This means that a corpus for 14 or more recurring solutions would amount to a six-figure number. Moreover, this only indicates the potential of a generic grammar of this kind using subdivision as its generative tool. A great deal of designs can be generated, as shown in Figure 7.13, displaying a partial tree diagram with the delineation of possible divisions. This grammar can

potentially generate any solution that is contained within a rectangular outline. However, this can be made specific by imposing conditions on the layouts.

Figures 7.18 and 7.19 show the possible patterns allowed by the Palladian villas and Malagueira houses. They are incorporated into the shape rule parameterisation as shown in the shape rule sequence demonstrated in Table 7.1. These patterns allow for underlying 3x2, 3x3, 3x4, 5x5, 5x3, 5x4, and 5x5 grids. Despite the abstract grid principle, this is only a rationalisation used to maintain the proportional ratio of the divisions, since in practice only the desired divisions are performed. Malagueira allows for 8 basic layouts derived from vertical and horizontal divisions. In this way, the basic layout is determined in three divisions.

This corpus only focuses on configuration, and the shapes and proportions vary parametrically. The diagram represented in 7.13 replicates the possible subdivisions that occur using the subdivision grammar by mapping the horizontal and vertical divisions.

The solutions in red represent potential Palladian villa solutions, in green Prairie house solutions and in blue Malagueira solutions.

It is possible to predict the 'zones' in the diagram into which the different types fit. The Malagueira houses occupy the left branch, while the Palladian villas occupy the centre and the Prairie houses the right. The Palladian villas seem to share common features with the two other types. Nevertheless, many other solutions that do not fit these three styles can be found. In addition, other solutions not necessarily present in the same zone may meet the criteria defined and some in early design stages may respond to more than one style

To the best of our knowledge, all previous work has implied a unique grammar that describes a particular corpus of designs or, for that matter, an alternative grammar for a grammar already developed, even though the range of work produced by the grammar also has intrinsically common features. This implies that finding and studying this grammar tells us something about the essence of the corpus of work.

This work refutes certain common assumptions regarding shape grammars, namely the uniqueness of the design style that one grammar can produce. Given that there is more than one way to reproduce designs, more than one suitable grammar and the fact that one grammar can produce more than one style, many different representations are potentially viable.

This represents a breakthrough in shape grammar methodology and research. Shape grammars are no longer exclusive, but can potentially be manipulated to generate a larger corpus of new designs. This allows for efficiency in exploring shapes and analysing results, thus widening the scope of grammars. The exploration of a new corpus of designs and design hybrids gives rise to many possibilities. It is expected that the mutation of these design styles or the overlapping of rules will produce new consistent designs with a new hybrid style. Moreover, computerised implementation will represent a positive development, allowing for the exploration of design solutions and even the enumeration of design corpus results. The potential of this generic grammar will be fully tested with a computerised tool, as was the case with previous work developed for housing shape grammars, such as the ABC system and the Haiti Gingerbread House grammar (Benros et al. 2011).

8. Hybrid designed by a generic shape grammar	295
8.1 A forth grammar generated by the generic shape grammar	295
8.2 Design hybrids amongst housing designs	304
8.3 Design hybrids within different shape grammars	316
8.4 Results	318
8.5 Hybrid questionnaires and user perception of design	337

8. Hybrid designed by a generic shape grammar

The present chapter focus on a set of solutions not covered in the previous chapter on the generic grammar. The previous chapter replicated existing solutions, new solutions that could fit within the design criteria of each language and showcased designs for each language. By doing so it allowed for a third specimen of design not discussed yet – the hybrid. The hybrid is part of a set of new designs that brace across styles or languages and share commonalities between both parent languages. A non-restrictive grammar would allow this corpus and since its design could assist describing how users perceive the graphical representation of design this is the topic of the following chapter.

8.1 A fourth grammar generated by the generic shape grammar

The concept of hybrid was introduced in English language to describe a cross breeding between two plants or animals of different species or varieties. The result would be a combination of the main dominant features of the two species into one new specimen.

In the animal world a good example are mules. These are the result of the cross breeding of two domestic animals, a female horse and a male donkey. Not coincidentally, the word mule derives from the German language and means 'to be reinforced'. The animal originated shares commonalities with both horses and donkeys but is in many ways distinctive. It is a good robust domestic animal (reinforced as the word claims). Most are physically closer to the horse built, height and agility, but share with the donkey species the capacity to carry weight, the strength, stubbornness and resilience, and even some physical features such as the long ears and the head shape. As many other hybrid species the mules have one great disadvantage, they are sterile and incapable to reproduce which certainly consists as a major setback in biology.

In design, hybrids are envisioned as design solutions that are derived from two parent influences. Usually the designer is driven by a selection of the best features of each specie (instead of the dominant features) and creates a design which attempts to be a merge between the two species. The previous chapters described a generic grammar to produce designs of 'three parent' design languages. This generic grammar enables the design of solutions within the corpus of each language, it also enables the design of new solutions.

What has not been described so far is what happens in the interstitial spaces amongst the three languages. The three grammars are district and directed by different features, geometries, proportions and spatial relations. However, overlaps occur between certain designs. For example, in chapter 7 it is shown the derivation of the Winslow prairie house (Koning & Eizenberg 1981) and is mentioned the resemblance of this house with some Palladian designs (Stiny & Mitchell 1978). (Figure 8.1) Within these features one can highlight a rectangular containment shape (generally not evident in prairie houses), a level of symmetry, a subtle

suggestion of underlying 3x3 grid, the alignment of external features with the symmetry axis such as stairs and terraces and more surprisingly a central room.

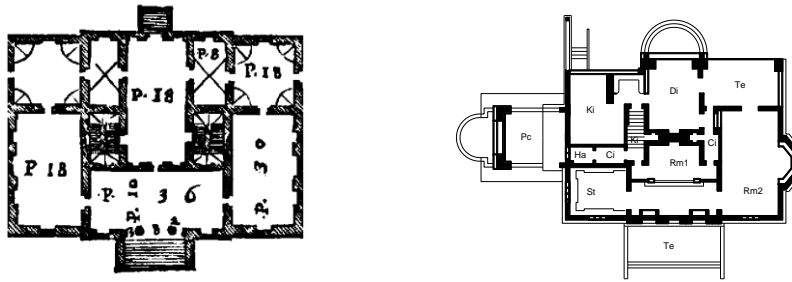


Figure 8. 1: Representation of villa Poiana from Palladio (Palladio 2002) and Prairie Winslow House from Frank Lloyd Wright were some visible similarities can be drawn (Koning & Eizenberg 1981)

This central room resembles the central space patent in Palladian villas, the space commonly used for entertainment. There are not many prairie houses with a central room since most of the focus is made on the fireplace both as geometric centre and as origin of design. In this case the focal point fireplace occupies the centre but shares this position with the adjacent living space. In the figure Winslow house is shown next to villa Poiana which stands today in the Vicenza area. This was selected as the Palladian villa that draws a closer parallel. They share an approximation in terms of proportion, position of the main entrance and external staircase, internal structure based on an underlying 3x3 grid, position of living areas around the perimeter and a special focus on the main central entertainment area. Other aspects can be observed, the suggestion of symmetry, the relative position of circulation spaces and internal stairs in the smaller mid bays and finally the array of smaller external elements around the outline of each house such as terraces, loggias and external stairs.

Despite all similarities Winslow house cannot be considered a hybrid. This is a house that in the generic grammar tree might occupy a mid-position between the Palladian grammar and the prairie houses grammar.

However, this close proximity of gender could hint at some hybrid examples. Examples that sit between languages and could easily be adopted by either language.

The idea of using the generic grammar for generating a hybrid design is in architecture context a polemic one. The three parent design languages selected represent distinct single housing genres from different eras, scopes and influences. The creation of a hybrid using the work of any of the three great architects that authored them does not pose design relevance. Hardly anyone would be interested in owning a hybrid Palladian/Prairie house and it would be difficult to assess which would be the dominant features of each style.

The purpose of this section is to test whether people's aesthetic judgements coincide with the grammar corpora results. The intuition and overall perception of design is mostly based on aesthetic assumptions based on the graphic representation of the design language solutions. the development of hybrids assist determining the boundaries of these perceived similarities. It

allows us to discuss the role of design similarities and differences and the boundaries of each design language. In addition the study of hybrid grammars allowed for relevant questions to be posed:

1. If style can be 'quantified' and compared can certain designs have relevance and sit among two distinct languages whilst maintain coherence
2. Can a design inherit the dominant relevant features of two (or more) parent languages and mutate as a stronger language
3. Can hybrid designs trespass the value of the parent languages and stand on its own

To test the hybrid theory a strategy was put into place.

The generic grammar was again validated by a fourth design language. This language previously described as a grammar covers the language of the bungalows of Buffalo in the state of New York in the USA (Downing & Flemming 1981). This grammar describes single housing and is based on a universe of houses designed and built in the beginning of the twentieth century. These houses share a set of characteristics which can define a style but were designed within a span of two or three decades by different designers in a confined geographical area. Unlike the Palladian villas, prairie houses and Malagueira, these houses are not originated through conception of a specific designer. In a way they reflect a regional vernacular style in which the internal spaces are well defined, its particular use and exterior provides a set of commonalities that creates a consistent street view of the group of houses.

The Buffalo bungalows were selected for the generation of a hybrid design.

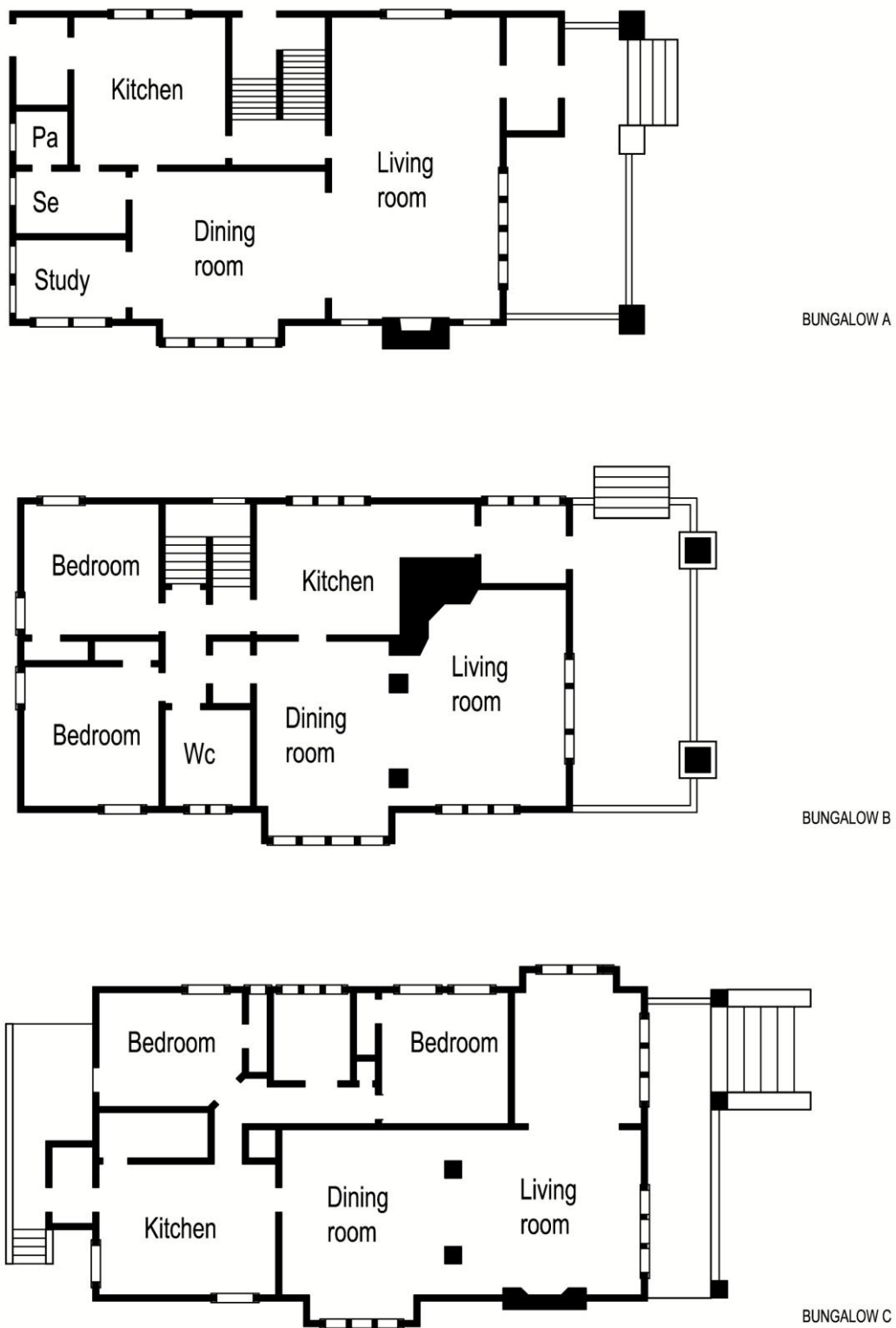


Figure 8. 2: Buffalo bungalow A, B and C as illustrated in the Buffalo bungalow's shape grammar (Downing & Flemming 1981)

The term bungalow was initially used to describe a small holiday house in the country side or by the coast of humble features that serve its purpose as a dwelling but could only be used in part of the year. Its thin wall construction did not incorporate thermal insulation. The house was free-standing, one storied, wooden structures sustained by piers, surrounded by verandas or porches and its whole perimeter covered by a multi-volume pitched roof structure.

This genre became popular in upstate New York as a holiday home and in the beginning of the twentieth century made its transition to the suburb as a more permanent residence. As mentioned by (Downing & Flemming 1981) this genre became so popular that several publications illustrated its layout and publicized it thorough the USA crossing regional boundaries.

The authors produced a grammar based on a set of examples that were built between 1915 and 1924. In this case set all houses were selected from the same neighbourhood and share evident commonalities. Figure 8.2 showcases three of the houses illustrated and named as bungalow A, B and C.

The common features identified are:

- Single housing centred within the plot
- Rectangular floorplan and outline
- Orthogonal spatial division
- Emphasis on the living space
- Reduction of circulation spaces and incentive on interspace communicability
- Inclusion of porches and verandas as transition between private and public areas
- Compact floorplan that derives from rectangular packing of spaces
- Inherent common 3x2 grid but other grids are also allowed (2x4, 3x3, 3x4)

Bungalow A showcases the aforementioned characteristics and represents the smallest of the three houses. It illustrates a small front porch and suggests an inherent 3x2 grid. Bungalow B shares some of the spatial organization of A, but includes in the ground floor two bedrooms and a bathroom being a self-contained independent floor which could serve as a small bungalow in itself.

Bungalow C illustrates a larger construction with larger proportions, two porches and two sets of external stairs, two ground floor bedrooms and a full bathroom to join the ample living dining space and full kitchen. (Figure 8.2)

Similarly to Malagueira these houses are set within a rectangular plot orientated perpendicular to the main road. This creates a main access and rear back yard.

In the bungalows of Buffalo grammar a grid formulation is proposed. This grid is not formalized conventionally but using independent shape rules and schematas that can be articulated in different combinations. The method is less elaborate than the Palladian grammar which is assisted by intricate copy rules, instead the method proposes first the assignment of an enclosure followed by the internal division. The allowed grids vary between a 2x2 up to 3x2 even though some houses could be supported by a larger 3x3 grid.

A set of predicted patterns can be illustrated, summing the possible combinations. This can also be easily illustrated using the subdivision method. Most of these patterns resemble the Malagueira supported patterns as illustrated below.

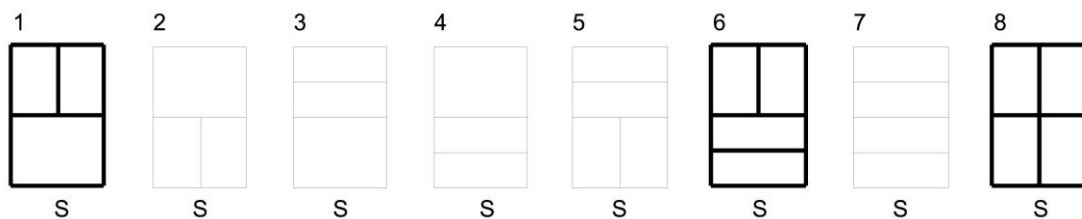


Figure 8. 3: Malagueira supported patterns (Duarte 2001)

The Buffalo bungalows grammar is used below as validation of the generic grammar originally developed targeting three original grammars the Palladian villas, prairie houses and Malagueira. Bungalows A, B and C illustrated in the original grammar were generated using the set of generic shape rules and the generic formalism. Figures 8.4 to 8.6 illustrate the derivation of these bungalows.

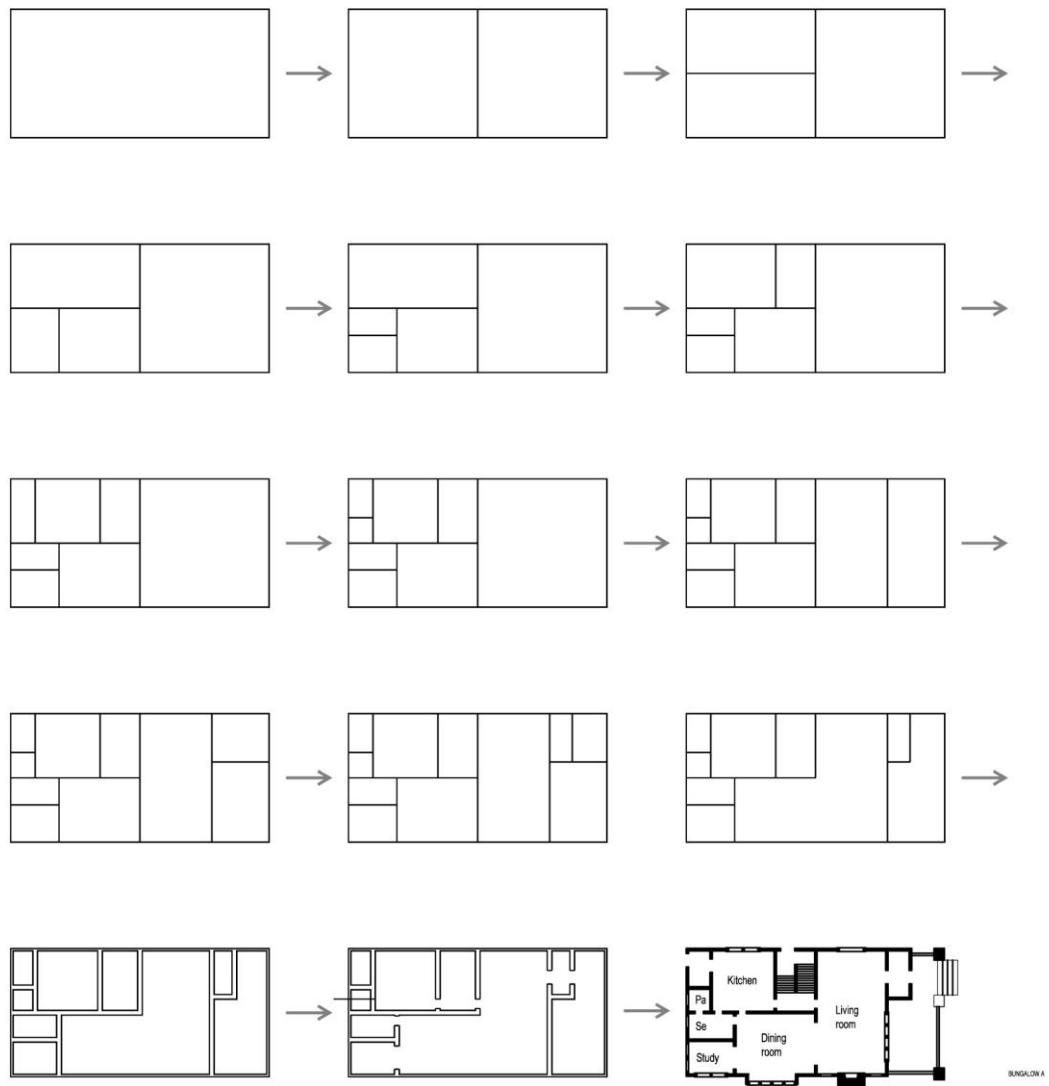


Figure 8. 4: Buffalo bungalow A derivation using the generic grammar formulation

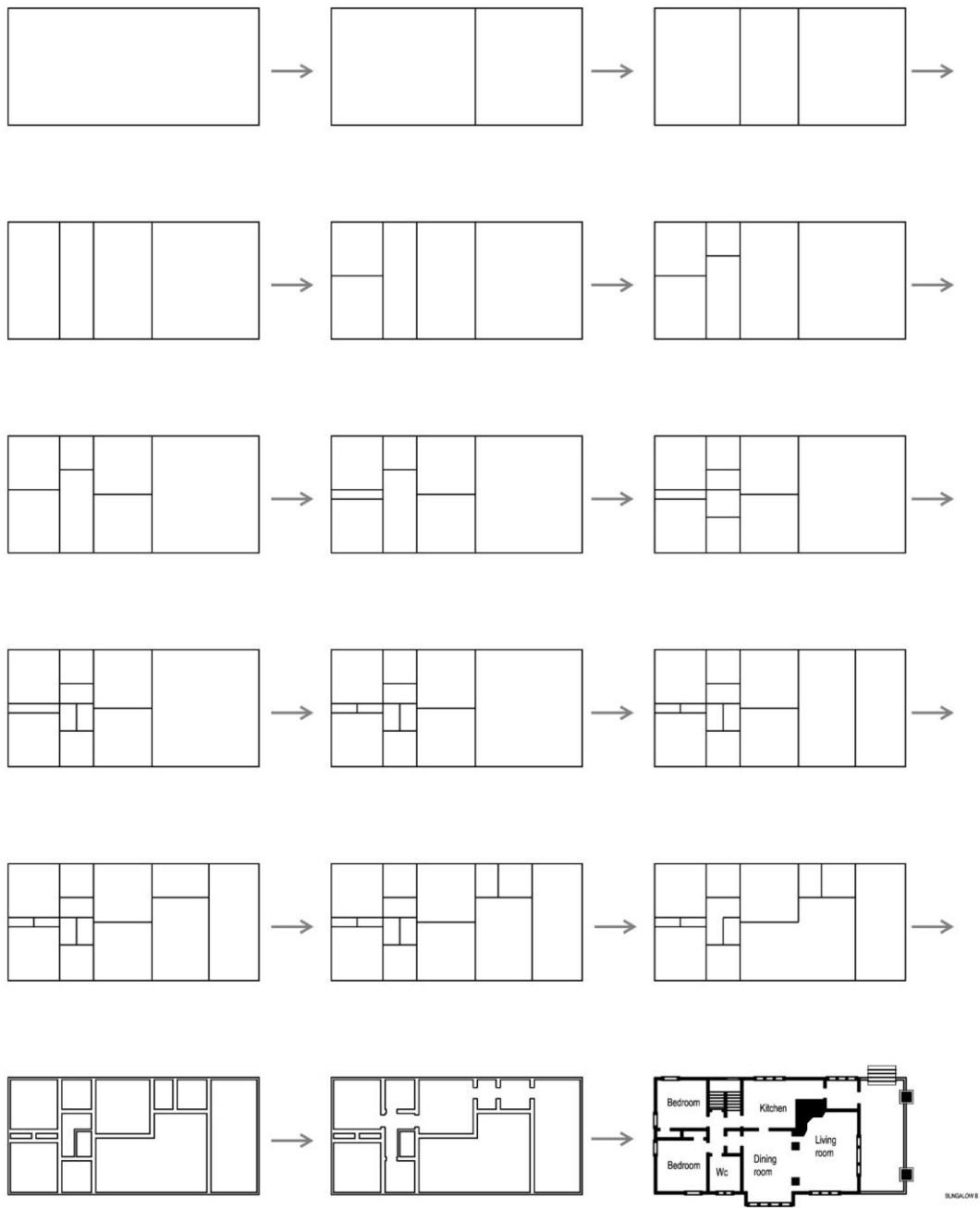


Figure 8. 5: Buffalo bungalow B derivation using the generic grammar formulation

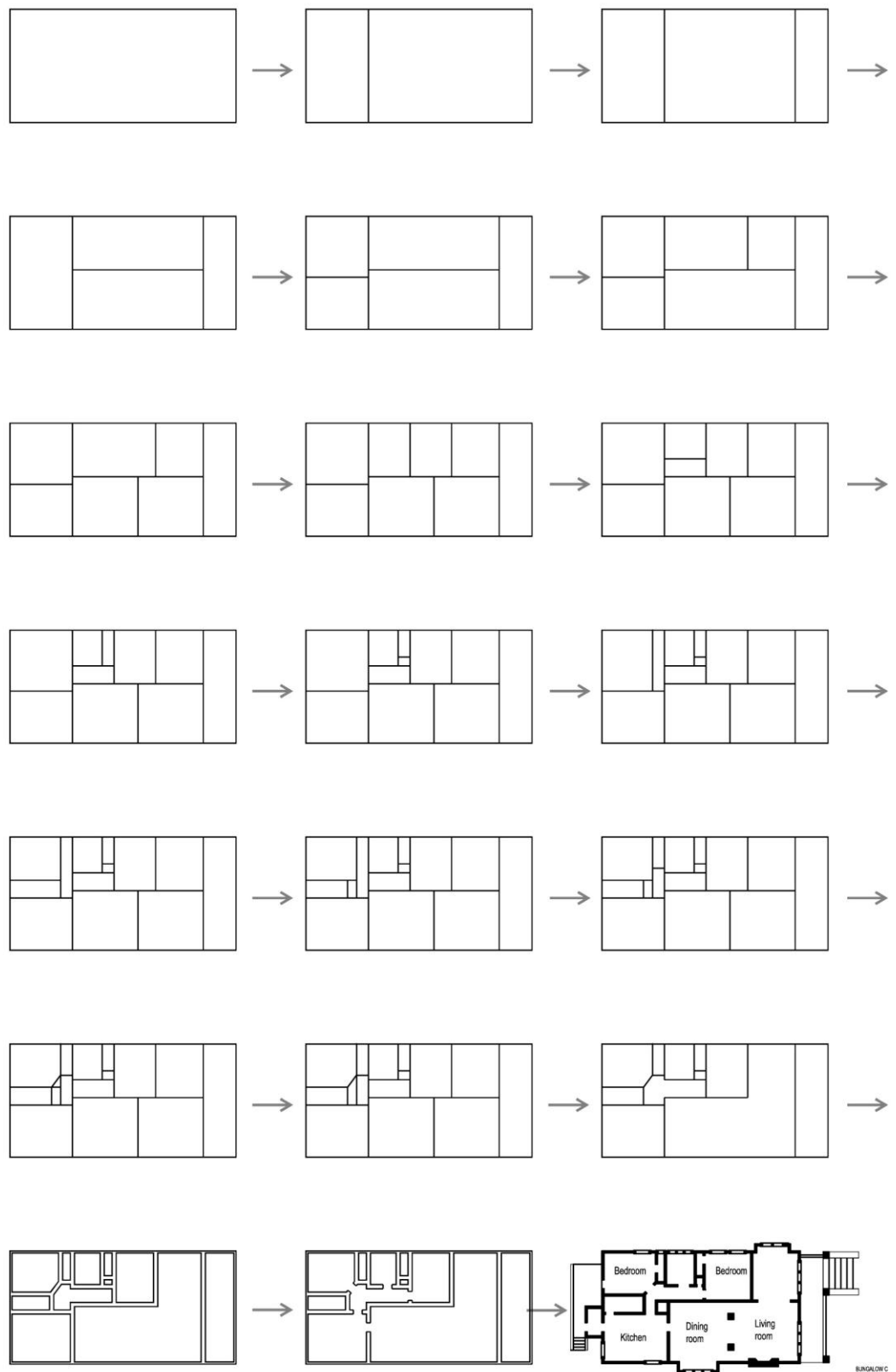


Figure 8. 6: Buffalo bungalow C derivation using the generic grammar formulation

8.2 Design hybrids amongst housing designs

The generic grammar as showcased in chapter 7 allowed the generation of a set of language related houses from the existing corpus. It also allowed for the design of new solutions that can be considered in the part of the new corpus of solutions. However, this generic grammar can also prompt the generation of solutions that fall between categories. Understandingly the grammar is restricted by the use labels and parameterization to condition the allowed designs and avoid unexpected designs. There are two reasons that would support the design of these solutions that fall between categories or hybrids, one is related with design exploration and the second for research purposes.

There are some issues that can be controversial when dealing with hybrid designs. How could one manipulate the design of an acclaimed designer such as Palladio, Frank Lloyd Wright or Siza and combine it with some other design to generate a new solution. Additionally, the question arises of what consistency or even relevance to fuse two designs such as any of these with so many differences and asymmetries into one new hybrid design. In housing design context, function, language and use are key for design consistency. If the same formalism can be manipulated to create all of the above using the same shape rules it can also be manipulated to design a design that falls in between, but that is not its purpose. Is important to understand how these hybrid designs would behave, look and feel.

As an experiment is introduced a fourth language of the Buffalo Bungalows.

This language presents a good opportunity for the study of hybrids in this research:

- The grammar describes a language that is distinct but is not generated by one single designer, but a regional style than a personal creation which does not impose any sense of ownership or authorship
- The bungalows share commonalities with other houses such as Malagueira
- The style is well defined and illustrated showcasing a set of well-defined functions and spaces of single housing
- The existence of style commonalities allows for the creation of consistent design solutions that are operable and can stand alone

A set of predicted patterns can be illustrated, summing the possible combinations. This can also be achieved using the subdivision method. Most of these patterns resemble the Malagueira supported patterns as illustrated below.

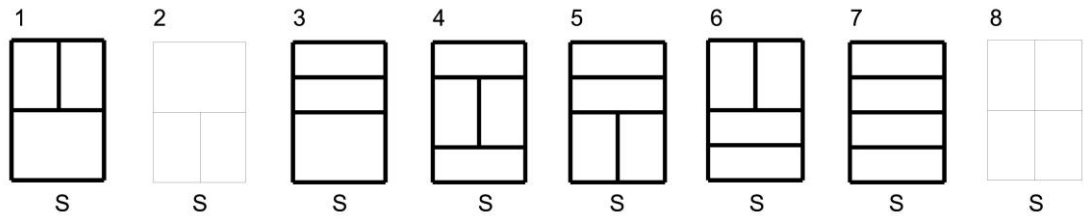


Figure 8. 7: Buffalo bungalows supported patterns

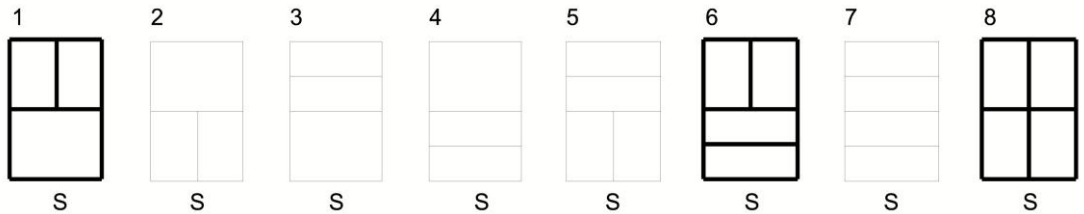


Figure 8. 8: Malagueira supported patterns (Duarte 2001)

Above in Figures 8.7 and 8.8, the bold highlighted lines showcase patterns that were used in built examples and are integrant part of each respective grammar. The single regular line patterns are represented in the grammar and reflect the set of new designs that could be considered as family related but were not tested by the original system.

This proximity between supported patterns inspired the development of a hybrid design. Ideally a hybrid is generated from parent languages that share commonalities. The outcome of complete distinct parent languages would certainly generate designs that would lack in consistency and would require severe redesign to achieve a working solution.

Observation of the allowed patterns for both house typologies revealed:

- Both house types allow for a rectangular containment
- Common underlying grids and divisions are allowed by both languages
- Both languages propose single housing despite Buffalo allows for a house detached and isolated within the plot and Malagueira for a full occupation of the available plot
- There is a distinction made between street side or access and back yard
- The outline proportion and dimensions have similarities, whilst Malagueira proposes a 12x8m Buffalo varies from 36'x24' (approximately 8.5x8.5m)
- Both styles allow a space allocation of the basic living functions: living, dining, kitchen, bathroom, bedroom and circulation (stairs, corridor or hall)
- Both house types allow for construction of up to two storeys
- Both languages have a design emphasis on the orthogonal layout but allow particular and localized angled layouts

Each style also presented specificities and particular features:

- Fixed house plot in Malagueira versus Buffalo bungalow with a varied dimension

- External space provided by Malagueira resembles a patio of that occupies 25 to 30% of the available plot, whilst Buffalo bungalows provide ample porches or verandas
- Malagueira allows fenestration from each space through either the main street or the internal patio, the house occupies the entire plot, while Buffalo bungalows are centred in the plot and views are displaced in the four sides
- Both styles showcase spatial proportions that are particular, living spaces in Malagueira are usually rectangular of 2:3 proportions while circulation spaces have different ratios such as 1:2 or 1:3

The major difference and particularities between the two languages are key in the development of the hybrid design. Three hybrid designs were developed and are illustrated in Figures 8.9 to 8.11, respectively these designs were generated using bungalows A, B and C as design principle and applying some of the selected dominant characteristics of the Malagueira language. The design of these hybrid solutions used the commonalities between languages as guidelines and applied some of the selected distinct characteristics of Malagueira.

The hybrid design used the following design principles:

1. 12x8m house plot
2. Segregated access from street side and adoption of backyard
3. No side fenestration
4. Integration of a patio instead of the porch
5. Basic spatial organization derived follows bungalow's original example
6. Malagueira's outline of 12x8m as a criteria
7. Subdivision process to generate spaces applied Malagueira patterns and proportions

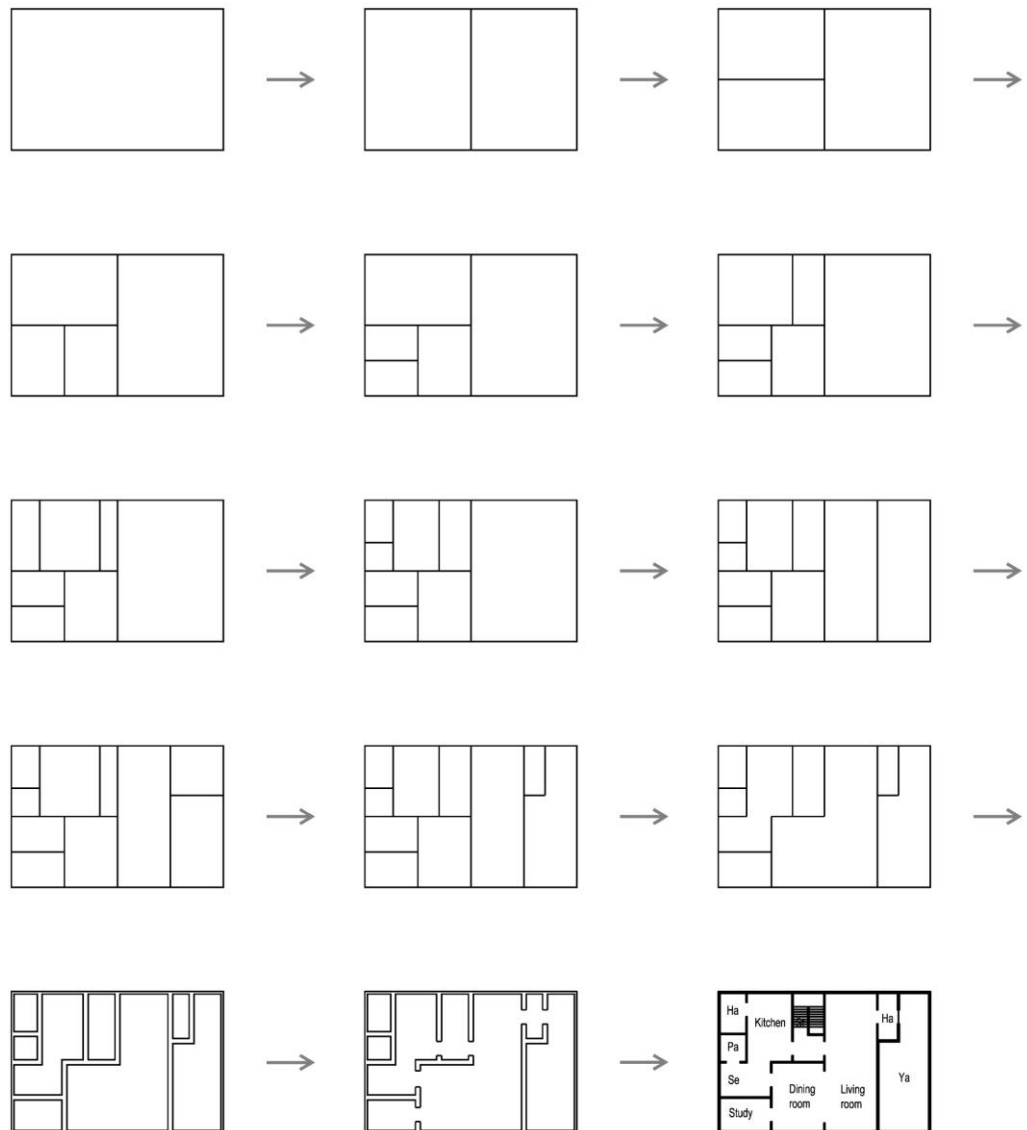


Figure 8. 9: Buffalo bungalow A hybrid - A derivation using the generic grammar formulation (pattern 1 from Malagueira typical patterns can be used to design Buffalo Bungallow A as illustrated in Figure 8.7)

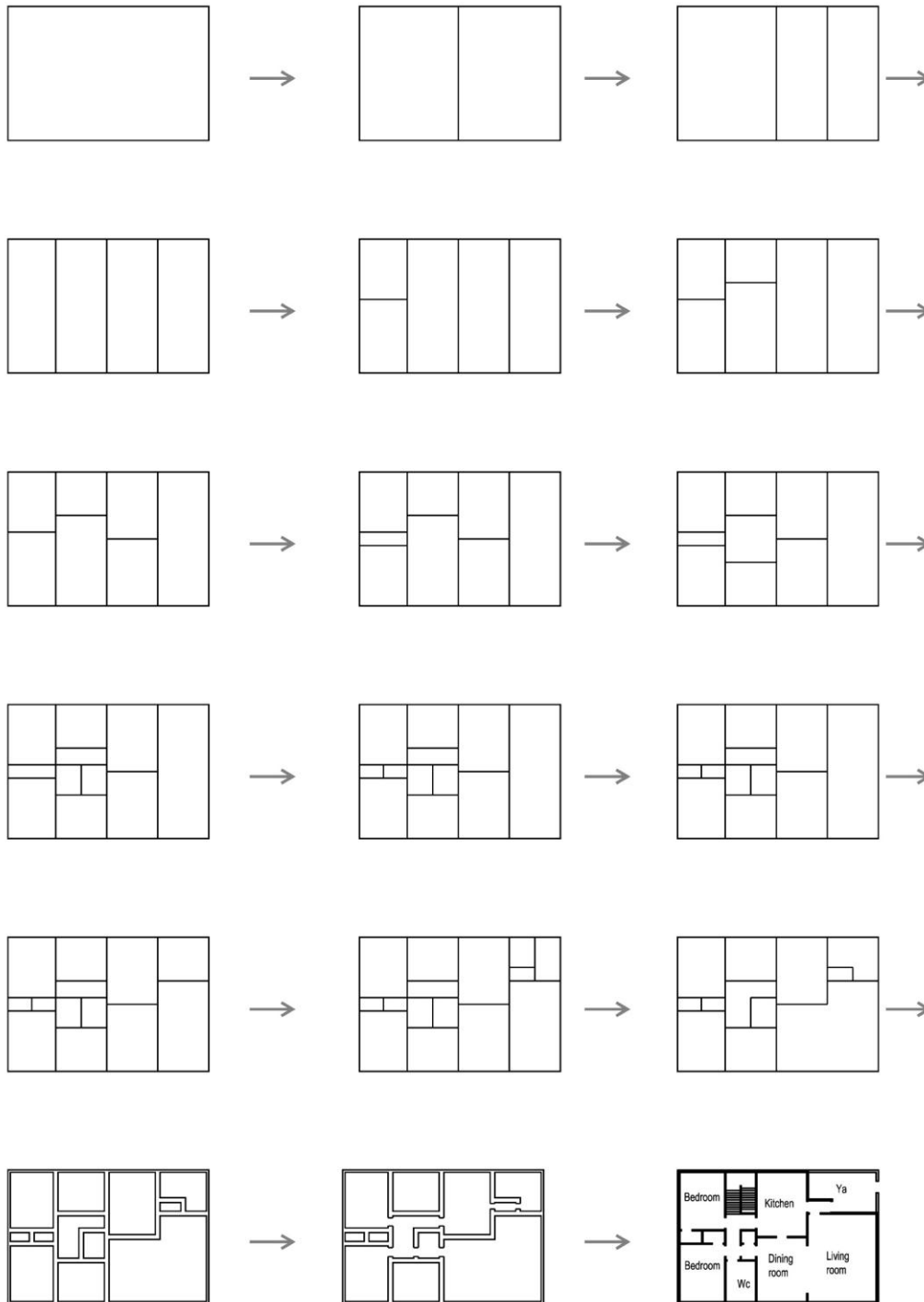


Figure 8. 10: Buffalo bungalow B hybrid – derivation using the generic grammar formulation

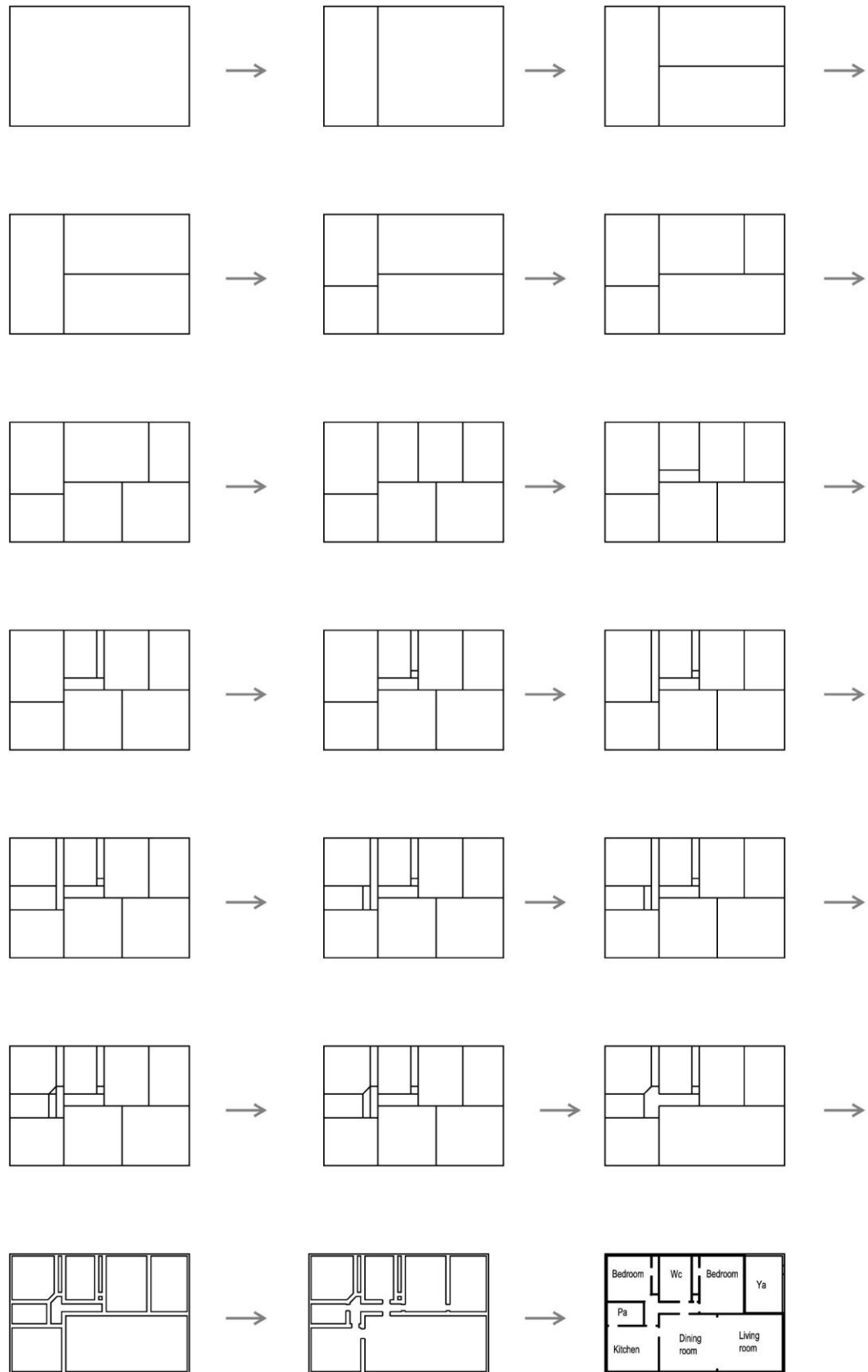


Figure 8. 11: Buffalo bungalow C hybrid - derivation using the generic grammar formulation (a typical Malagueira pattern '8' can be used to represent Buffalo bungalow B as shown in Figure 8.7)

Figures 8.9 to 8.11 demonstrate the derivation of Hybrids A, B and C.

Like other examples created by the generic grammar these were generated using the same principles and generic shape rules and through the same stages of design:

1. Define boundary
2. Spatial compartmentation through subdivision
3. Concatenation
4. Wall thickening
5. Functional assignment
6. Creation of openings
7. Detailing
8. Design termination

For the purposes of hybrid design, and to expedite the process some intermediate spaces were illustrated above. The hybrids were generated using the same generic rules as the previous designs tested with no need to add any additional feature or shape rules.

The results are solutions that could easily fit both styles but are less distinguished. This means that some of the characteristics from each language were stripped away. Most hybrids feel almost anonymous despite sharing some familiarities.

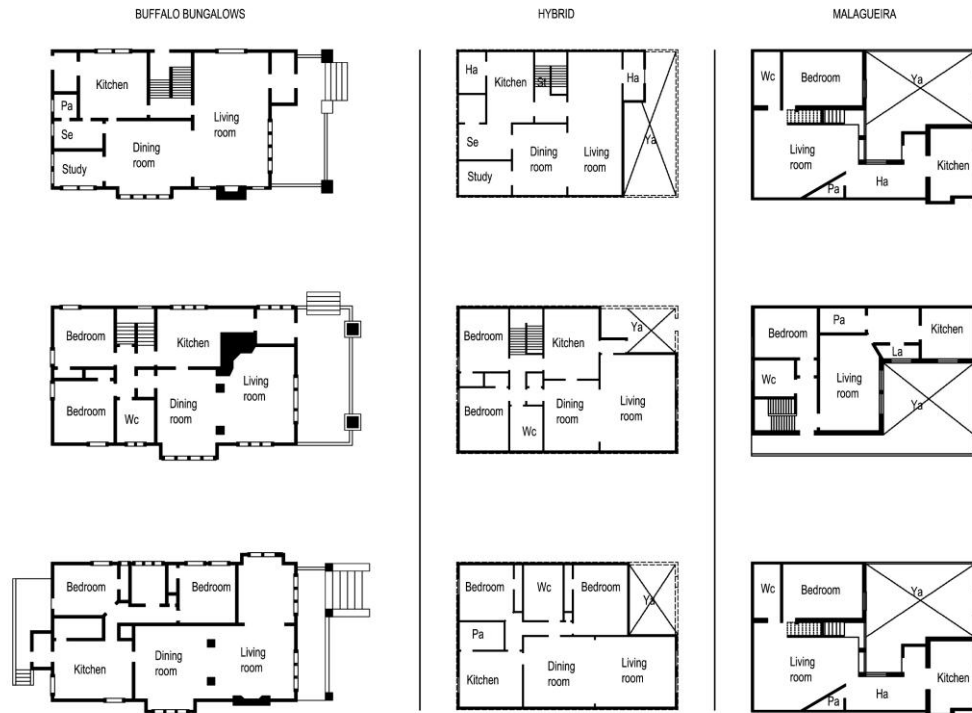


Figure 8. 12: Hybrid design comparison

(Hybrids retain Malagueira envelope and Buffalo's internal distribution)

In the illustration above (Fig. 8.12) three bungalows were illustrated on the left column corresponding to bungalows A, B and C. These were the starting point for the development of three hybrid solutions (illustrated in the centre column) and as reference the second parent language Malagueira with houses Ab and Bb.

Each column showcases one style, on the left the Buffalo bungalows in the middle a hybrid design and the right column shows Malagueira houses examples. Buffalo bungalows A, B and C are shown on the first second and third row respectively.

The left column is a distinctively rich array of volumes. To the main rectangular outline a series of small volumes are plugged-in to create recessed spaces and bow windows. The porches are dominant and are segregated from the outside space by a sequence of steps. Most windows project outwards and look out from the sides of the construction.

Distinctively the right column exhibits a set of particularities. The courtyard or patio is dominant in all houses, occupying a significant area. Fenestration occurs either towards the patio or towards the road which determines most of the internal spaces locations. The fixed outline and ratio also dictates the house limits creating a constant element for all solutions.

The hybrids on the other hand are placed in the centre for better comparison and evaluation. They resemble both of the previous cases and at the same time they do not. The fine negotiation of the two languages resulted into a solution that does not compromise in any particularity or quirk of style but could easily fit both ways. When compared with the bungalows the design is more compact and contained, sharing more of this containment feature with

Malagueira. When observed spatially it claims similarities with the bungalows from which inherited the layout. Nevertheless, the internal spatial proportions are Malagueira like and therefore it could easily sustain the Malagueira tag.

The courtyards replaced the verandas, stripping away the great feature of the bungalows.

Hybridation is possible by the generic structure but is it desirable? Does it pose an advantage?

The total space covered by the generic grammar is larger than the three parent languages that originated it. The integration of the Buffalo bungalows is an example of this and the generation of Malagueira and Buffalo hybrids and additional proof.

A more abstract version of hybrids is the complete set of new designs presented through the derivation and a number of iterations of the division rule. These abstract representations showcase the across the board examples of derived generic grammar houses.



Figure 8. 13:

Hybrid design comparison

(Hybrids retain Buffalo's envelope and Malagueira internal distribution)

In the illustration above (Fig. 8.13) the same three bungalows A, B and C were presented and accompanied by the same Malagueira examples. Only this time the methodology changed. The directing features of style were swapped.

This way:

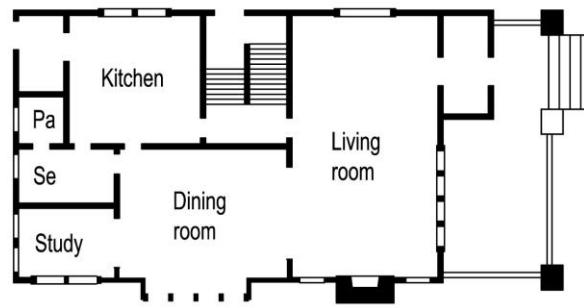
- The Buffalo bungalow's outline was the retained feature
- The Malagueira internal layout was maintained and adapted in accordance
- Adjustments between envelope and interior were performed on the latter

- External features such as fenestrations, doors, terraces and stairs were retained

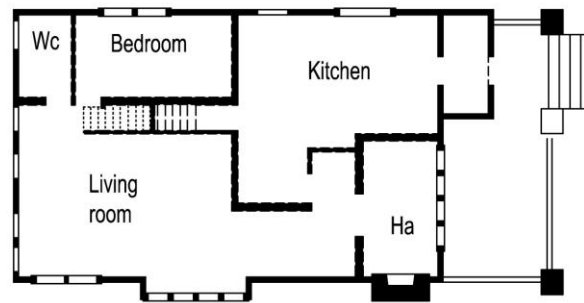
The results are revealing. Unlike the previous example (Fig. 8.12) that lacked some character and definition the latest hybrid floorplans resemble the original Buffalo language whilst proposing a different configuration. All three hybrids propose consistent layouts with coherent spatial distributions. They also position themselves closer to the Buffalo language than the previous hybrid design positioned to the Malagueira Language.

This can be better shown in picture 8.14 where the two hybrid solutions that derive both from the Buffalo bungalow A and Malagueira house type Ab are shown next to the original designs.

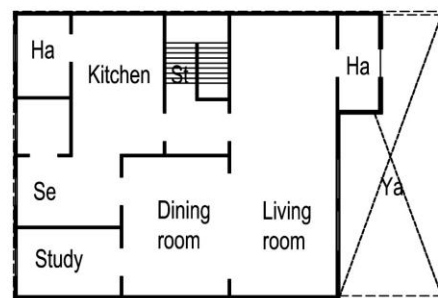
BUFFALO BUNGALOWS



HYBRID



HYBRID



MALAGUEIRA

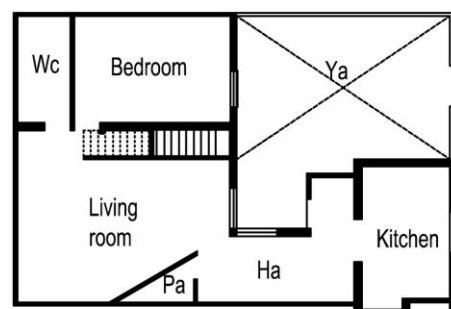
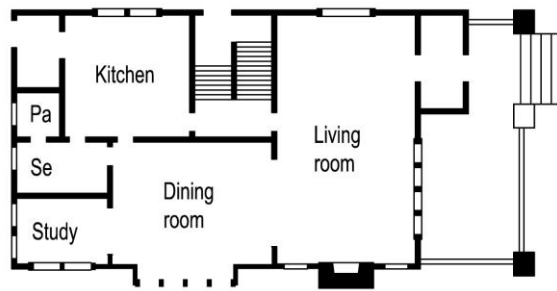
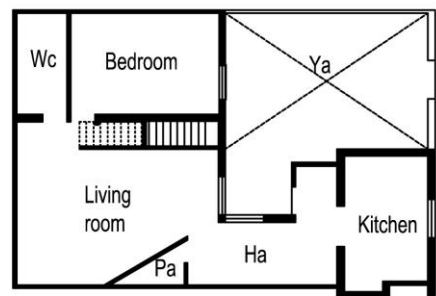


Figure 8. 14: Hybrid design comparison (top to bottom: Buffalo bungalow A, hybrid type 1, hybrid type 2 and Malagueira house Ab (Duarte 2005))

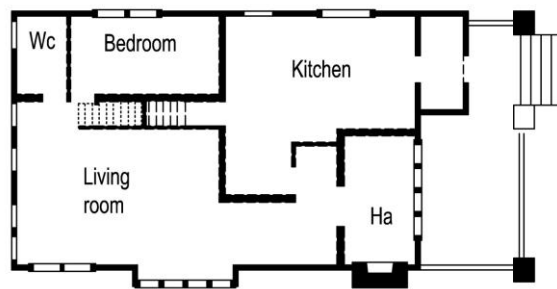
BUFFALO BUNGALOWS



MALAGUEIRA



HYBRID



HYBRID

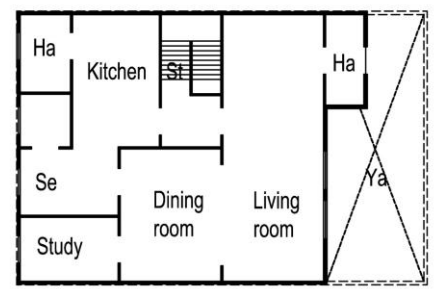


Figure 8. 15: Hybrid design comparison (left to right, top to bottom: Buffalo bungalow A, hybrid type 1, Malagueira house Ab (Duarte 2005) and hybrid type 2)

8.3 Design hybrids within different shape grammars

Palladian villas (Stiny & Mitchell 1978), Prairie houses (Koning & Eizenberg 1981) and Malagueira (Duarte 2001) were also put into the test. Using the three languages six new hybrids were created as shown in Figure 8.16. The image shows a table like arrangement where three original houses from each grammar were shown and maintained.

To represent a Palladian example, villa Emo was selected. As a Prairie house, Winslow was picked, and finally as a Malagueira house example was selected house-type Ab. These houses were selected observing three criteria:

- All houses were self-contained within a rectangular outline
- All houses presented an orthogonal layout
- Houses of similar size and proportions were selected to facilitate the experiment, however Malagueira had to be scaled up from its original 12x8 m ratio to allow a successful experiment

The methodology followed the sequence:

1. Each original house was positioned in three separate rows representing a language
2. Each column was responsible for the second parent language for each hybridization
3. The hybrid design would be generated by the envelope proposed by the particular 'row' and the internal layout proposed by the particular column
4. The envelope and outline from one of the parent styles was retained any adjustments occurred were performed in the interior
5. Exterior features such as verandas, porches, stepped accesses were maintained from the original style
6. Whilst keeping the outline of each parent style does not impose particular demands, the interior was generated using the original design as model, but any adjustments followed the language spatial rules (of symmetry in the case of Palladio, of proportion in case of Malagueira e.g.)

The resulting designs show new houses designs that could be mistaken by a new language solution, but also designs that lack in consistency. Some languages share many commonalities resulting into credible examples. The new examples were named after the two parent languages where the first language mentioned refers to the language responsible for the outline and the second the language responsible for the spatial distribution.

Six new hybrid designs were generated:

- Palladian x Prairie house
- Palladian x Malagueira house
- Prairie x Palladian house
- Prairie x Malagueira house
- Malagueira x Palladian house
- Malagueira x Prairie house

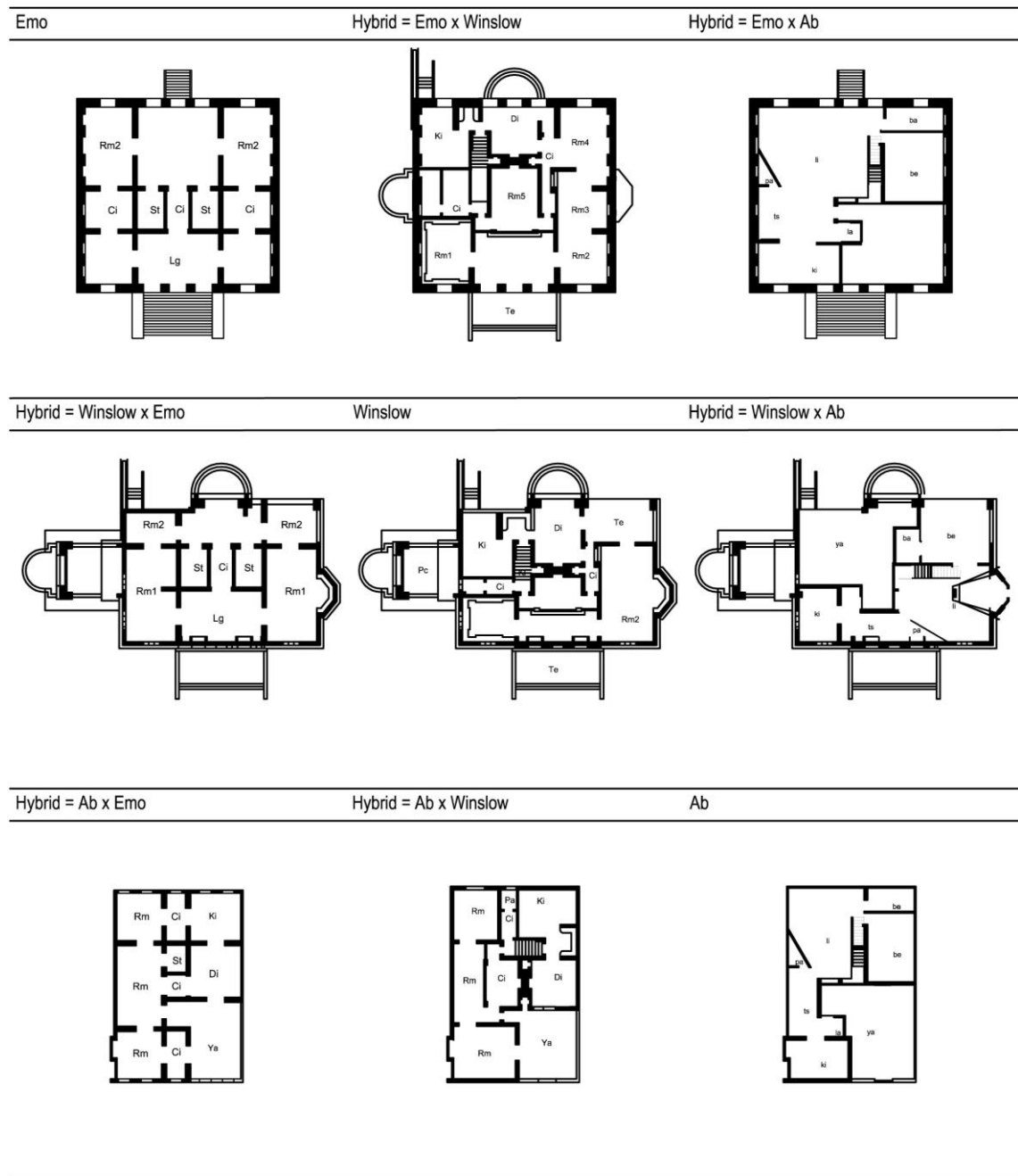


Figure 8. 16: Hybrid design comparison

(left to right, top to bottom: Palladian Villa Emo, hybrid design Emo x Winslow, hybrid Emo x Ab, hybrid Winslow x Emo, Winslow prairie house, hybrid Winslow x Ab, hybrid Ab x Emo, hybrid Ab x Winslow, and Malagueira house Ab). The original designs occupy the diagonal spine of this table

8.4 Results

Figures 8.17 to 8.19 showcase in detail the results attained from the experiment.

The Palladian hybrids are shown in Figure 8.17. The top example illustrated is the original design for villa Emo, middle and bottom respond to hybrid designs attained from cross overs with Prairie houses language and Malagueira.

Due to its regular and grid like layout the Palladian x Prairie hybrid seems to work well as a house. It showcases coherence both in the interface with the envelope and within the interior. The original house that inspire the interior layout also shows a central room which in many ways resemble the design criteria of the Palladian villas. The same cannot be said for the latter example. The Palladian x Malagueira design is awkward and forced. The house interior is deformed from its optimal rectangular outline to match the squarish outline. The house works but does not fit in either language criteria. It does not set closer to either language and certainly does not stand on its own.

Figure 8.18 shows the Prairie houses hybrids. The top floorplan includes the design for Winslow house and this is followed by the Prairie x Palladian hybrid and by the Prairie x Malagueira example. Again Palladian villas and Prairie houses seem to demonstrate several similarities which facilitates the hybrid design. The resulting house is coherent, showcases a working solution and could be easily identified with both styles. The latter example proposes a difficult solution that lacks in consistency and character. The interior layout was deformed to suit the envelope and the yard positioned coincident to the original terrace that Winslow house proposed. Despite the efforts the resulting design could hardly be affiliated with either language.

The last example shown is the Malagueira hybrids in Figure 8.19. This image illustrates on top the Malagueira house Ab, the Malagueira x Palladian example and in the bottom the Prairie house hybrid. Surprisingly most examples seem to work as a house. The insertion of a typically square outline into a rectangular containment seems to be more successful than the opposite. The deformation of the internal spaces and the modification of the internal proportions in this case does not seem to affect the main style features both in Palladian and in the Prairie house hybrids. Instead both examples of hybrids show a level of coherence and even adaptability with the new circumstances. Both languages seem to accommodate well the new outline and even the internal patio and entrance recess.

Some conclusions can be drawn from the observation of the hybrid designs:

- Certain languages propose design features that are in fact similar but shown within different parameterizations
- Proportions and overall size are a key language feature
- Internal layout and spatial adjacency seems to be a particular element of each language however it can be manipulated to suit
- Certain languages showcase several design similarities which are easily disguised by external features and ornamentation
- The hybrid is more acceptable the closer it is from the original design
- Two parent languages that share similarities originate more consistent designs

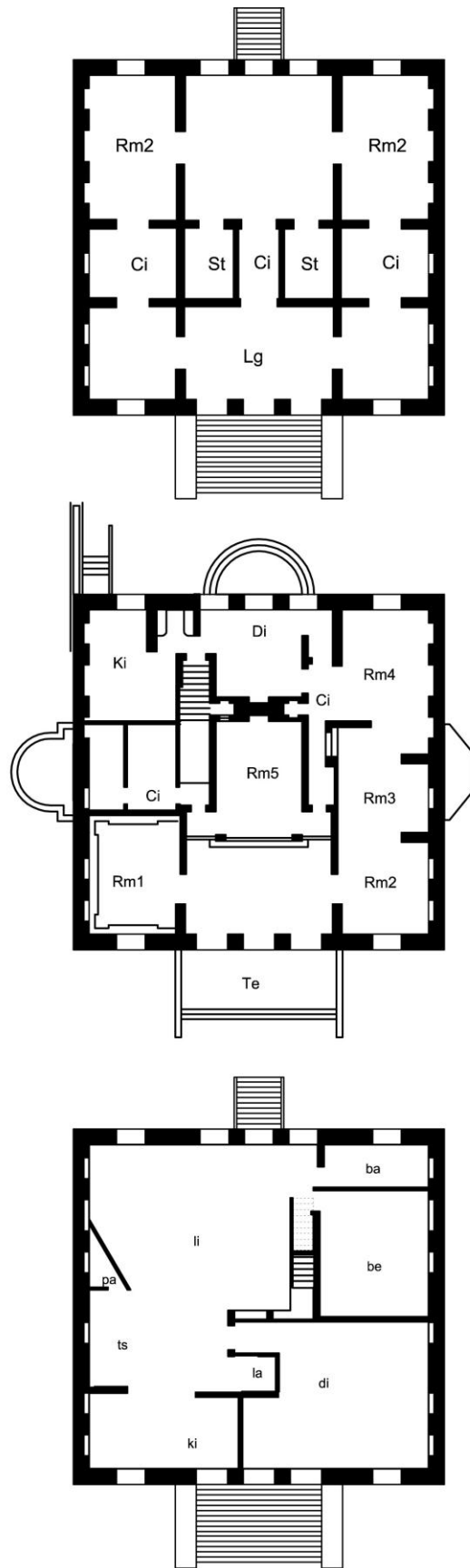


Figure 8. 17: Hybrid design comparison (top to bottom: Palladian Villa Emo, hybrid design Emo x Winslow, and hybrid Emo x Ab)

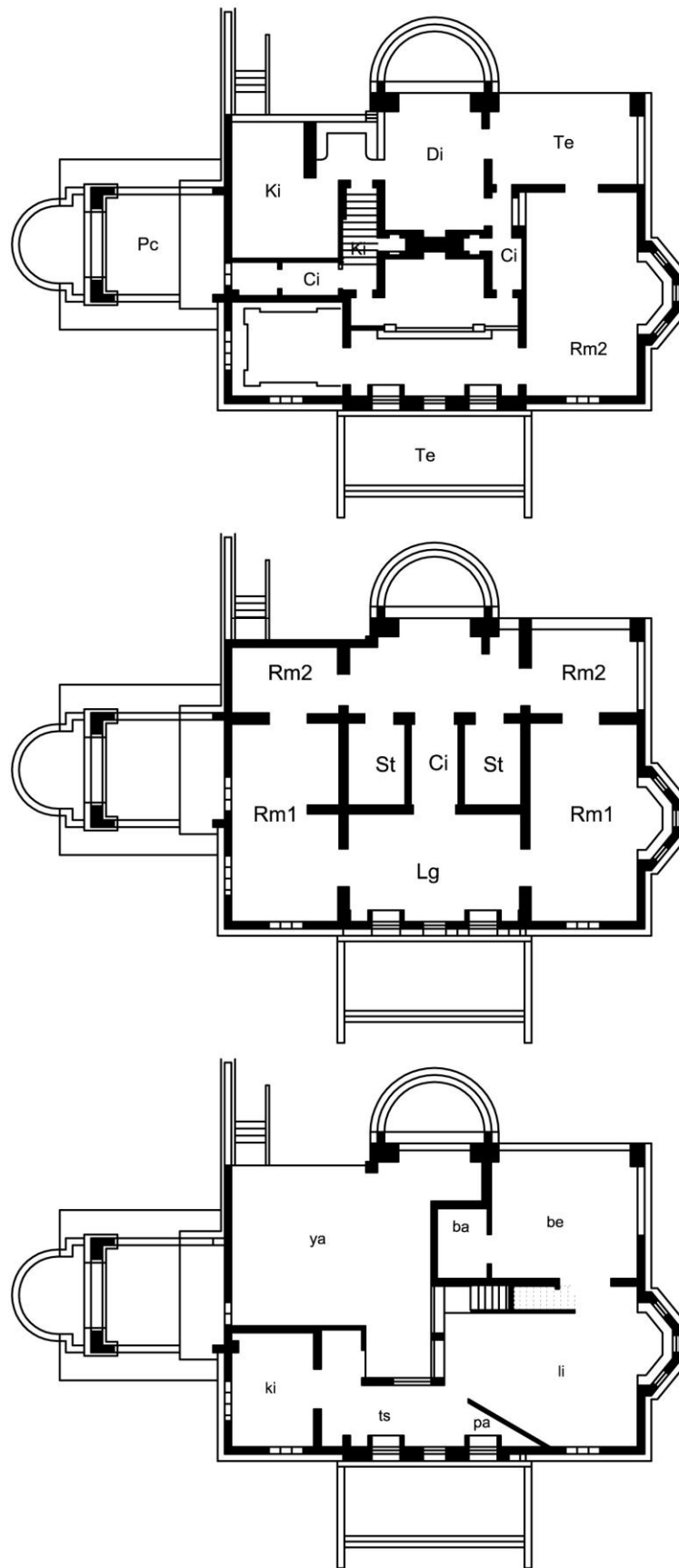


Figure 8. 18: Hybrid design comparison (top to bottom: Winslow prairie house, hybrid Winslow x Emo, and hybrid Winslow x Ab.)

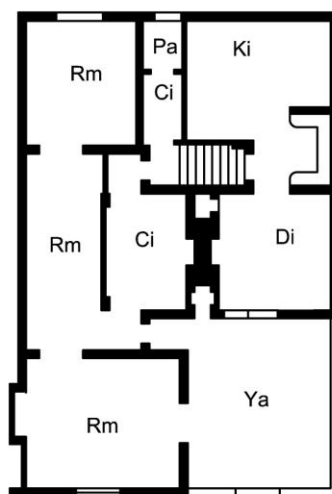
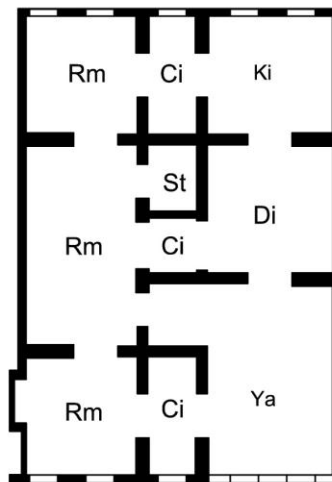
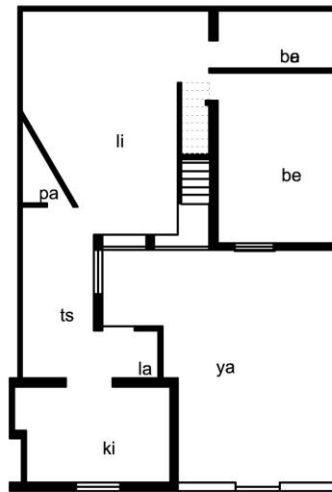


Figure 8. 19: Hybrid design comparison (top to bottom: Malagueira house Ab, hybrid Ab x Emo, and hybrid Ab x Winslow)

A hybrid grammar could be formulated from these findings, however the formulation of this hybrid grammar is indistinct from the generic grammar formulation illustrated in Table 8.2.

As shown this grammar does not differ from the formulation presented in previous chapters. Alike the Palladian, Prairie and Malagueira language this hybrid grammar uses the same shape rules illustrated before. The true distinction is the parameters used, or to be precise, the variable intervals allowed.

For each style a particular interval is defined and within this interval the particular styles are defined. The hybrid language, a specific one that combines the Buffalo bungalows with Malagueira or any of the examples presented crossing Palladio, Malagueira and Prairie houses showcase than the great difference between each language when applied to housing is the proportional difference between floorplans. Within the fine tuning of these intervals lies the harmonic balance of each design language.

The table presents a broad generalization of the hybrid grammar, which could be extrapolated as the generic grammar. However, a more precise formulation could be attained by replacing for each rule the parameterisations of each language. For instance to replicate examples such as the Malagueira x Palladian hybrid:

1. Outline containment derived from Malagueira proportions $X = 8\text{ m}$ $Y = 12\text{ m}$ permitted ratio 2:3
2. Internal layout – vertical subdivision following Palladian language: $y/x = n/m$; $m = [3,5,7]$; $n = [2,3,4,5]$; $y = y_1 + y_2$ $y_1 = y/n$ v $y_1 = y/3n$; $y_1 = y_2$ (for $n=2$ v $n=4$)
3. Internal layout – horizontal subdivision following Palladian language: $y/x = n/m$; $m = [3,5,7]$; $n = [2,3,4,5]$; $X = 2.X_1 + X_2$; $X_1 = n$ v $X_1 = 3.n/2$ V $X_2 = 2n$; $y_1 = y_2$ (for $n=2$ v $n=4$)
4. Space merging following Palladian grammar
5. External wall thickening following Malagueira grammar and constructive system
6. Detailing stages to proceed following external features of Malagueira language

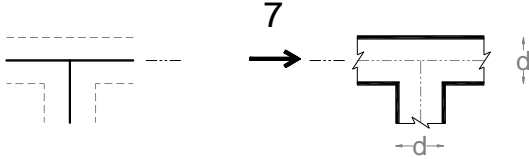
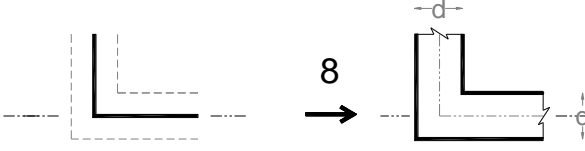
Table 8.1: Generic grammar formulation

STAGE 1	BOUNDARY DEFINITION	
RULE 1	ADDING BOUNDARY	PARAMETERISATION
Rectangular Outline Shape insertion		$y/x = n/m$ $m = [u,v]$ $n = [u,v]$ $u \wedge v \in I$
STAGE 2	SPATIAL SUBDIVISION	
RULE 2	HORIZONTAL SUBDIVISION	PARAMETERISATION
Horizontal subdivision		$[x, y]$ $Y = y1 + y2$
RULE 3	VERTICAL SUBDIVISION	PARAMETERISATION
Vertical subdivision		$[x, y]$ $x = x1 + x2$

STAGE 3	SPACE MERGING	
RULE 4	HORIZONTAL MERGING	PARAMETERISATION
Horizontal merging		$X=n$

RULE 5	VERTICAL MERGING	PARAMETERISATION
Vertical merging		$Y=m$

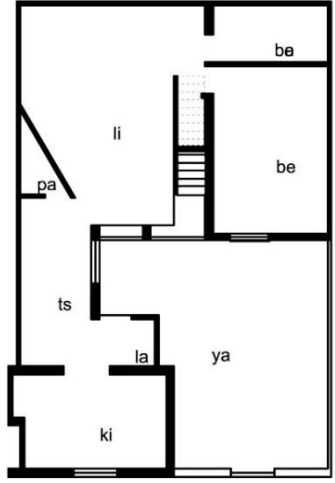
STAGE 4	WALL THICKENING	
RULE 6	SINGLE WALL	PARAMETERISATION
Offset for wall thickening		$d_{ext} = x$ $d_{int} = x$

RULE 7	WALL 'T' JUNCTION	PARAMETERISATION
Creation of wall junction		$d_{ext} = x$ $d_{int} = x$
RULE 8	WALL CORNER JUNCTION	PARAMETERISATION
Creation of wall corner		$d_{ext} = x$ $d_{int} = x$
STAGE 5-8	LANGUAGE SPECIFIC STAGES	
RULE N TO M	Language Specific Shape Rules	PARAMETERISATION

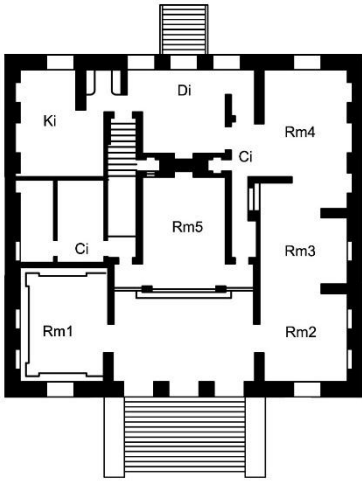
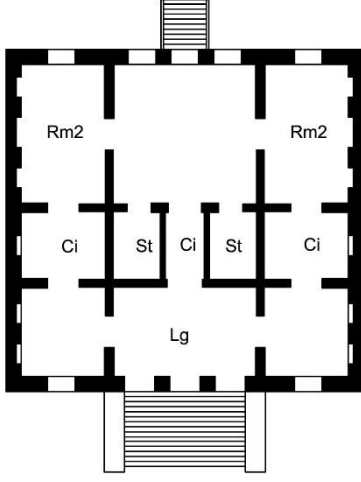
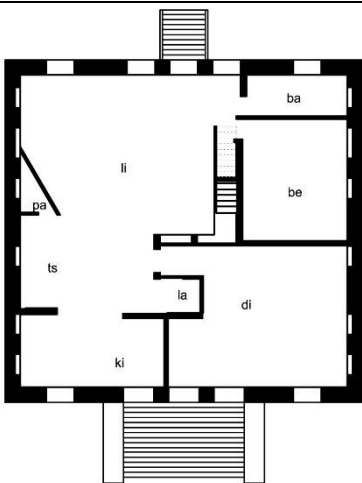
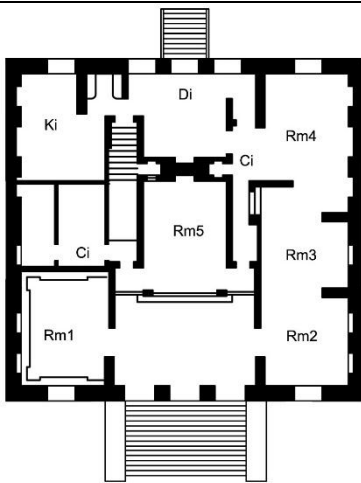
The study of hybrids allows to formulate the following conclusions:

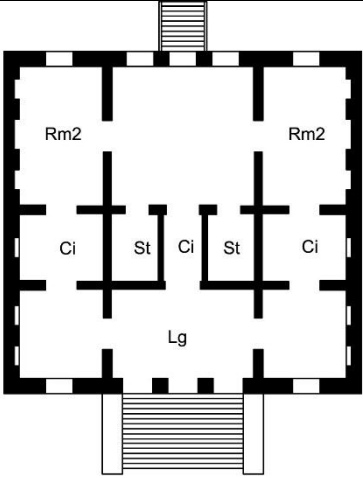
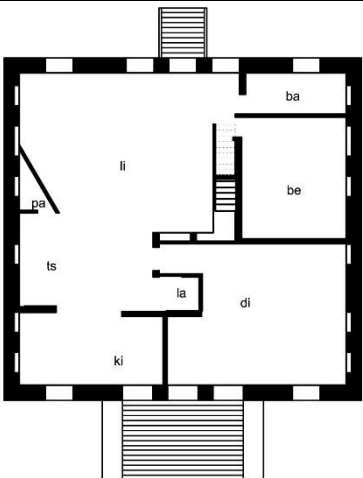
- The generation of hybrids can be performed using the generic grammar, its shape rules and particular language conditions such as specific parameterizations and intervals of values
- Hybrids can be generated from parent languages, its success and coherence is linked to the style proximity of the original languages
- There is an interstitial space occupied by hybrid grammars that sit between the parent grammars that originated the generic grammar. This space can be identified and 'quantified' by the intervals of values that originate the 'hybrid language' at stake
- The generic grammar is 'per se' a hybrid grammar. It is a grammar that enables amongst other solutions a specific style or language, but a particular hybrid language is a sub-group of the generic formulation
- Hybrids have no place in the design domain but its study and exploration have interest for both analysis and design exploration

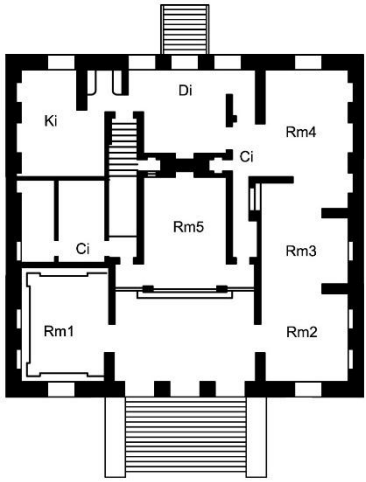
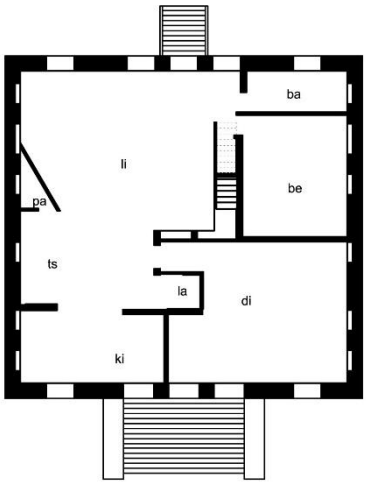
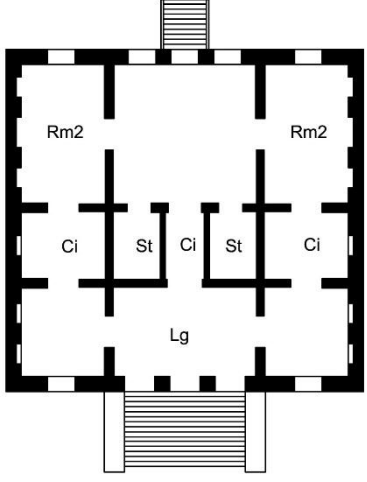
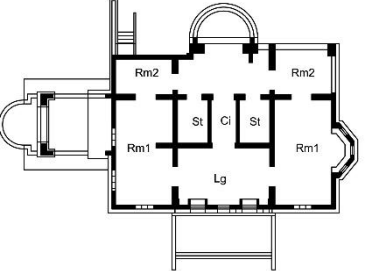
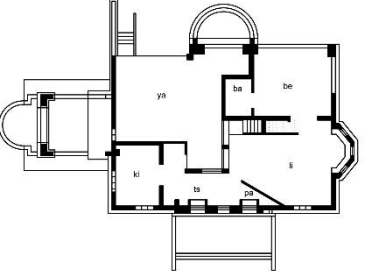
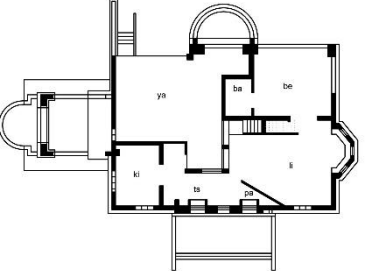
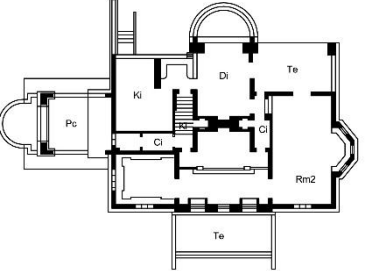
Table 8. 1: Hybrid corpus questionnaire

1.	Please indicate which of these houses' floorplans holds greatest aesthetic value.	
		
a. 54%		b. 46%
2.	Please indicate which of these houses' floorplans holds greatest aesthetic value.	
		
a. 55%		b. 45%

3.	Please indicate which of these houses' floorplans holds greatest aesthetic value.	
		
a. 54%		b. 46%
4.	From the following 3 floorplans only one represents an existing house, please select the most likely option	
		
a. 63%	b. 15%	c. 22%

5.	Please indicate which of these houses' floorplans holds greatest aesthetic value.	
		
a. 47%		b. 53%
6.	Please indicate which of these houses' floorplans holds greatest aesthetic value.	
		
a. 54%		b. 46%

7.	Please indicate which of these houses' floorplans holds greatest aesthetic value.	
		
a. 55%		b. 45%

8.	From the following 3 floorplans only one represents an existing house, please select the most likely option	
		
a. 56%	b. 13%	c. 31%
9.	Please indicate which of these houses' floorplans holds greatest aesthetic value.	
		
a. 45%		b. 55%
10.	Please indicate which of these houses' floorplans holds greatest aesthetic value.	
		
a) 51%		b. 49%

11.	Please indicate which of these houses' floorplans holds greatest aesthetic value.	
a. 45%		b. 55%
12.	From the following 3 floorplans only one represents an existing house, please select the most likely option	
a. 21%	b. 56%	c. 23%

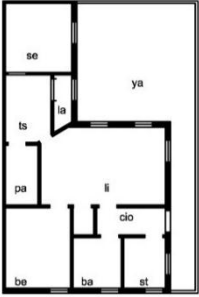
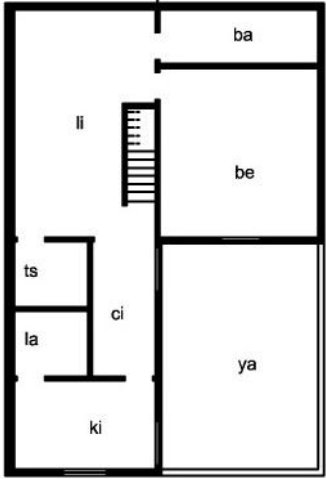
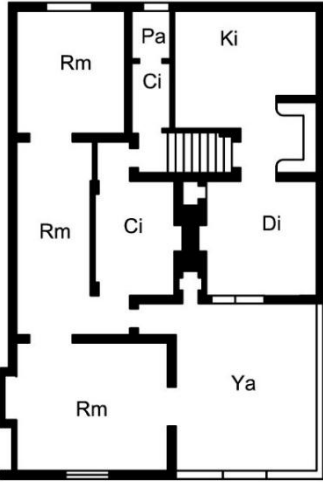
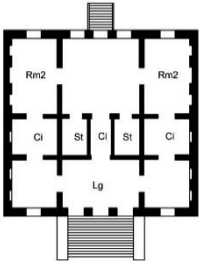
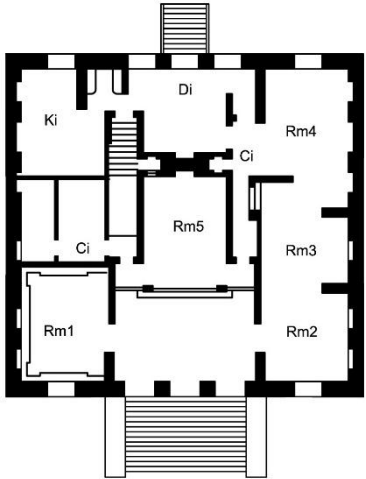
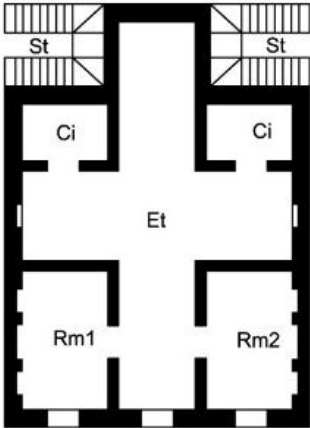
Table 8.1 illustrates a survey conducted for a universe of 103 participants to evaluate the differences between perceived design and real design language characteristics.

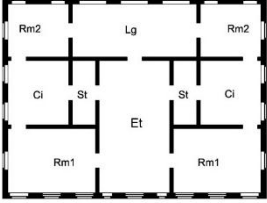
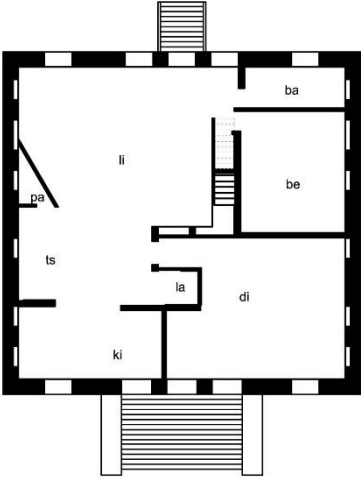
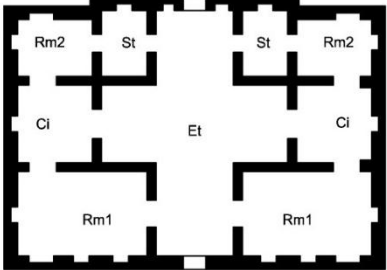
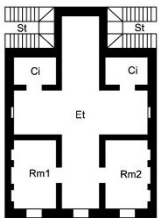
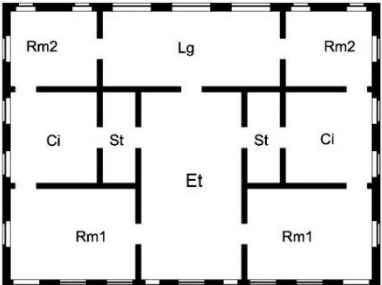
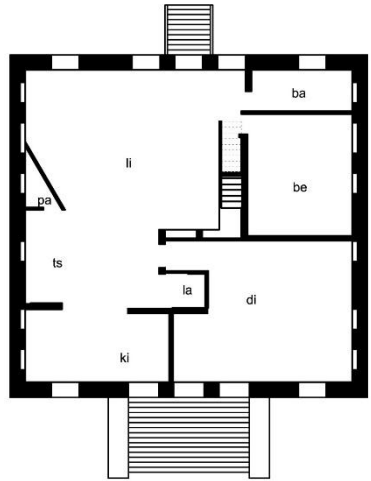
Ain this experiment a series of hybrid designs was mixed with real house examples.

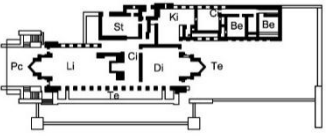
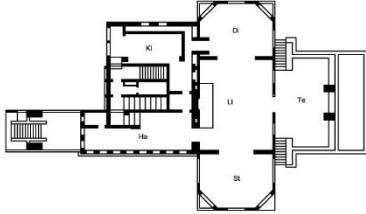
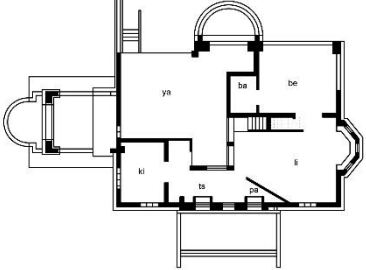
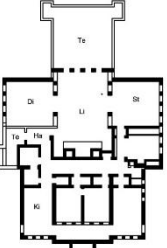
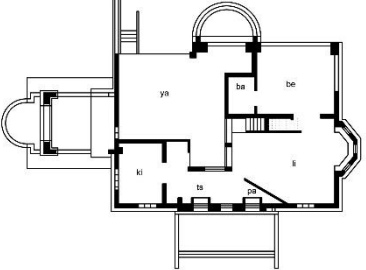
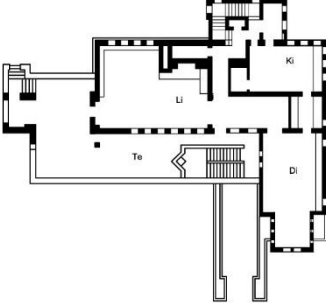
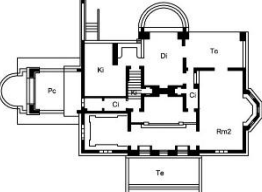
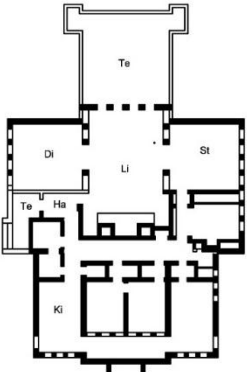
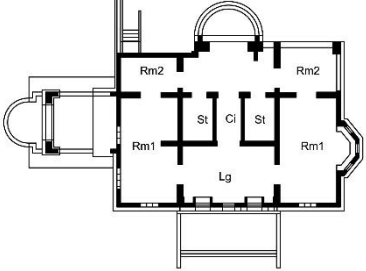
These houses were then shown to a range of expert and non-expert designers to assess how design languages were perceived.

Table 8. 2: Second questionnaire on hybrid families

1.	<p>Observe the house floorplan on the left.</p> <p>Please choose from the examples a) and b) the house that is more similar.</p>	
		
	a. 81%	b. 19%
2.	<p>Observe the house floorplan on the left.</p> <p>Please choose from the examples a) and b) the house that is more similar.</p>	
		
	a. 80%	b. 20%

3.	<p>Observe the house floorplan on the left.</p> <p>Please choose from the examples a) and b) the house that is more similar.</p>	
		
	a. 60%	b. 40%
4.	<p>Observe the house floorplan on the left.</p> <p>Please choose from the examples a) and b) the house that is more similar.</p>	
		
	a. 60%	b. 40%

5.	<p>Observe the house floorplan on the left.</p> <p>Please choose from the examples a) and b) the house that is more similar.</p>	
		
	a. 11%	b. 89%
6.	<p>Observe the house floorplan on the left.</p> <p>Please choose from the examples a) and b) the house that is more similar.</p>	
		
	a. 97%	b. 3%

7.	<p>Observe the house floorplan on the top.</p> <p>Please choose from the a) and b) examples the house which shares larger similarities to the house above</p>	
		
	a. 60%	b. 40%
8.	<p>Observe the house floorplan on the left.</p> <p>Please choose from the a) and b) examples the house which shares larger similarities to the house above</p>	
		
	a. 56%	b. 44%
9.	<p>Observe the house floorplan on the left.</p> <p>Please choose from the a) and b) examples the house which shares larger similarities to the house above</p>	
		
	a. 13%	b. 87%

8.5 Hybrid questionnaires and user perception of design

In the first questionnaire the users were asked two types of questions, first to identify the option with greatest aesthetic value and second to determine amongst a set of three houses which one was the most likely to be a real design amongst hybrids.

Surprisingly a great percentage of users seem to relate to hybrid examples rather than the existing houses. As an example is the hybrid Malagueira/Palladian house shown in example 3) and the Prairie/Malagueira hybrid shown in example 1) both scoring above 50%.

The most known floorplan is without doubt the Palladian villas and these have scored always positively even when compared with other hybrid examples as shown in examples 5-7. However, when asked which is the villa most likely to be an existing house the real villa Emo was not the selected in the set shared with hybrids and scoring only 31%.

The Prairie houses also showed surprising results. Famously selected as some of the most influential houses and floorplans of the twentieth century, most questionnaire results found a greater aesthetic value in the hybrid examples rather than the existing ones as seen in questions 9-11. However, when asked to identify the real solution the right floorplan of the Winslow house was clearly selected with a 56% score.

This questionnaire was taken by 103 users both experts in the architectural field and laymen.

The second questionnaire (shown in Table 8.3) targeted the user perception of design languages families.

The questionnaire was prepared by mixing a set of existing houses designed by the original architects and hybrid houses generated by the grammar. The user was asked to select from the examples the house that shared more similarities with the given example.

The first example was an existing Malagueira house juxtaposed with 1) an hybrid Malagueira/Prairie and 2) an existing Malagueira house. 82% of the responses found the hybrid more similar to the existing house than the real house exposed. When compared hybrid response a) which scored 81% versus the real Malagueira house response b) seems to be an outlier to the hypothesis formulated in this research. When compared to the existing house the fact is that the hybrid shares more architectural similarities. The position of the yard, the overall internal distribution and even relationship with the boundary. As we will observe in the next section, response a) occupies a location closer in the parametric space from the existing design than response b) (consult Fig. 9.14). This explains why it showcases more similarities than the other response: Our intuition plays an important role in the perception of the parametric space of a shape grammar and its corpora, sometimes even more relevant than formal boundaries of the shape grammar.

Surprisingly the next example 2) an existing Malagueira house is then compared with another a) Malagueira existing house and a b) Malagueira/Palladio hybrid. The rigidity of the hybrid design

did not mislead the users which voted in unanimously (80%) in the existing example a). This is corroborated further on the study of the parametric space by (Fig. 9.14).

The examples with Palladian villas are resounding as most users can clearly identify the clear orthogonality and symmetry of each house. The unanimous majority was able to identify the Palladian style without difficulties.

However, the Prairie houses were an easy prey as most users got distracted by similarities amongst the hybrids presented and all without exception picked the hybrids as the most likely family related options in detriment to the existing Prairie houses.

A couple of conclusions can be extracted from the telling results of these two questionnaires:

1. The generic grammar allows for hybrid solutions that fall within the set of criteria of the original designs as shown in questionnaire 1
2. Hybrids share clear language commonalities that can be intuitively perceived as shown in the questionnaire 2

In fact, the first questionnaire tested the perception of observers with regards to the consistency of the design of both hybrids and existing solutions. Aesthetics had an important role in this analysis however feasibility and credibility of design had a greater weight.

The second questionnaire focused mainly on the perception of similarities of designs within a design language. Clearly observers identified similarities rather than feasibility. In a universe of participants both experts in the subjects and laymen, they were consistently asked to select houses that were similar. The results show that the observers chose the similar examples despite some being hybrid solutions and others real existing solutions, and chose it in great majority. This shows that even hybrid solutions carry genetic data that informs the graphic representation of each house. This was intuitively perceived as the examples shows.

The generic grammar proved in the last chapter to be efficient generating: 1) the existing corpus of solutions of a given language 2) a new set of solution that fit within the criteria 3) new solutions from different languages. In this chapter, the idea of hybrids was explored, hybrids being solutions that share genetic material with two or more parent languages. Hybrids were generated through the generic grammar whilst using set of parameters of two specific languages: Palladian/Malagueira, Palladian/Prairie, Prairie/ Palladian, Prairie/ Malagueira, Malagueira/Prairie and Malagueira/Palladian. Most of the hybrid results are coherent and operative as houses floorplans. They have a logical spatial sequence and a harmonic architectural spatial layout. Stylistically they sit either more towards one or the other language. This is probably the reason why the questionnaire users often picked it despite the real examples, because they proposed a certain graphical familiarity and most often than not people choose designs they are familiar with. The hybrids proved to be operative design solutions and to be perceived stylistically consistent.

The parameters they share with the parent languages sufficed to bridge the differences. This seems to prove the hypothesis two which claimed that 'Design solutions from the same design family share graphic commonalities that can be perceived by non-expert users.'

9.	Validation	341
9.1	Case study	
	5 Palladian villas, 5 Prairie and 5 Malagueira houses	343
9.2	Methodology used	345
9.3	Comparison and analysis	356
9.4	Results	359

9. Validation

The previous chapter used qualitative methods to assess the generic grammar formulation. To do so a set of questions were placed in a questionnaire form and expert and non-expert observers assessed results of corpus of the generic grammar, particularly hybrid examples.

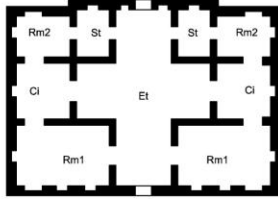
This present chapter aims to complement the previous section by assessing the grammar but this time using quantitative methods. Without relying on third parties or observers the proposal aims to assess and validate the grammar by using measurable methods to test the success of the corpus of solutions generated by the generic grammar. These solutions comprehend existing examples recreated by the grammar and hybrid examples that borrow parameterizations from 2 parent languages.

More so this chapter proposes to test the basic core of the generic grammar formulation by testing the basic set of the core of the shape rules. The rules linked with the first 4 stages of design which coincidentally are the stages responsible for the primary definition of design. The test of the primary shape rules will assess the parameters used by the grammar in each stage to generate 15 of the houses shown in the study case. For each rule the parameters are shown, compared and plotted in a chart to be easily demonstrated.

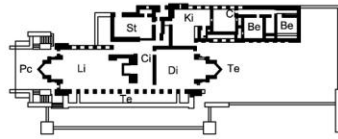
The results of each analysis are then compiled using statistical methodologies to compare multiple components and then performed a component reduction to help illustrate this in Cartesian graph using simple bi-dimensionality.

This quantitative method will assist in the analysis of the proposed parametric space allowed for each design language.

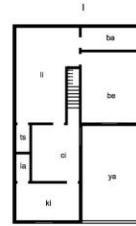
Malcontenta



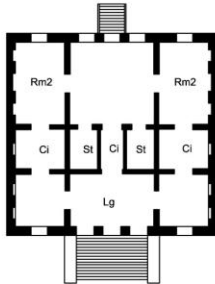
Robie



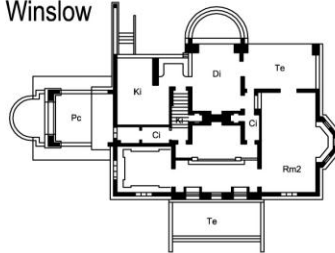
Aa



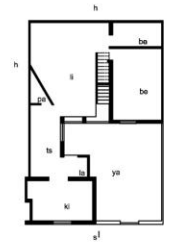
Emo



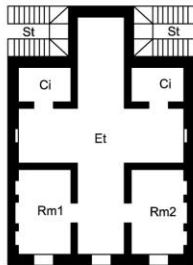
Winslow



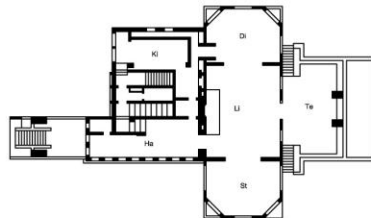
Ab



Barbaro



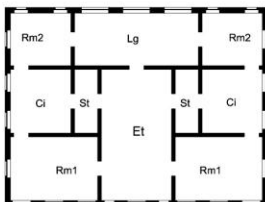
Henderson



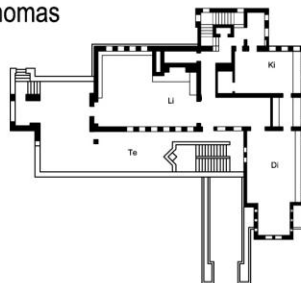
Ac



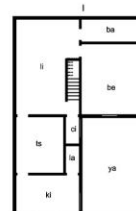
Zeno



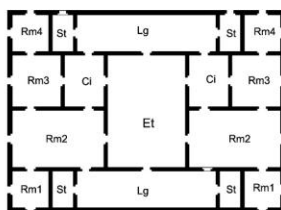
Thomas



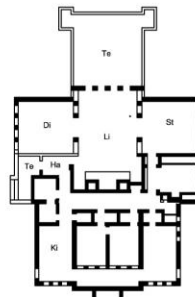
Ad



Valmarana



Cheney



Bb



Figure 9. 1: Case study set of 5 Palladian villas, 5 prairie houses and 5 Malagueira
From top to bottom left to right we can find villas Malcontenta (1550), Emo (1558) , Barbaro (1550), Zeno, Valmarana (1563), Prairie houses Robie (1910), Winslow (1894), Henderson (1901), Thomas (1901) and Cheney (1903) and Malagueira house types Aa, Ab, Ac, Ad and Bb following Duarte's classification system (Duarte 2001)

9.1 Case study 5 Palladian villas, 5 Prairie and 5 Malagueira houses

This thesis focused on shape grammars as formulations consisting of transformation rules that describe design. Previous studies focused on recreating the style of family-related solutions and these grammars constituted part of this case study (Stiny & Mitchell 1978; Koning & Eizenberg 1981; Duarte 2001).

Figure 9.1 illustrates the case study and the 15 houses used for the purposes of validation of the generic grammar presented. Five representative houses of each design language were selected. The houses selected reflect some of the examples mentioned in other sections of this study like the Palladian villas, houses originally created by each designer, but also studied and recreated by other shape grammarians. The corpus of houses shown above shows an array of different archetypal houses for each language. It was an important requisite the inclusion of representative houses from each style. The Palladian villas illustrated with 3x3, 3x4, 3x5 and 4x5 cells, covered most of the existing types. The prairie houses selected show the typical 'butterfly' cross-shaped pattern evident in Robie, Henderson and Thomas house and additional compact examples that diverge slightly from the typical norm such as Winslow and Cheney house. Within the Malagueira examples are illustrated both houses type A and B which showcase a different array of courtyard locations.

Diversity of the case study was the main criteria in the selection of these examples.

Within the extensive corpus of existing solutions were selected the following houses of each style:

1. Andrea Palladio - Palladian villas:
 - a. villa Malcontenta (1550)
 - b. villa Emo (1558)
 - c. villa Barbaro (1550)
 - d. villa Zeno (15??)
 - e. villa Valmarana (1563)
2. Frank Lloyd Wright - Prairie houses:
 - a. Prairie houses Robie (1910)
 - b. Winslow (1894)
 - c. Henderson (1901)
 - d. Thomas (1901)
 - e. Cheney (1903)
3. Alvaro Siza Vieira – Malagueira houses (built between 1977 to 1999):
 - a. house type Aa
 - b. house type Ab
 - c. house type Ac
 - d. house type Ad
 - e. house type Bb

Usually style and language is determined by empirical observation and analysis of the key elements of a building. This is a rather inflexible and subjective method. The intention is to use

the representation of these houses to derive a more accurate methodology. In addition, the shape rule system used for the generic grammar was also put to the test.

Other methods in the past, were tested to derive representations from original design examples and illustrate it in a more systematic manner. Style was proposed to be represented by feature space archetypes because “unlike a fixed symbolic representation, both the measurements of features that define style and the selection of those features themselves can be performed by machine (...) automatically from a set of examples”. (Hanna 2007)

9.2 Methodology used

Previous studies on representing feature space archetypes were based on the mapping of design solutions in a high dimensional space. To do so, a spatial reduction is performed and bi-dimensional chart visualized. Drawings of each design solution are tested, particularly the plan, measured in a lower dimensional space and using features that particular to the style. The difficulty is the representation of these multiple features through a meaningful vehicle.

This is achieved through dimensionality reduction commonly used in statistics, linear algebra, face recognition, and other fields. The process infers distinguishing features from a multi-variable set of large data. Statistical methods are then used to treat and organize the large data. Principle component analysis (PCA) is a statistic procedure that uses orthogonal transformation to convert unrelated variables into a set of linearly orientated elements, hence its denomination. An eigenvalue data matrix can be centralized or normalized from those values.

This set of variables is organized through a transformation. The first principal component has the largest variance followed successfully by the others with an orthogonal constraint. This process is denominated eigenvalue decomposition.

PCA provides a useful tool for large data visualization supplying one axis per variable in the high-dimensional space and 'projecting' it to a lower dimensional space by using the first few principal components.

Recently the space syntax group used PCA allied to their research on spatial fruition and people's movement through space. Their original representation and spatial analysis identifies spaces in plan and creates a network of vectors that show visibility amongst different spaces. This analysis is represented through a visibility graph to determine the flow through space by determining visually unobstructed routes. This has implications on how space is connected and how well integrated each space is from another (Hillier 1989). Other type of analysis is the axial graph which links the extreme corners of an internal space and reads the numerical value of each room diagonal. These elements, connectivity and integration are numerically measured and compared. Recent studies from space syntax used then PCA to compare the data originated from these different analyses.

This method was used to explore how to represent feature space archetypes. To achieve it 24 plans were selected from a list of iconic buildings of the twentieth century. For each floorplan axial graphs were constructed and the numerical values recorded. Adjacency graphs were also executed. These measured the adjacencies amongst adjacent spaces and recorded these numeric values. Both axial and adjacencies graphs were built using an open source software developed by space syntax – Depthmap. For each house a matrix of adjacency data was created and the eigenvalues of these ordered in decreasing degree. Each house was then represented by a 'feature vector' and each vector plotted into a high dimensional feature space. The dimensions of the feature space vector have no meaning outside statistics and its value only determined by PCA (Hanna 2007). These values are represented in a chart by two orthogonal axes one for component 1 and one for 2.

Figure 9.2 illustrates 24 buildings from the twentieth century represented by a feature space. This observed that “an approximate chronological order from left to right, seen clearly in the buildings by Wright, the Villa Savoye and contemporary Maison de Verre are next to one another, and van der Rohe’s two works are virtually overlapping.” (...) “Proximity in this feature space is meant to suggest those buildings that are more similar to one another by degrees.” (...) “The fact that buildings of similar styles do fall near to one another in the reduced feature space confirms that the features indicated by PCA are at least statistically related to style. Archetypes based on such space may be used as style descriptors”. (Hanna 2007).

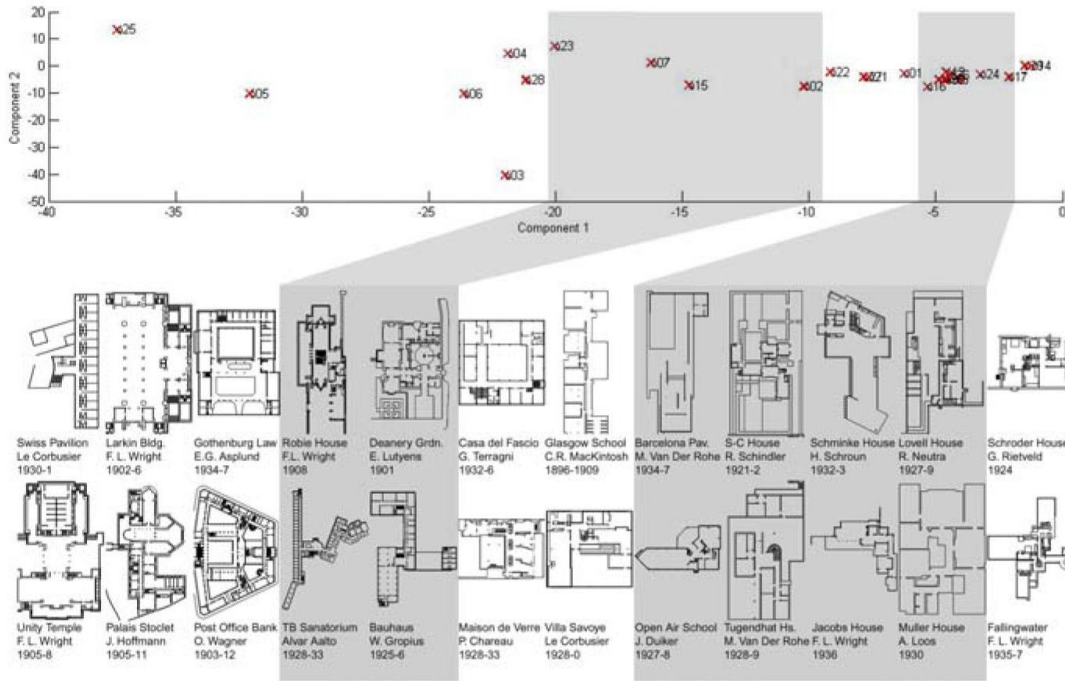


Figure 9. 2: 24 twentieth century buildings plotted into feature space (Hanna 2007)

This work allows us to observe the following five points:

1. unrelated building designs can be compared in quantative manner
2. style can be represented by quantative methods
3. there is a correlation between style and the graphical representation of a floorplan an this can be compared
4. different buildings that share chronological and style commonalities can be objectively analysed, compared and arranged by degrees of similarities
5. PCA and bi-dimensional reduction means of analysis are an effective statistical procedure to evaluate graphic representation of floorplans

This prompted the idea of using this methodology to test the designs generated by the generic grammar and processes used. A set of values was extracted from each house throughout the generation process.

A three-step methodology was put in practice:

1. the analysis of the main house floorplan features and its comparison on a spreadsheet
2. the illustration of these isolated features within chart to allow proper comparison
3. the use of Principle Components Analysis to compute with several variables and consequent illustration of each villa as a data point in a chart

The first set of analyses used a straight forward system. For each house shape rules 1, 2 and 3 were tested. These three rules are responsible for the overall design of the house. Considering that the generic grammar uses a top-down approach is at early stages of design that the outline of each house is set.

Therefore, the ultimate validation of the formulation proposed is to test it against a selected group of predesigned houses of each style.

The first shape rule designs the bounding shape. For all languages explored these bounding shape is a rectangle with a certain width and height.

Table 9.1 shows the values measured for each house (x-width, y-height). From the observation of the table alone one can clearly identify some singularities. The five bottom examples of Malagueira houses all showcase the same x and y values. This does not come as a surprise since the house usually occupies most of the plot and this plot uses a fixed measurement of 12x8m at a typical 3:2 ratio.

This is later clearly plotted into a chart 9.2 and 9.3 where these values are arranged. For clarity Palladian villas are coloured in red, Prairie houses are represented by the green and Malagueira in blue.

From these charts, some patterns arise:

- Palladian villas are clustered in the centre of the chart, occupying a preferential zone
- Prairie houses are illustrated at a clear area within the chart but from all the examples illustrated they are more scattered and occupy the largest chart area
- Malagueira houses are the example of clustering, all the villas are positioned coincidentally on the same point
- Malagueira houses sizes and proportions share more commonalities with Palladian houses
- Prairie houses sizes and proportions share some commonalities with Palladian villas particularly with villa Valmarana which is quite atypical in size and number of rooms (almost resembling a small palazzo rather than a villa)
- Some prairie houses are also spread away from the expected cluster, that is the example of Cheney house, once more an atypical house from the style, showcasing a floorplan regularity and even some instances of symmetry

Table 9. 1: House boundary dimension following the application of rule 1

RULE 1 - BOUNDARY	X	Y
Palladio - Malcontenta	19.6	13.5
Palladio - Emo	22.2	21.2
Palladio - Barbaro	16.71	23.14
Palladio - Zeno	21.4	16.12
Palladio - Valmarana	30.6	22.2
Prairie - Robie	36.5	14
Prairie - Winslow	18.4	13.6
Prairie - Henderson	29.5	23.63
Prairie - Thomas	26.85	27.35
Prairie - Cheney	27.82	41.05
Malagueira - Aa	8	12
Malagueira - Ab	8	12
Malagueira - Ac	8	12
Malagueira - Ad	8	12
Malagueira - Bb	8	12

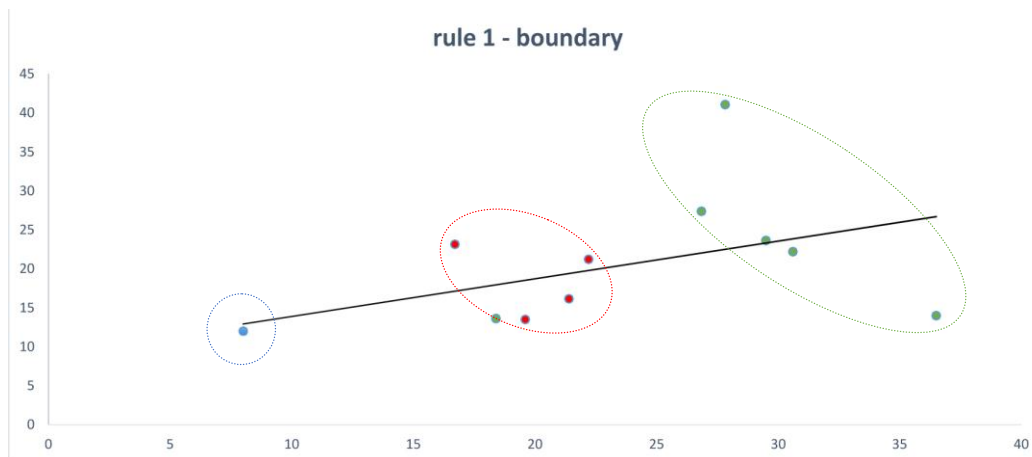
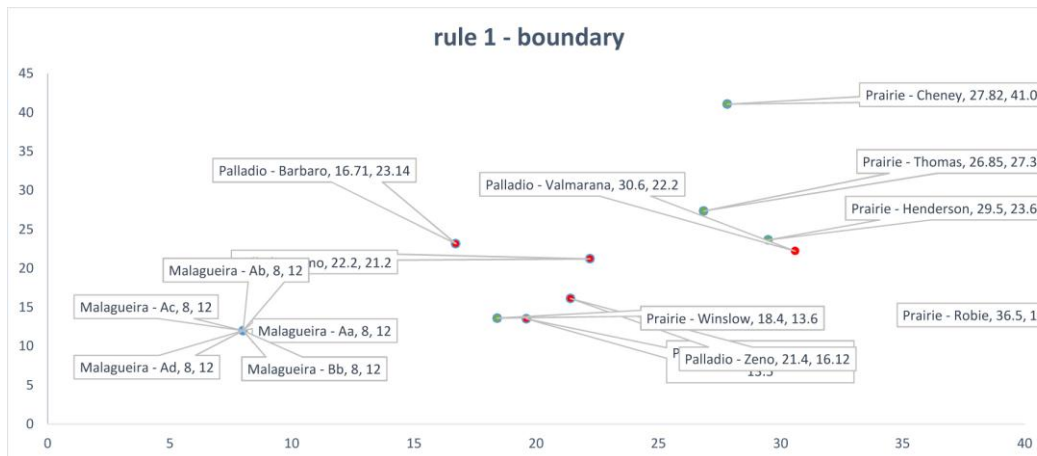


Figure 9. 3: Chart with the application of rule 1 with tags
 Figure 9. 4: Chart with the application of rule 1 without tags

The following section illustrates the same methodology being put into practice for rules 2 and 3. These rules cover the first and subsequent divisions to create the internal spatial layout. Shape rule 2 is responsible for the horizontal division whilst rule 3 for the vertical division.

For each house selected the overall dimensions and the horizontal or vertical division dimension was recorded respectively in tables 9.2 and 9.3.

When observing a complete floorplan, such as the ones presented in Figure 9.1, it might not be clear which division is being considered. The house is decomposed into the main elements or geometries. The main horizontal/vertical division regarded as the main division that would take place if this house would be generated by the generic grammar. Usually this would be a maximal linear division. For example, in a Malagueira house type the main horizontal division would be the first division taking place in the derivation process. This is also usually the maximal line. In the case of the Malagueira housetypes the first maximal line is the courtyard/interior division line.

As before, the horizontal and vertical divisions results were recorded into tables 9.2 and 9.3 and its results later illustrated in the charts 9.4-5 and 9.6-7.

From these charts, some observations can be drawn as stated below:

- Malagueira houses are consistently clustered reflecting the values reported in each chart and reflecting a clear style tendency while following the generic grammar, occupying a defined parametric space with consistent distances amongst design solutions
- Palladian villas occupy a central space within the chart and show a tendency for clustering even though if more subtle than the previous rule 1 example
- Prairie houses are the more asymmetric example studied with the largest spread
- Prairie houses examples occupy a top diagonal of the chart. This occurs since the relation at stake is the overall length versus division length for chart 9.4-5 and overall width versus division length. Usually these houses exhibit a large number of divisions and a generous overall size. This is reflected in the position occupied within the chart, while the central space is occupied by Palladian houses and the Malagueira the space next to the origin. Malagueira are the group of houses with smaller overall dimensions and a relatively small ratio of divisions (as shown in table 9.2-3)
- Some singular examples seem to occupy a mid-space between styles, such as Winslow house that consistently in charts 11-2 to 11-7 is showcased closer to the Palladian examples than its own style. This could be explained by the compactness of the floorplan, the regularity of the design and even some instances of symmetry. This is in fact an atypical floorplan that does not showcase the usual cross shaped pattern and asymmetry
- Likewise, Cheney prairie house stands out by its singularity. Also compact and showing a degree of symmetry it also refuses the cross shaped pattern associated with prairie

houses. Instead its floorplan is regular and rectangular orientated towards north/south. This will have an impact on the position occupied within the chart 9.4-9.5

- The Palladian villa Valmarana also is shown set away from its expected cluster zone. This large proportioned house is the largest villa within the case study. It incorporates the largest number of rooms and divisions. This will have an impact on the ratio length/division length, approximating it to a prairie house example

Table 9. 2: House value of horizontal division following the application of rule 2

RULE 2 - HORIZONTAL DIVISION	Y1	Y
Palladio - Malcontenta	4.9	13.5
Palladio - Emo	11.87	21.2
Palladio - Barbaro	8.35	23.14
Palladio - Zeno	10.95	16.12
Palladio - Valmarana	17.35	22.2
Prairie - Robie	9.57	14
Prairie - Winslow	9.56	13.6
Prairie - Henderson	5	23.63
Prairie - Thomas	10.78	27.35
Prairie - Cheney	11.75	41.05
Malagueira - Aa	6	12
Malagueira - Ab	6	12
Malagueira - Ac	6	12
Malagueira - Ad	6	12
Malagueira - Bb	6	12

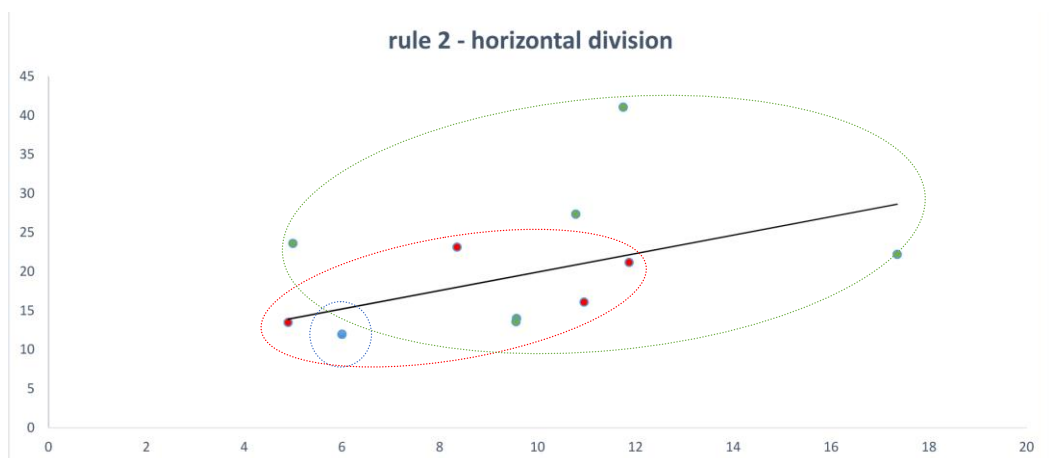
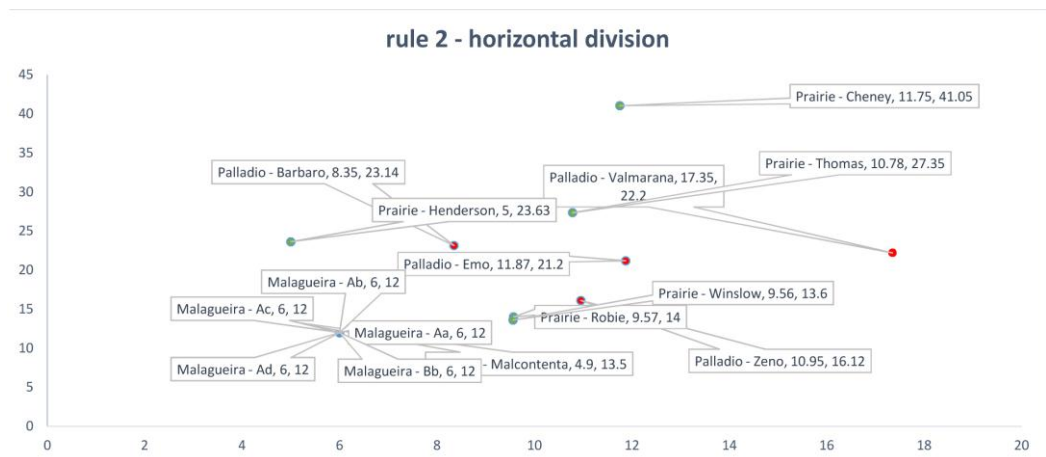


Figure 9. 5: Chart with the application of rule with tags

Figure 9. 6: Chart with the application of rule without tags

Table 9. 3: House value of horizontal division with application of rule 3

RULE 3 - VERTICAL DIVISION	X	X1
Palladio - Malcontenta	19.6	4.45
Palladio - Emo	22.2	6
Palladio - Barbaro	16.71	5.45
Palladio - Zeno	21.4	16.12
Palladio - Valmarana	30.6	10.81
Prairie - Robie	36.5	15.4
Prairie - Winslow	18.4	6.4
Prairie - Henderson	29.5	10.25
Prairie - Thomas	26.85	16
Prairie - Cheney	27.82	6.6
Malagueira - Aa	8	4
Malagueira - Ab	8	4
Malagueira - Ba	8	4
Malagueira - Bb	8	3
Malagueira - Aa	8	4

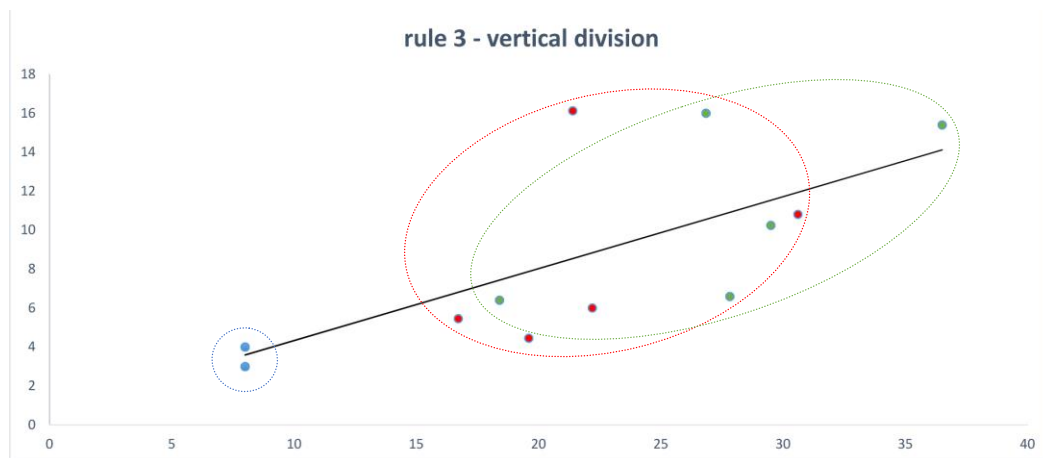
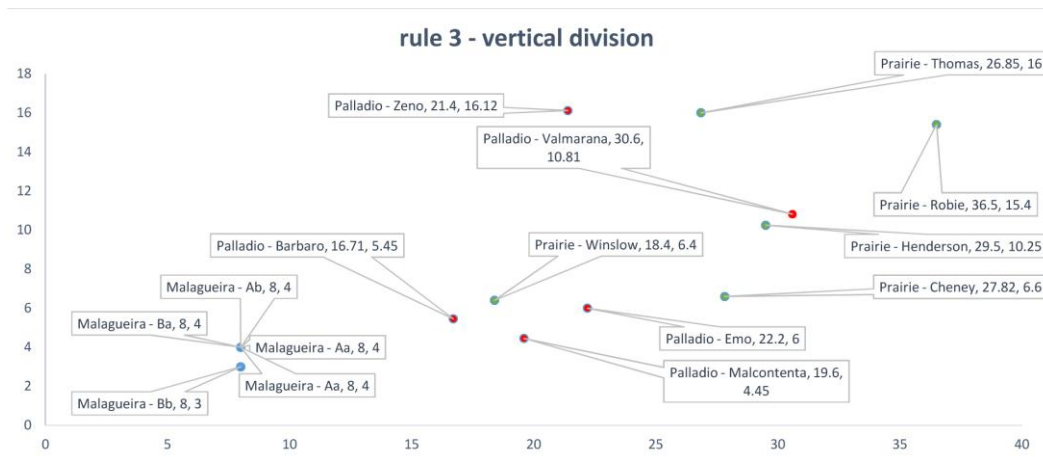


Figure 9. 7: Chart with the application of rule 3 with tags
 Figure 9. 8: Chart with the application of rule 3 without tags

The last chart presented in 9.9-10 does not reflect any of the shape rules within the generic grammar but a rule restriction on the number of recursions that are allowed for divisions.

The number of divisions occurred to generate each house reflects:

1. The complexity of the floorplan
2. The efficiency of the use of the subdivision process within a housetype
3. A specific pattern for each housetype or language

Whilst in the Palladian villas language the number of division is directly proportional to the house size and number of 'cells' - the greater the house larger the number of divisions. In houses such as Malagueira the number of divisions varies regardless of the house size since this is fixed. The variation is limited and the number of divisions is consistently limited from 6 to 10.

Prairie houses showcase large division numbers. This reflects the inherent complexity created by using the subdivision process to generate a floorplan. The number of divisions to design a floorplan is large in comparison. While the Palladian villas showcase a range between 7 to 16 divisions, prairie houses range from 12 to 21 iterations.

When these numbers are plotted into a chart illustrated in 9.9-10 the results show some emerging patterns:

- Malagueira houses consistently cluster in the bottom/origin of the chart showing a clear tendency. This is linked to the consistent number of divisions and a limited variation of the number of horizontal divisions within the corpus of solutions derived as delineated by the blue ellipsoidal line
- Palladian villas also show a clustering pattern occupying the immediate space above the Malagueira zone. These villas show a relation between size and number of divisions. The number of divisions varies with the size of the house. The larger the villa enclosure, the larger number of cells/rooms hence a larger number of divisions. This variation is consistent and varies proportionally. This cluster is highlighted in red
- Prairie houses exhibit a clustering pattern on the top right zone of the chart. This does not come as a surprise since most of these large proportioned houses require a large amount of division iterations to be generated. This is circled in green.
- Prairie houses also are the less atypical group represented within the chart. Each house is spread through a large region without showing a typical cluster such as the Palladian villas or the Malagueira houses, but a wider spread

As atypical examples are the two houses that stood out on previous examples:

- Villa Valmarana is shown close to the Palladian cluster (if not part), but is stands out in the periphery. Once more this can be attributed to its size, large number of rooms and consequent number of divisions, is atypical within the Palladian house types.

- Cheney house occupies a single location on the chart on top of the expected prairie houses cluster. Again, the atypical nature of this house, size and number of existing spaces drives this result. The use of symmetry is a singularity in the Prairie language.
- Thomas house in this analysis is located amongst two other languages. Its position in the chart is sandwiched by villa Valmarana on top and the Malagueira Aa house. This does not mean that Thomas house is apart from its cluster, but at a peripheral location mediating the borders with the other languages. This is an example of a house that its size and number of divisions could be found amongst the other case studies. Thomas is also the prairie house with the least iterations of divisions

Table 9. 4: House number of divisions versus horizontal divisions

HORIZ VS NUMBER DIVISIONS	HORIZ	DIV
Palladio - Malcontenta	2	8
Palladio - Emo	2	10
Palladio - Barbaro	3	7
Palladio - Zeno	2	7
Palladio - Valmarana	4	16
Prairie - Robie	6	18
Prairie - Winslow	8	12
Prairie - Henderson	7	12
Prairie - Thomas	4	13
Prairie - Cheney	14	21
Malagueira - Aa	3	6
Malagueira - Ab	4	7
Malagueira - Ba	3	6
Malagueira - Bb	3	5
Malagueira - Aa	4	10

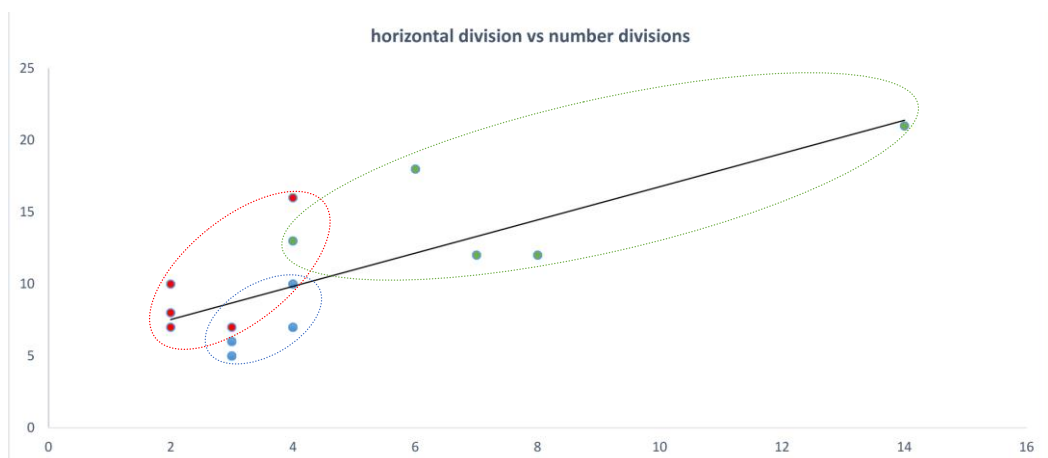
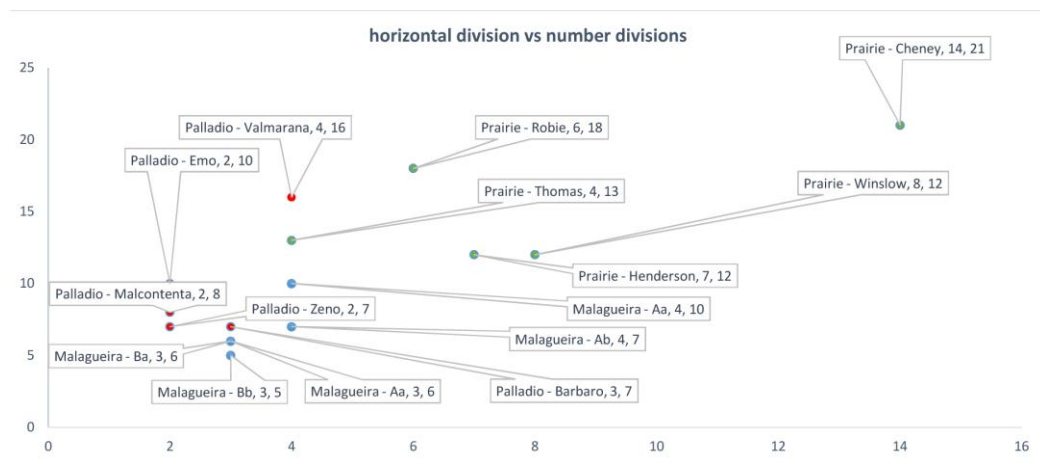


Figure 9. 9: Chart with the application of rule 1 with tags

Figure 9. 10: Chart with the application of rule 1

9.3 Comparison and analysis

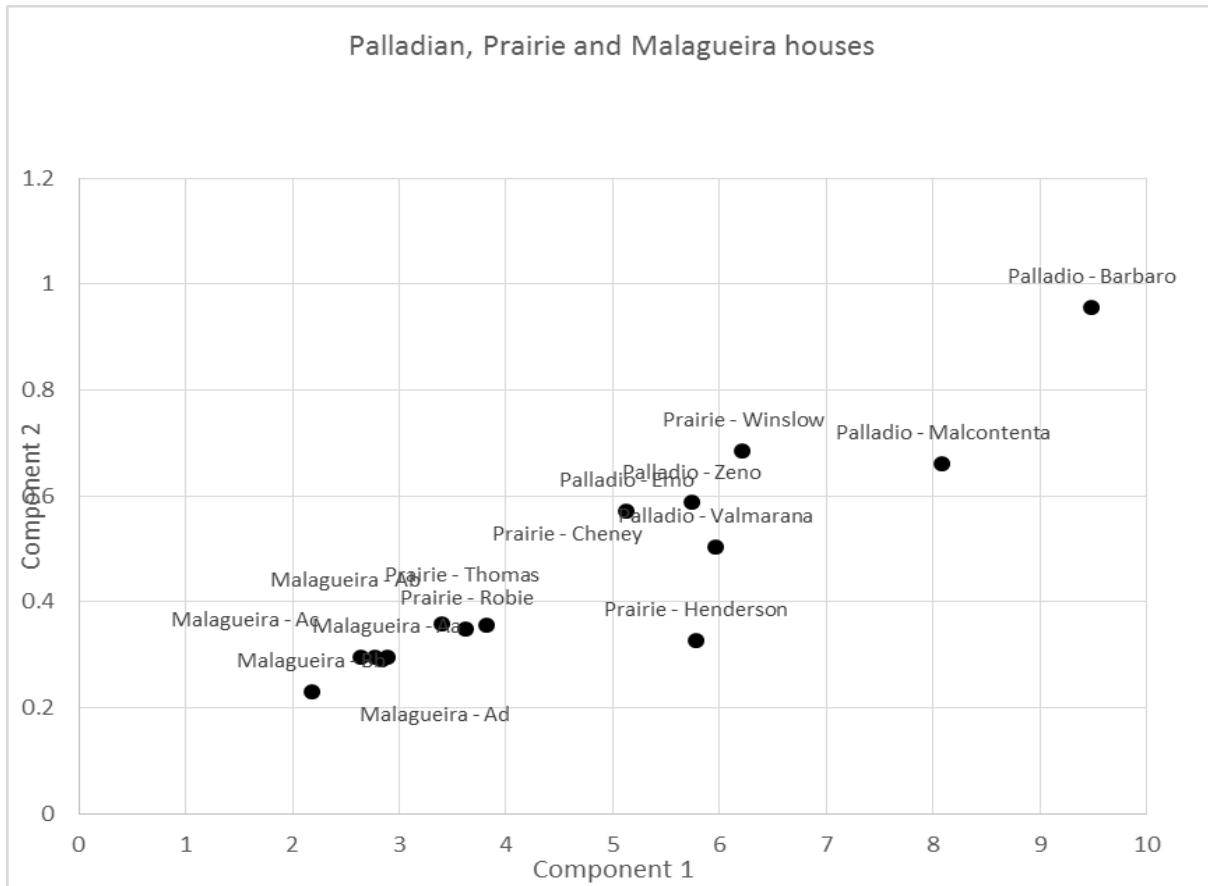


Figure 9. 11: Result comparison in Cartesian chart of the villas studied

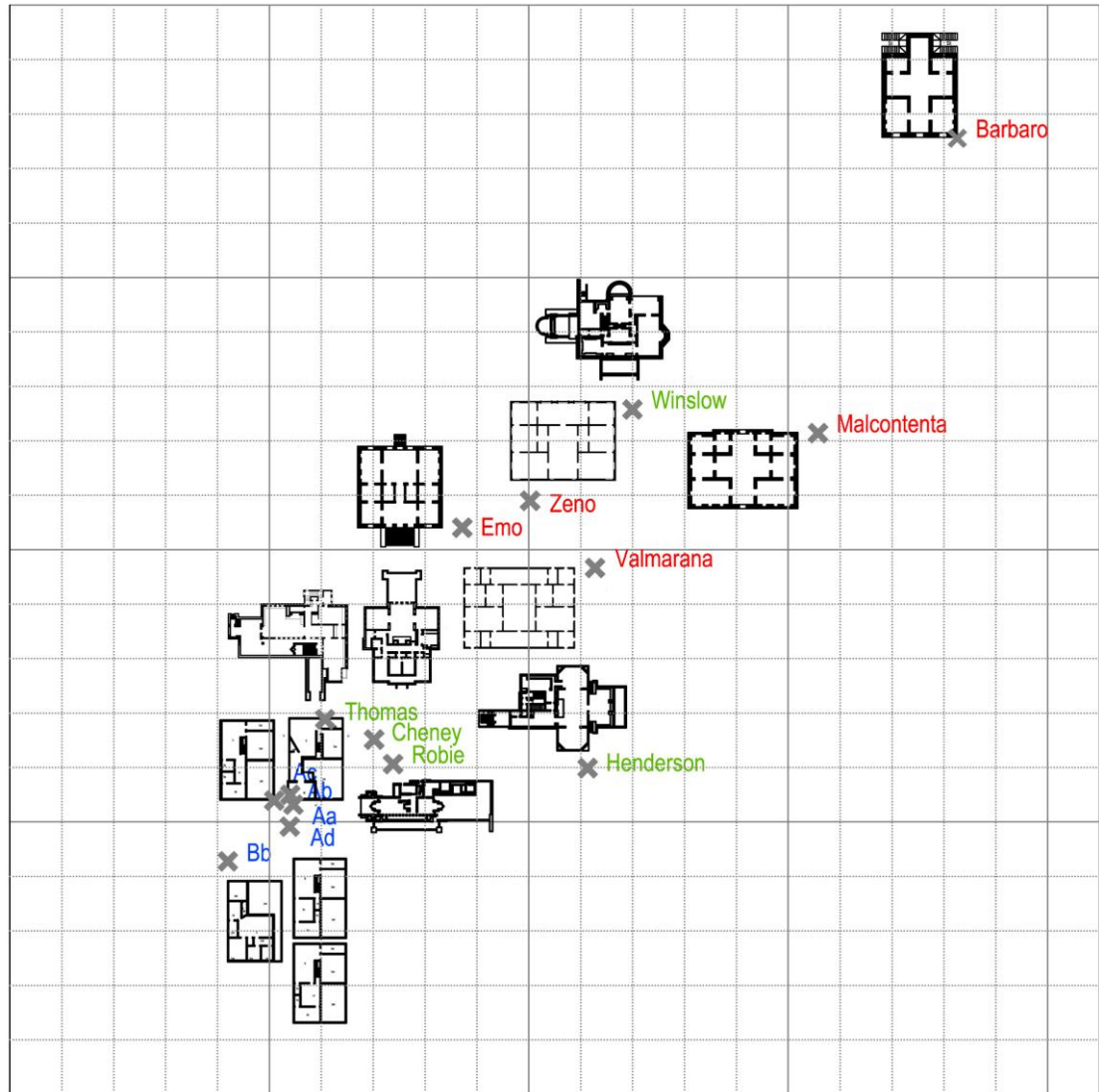


Figure 9. 12: Graphical representation of the villas studied plotted into a chart

Figures 9.11 to 9.12 illustrate the 15 houses used for the case study compared using quantitative methods. The variables analysed included:

- the outline proportional ratio
- the maximal horizontal division length
- the maximal vertical division width
- the number of divisions
- the number of rooms existing in each floorplan

These values were tested using Principal component analysis which compiled the variables reducing them from a high dimension to a Cartesian representation as shown in 9.11.

This chart illustrates each house through a point with two components. To better graphically illustrate these findings, chart 9.12 was produced. To each point the respective floorplan was associated and the proportion of each axis altered to allow for better clarity.

Overall certain conclusions can be observed for each type of housing language:

- a well-defined cluster is perceived for Malagueira houses occupying the bottom left area of the chart
- a cluster can be identified containing the prairie houses but one house stands out of the group and positions itself next to the Palladian examples; this cluster occupies the centre of the chart
- Prairie houses occupy the top right area of the chart, produce a larger spread in comparison to the other genres and also produces an example that stands out from the cluster

Two particular examples are identified away from its cluster these examples refer to:

- Villa Barbaro: it is in fact paradigmatic, is an isolated example of a 3x4 grid distribution and in terms of floorplan, size, orientation and number of rooms also positions itself quite far from the norm
- Winslow house: this prairie house was identified previously in other analysis as a paradigmatic example is compact, self-contained, nearly symmetrical and in many ways resembles a Palladian villa floorplan. For those reasons, Winslow is positioned closer to the Palladian examples rather than the prairie houses cluster

These findings allow us to conclude:

- It is possible to determine style, or design language, using quantitative methods
- Houses that share commonalities can be compared and contrasted using quantitative methods
- Bi-dimensional reduction is an effective process to compare different variables on a large data set
- Houses within a design language tend to gather in clusters that represent well defined boundaries
- These clusters occupy specific 'spaces' within the chart representation area

The inclusion of the generic grammar rules parameters as variables for this analysis had the following syllogism had to be proved:

1. If the shape rule variables were used for the Principal component analysis then the results would also test the validity of the generic grammar itself
2. The definition of design language and family of designs is demonstrated by a defined cluster of houses that integrate the family of designs
3. If the houses tested proved to gather in clusters the grammar is validated

The three clusters observed for Palladian villas, Prairie houses and Malagueira constitutes the validation of the generic grammar.

9.4 Results

The chapter discussed illustrated the results and of the generic grammar testing using quantitative methods. This allowed for testing the research question regarding 'how well does the parametric space of a shape grammar coincide with the design space expressed by the different corpora'. We believe that the results shown by the bi-dimensional reduction analysis points towards an understanding that design families share similarities defined by a shape grammar. Each shape grammar corresponds to a parametric space where the design corpora can be located. This is possible due to a restricted set of parameters defined by the parametric grammar rules and can be measured using statistical means.

To the generic grammar formulation, and to its primary shape rules, bi-dimensional reduction analysis was put in practice as a means to test the accuracy and precision of the rule set and consequent corpus.

Firstly the boundary rules were tested. To attain this, 15 existing houses, 5 from each language were assessed. The boundary rule was tested as is formulated in the generic grammar using the parameters required to regenerate the original houses. The results were telling. Not surprisingly the Malagueira houses formed a well-defined cluster (coincided in a point) expressing the uniformity of the plot. Whilst the Palladian villas, despite its variety of shapes and proportions, formed a defined cluster also occupying the centre of the chart and bridging between the other European language, Malagueira and the North-American prairie house style. The prairie houses style created a cluster positioned at the top right side of the chart. This first rule proved to be fundamental expressing the features of style and helping identify language clusters. It proved that despite differences and variety amongst a corpus of solutions, there are common traits that emerge through style. Some of these traits might not be perceived graphically with a naked eye but when analysed within a corpus of identifiable solutions is captured and expressed through a clustering effect as the chart illustrates.

Secondly, the other division rules revealed interesting aspects of style particularities. Rule 2 and rule 3 of the generic grammar showcased the application of the two forms of the division rule. The use of the division rule is crucial for the internal layout definition and for capturing the spatial distribution so particular to each house type. Again, but perhaps less predictable, Malagueira houses showcased a well-defined cluster, which occupies the bottom left of the

graph. All existing Malagueira houses, despite the variety of examples fall within a well-circumscribed cluster where variation is nearly imperceptible. The Palladian houses are shown with a higher degree of variation but still illustrated a clustering effect which occupies the centre of the chart. The Prairie houses proposed a wider spread of houses within the experiment but a clustering is still observed. It is apparent an overlap between Prairie houses and Palladian villas. As described existing examples of Palladian villas and Prairie houses, namely Villa Barbaro and Winslow house seem to fall away from the expected cluster widening the scope or range of the language. Both houses seem to fall between the two styles almost as an outsider. Both are particular amongst their own style as they seem to break some of the main principles of design they represent such as size, proportion and symmetry. That singularity is even more evident when the bi-dimensional reduction analysis graph and the components reduction is performed as shown in figures 9.11 and 9.12.

The bi-dimensional reduction analysis charts showed a defined cluster for Malagueira, where all examples fall within the circumscribed area, a Prairie house cluster defined by Prairie houses Cheney, Robie, Henderson and Thomas occupying a centre space in the chart and another well circumscribed Palladian villas cluster with Malcontenta, Zeno, Emo and Valmarana. Both the Barbaro villa and the Winslow house occupy spaces far from expected and detached from the expected clustering zone. This reflects the singularities of both houses. With Barbaro the size and proportion is atypical for a Palladian villa and with Winslow a boundary inscribed, compact and symmetrical prairie house also is atypical amongst existing examples.

The same applies when we include design solutions such as hybrids as shown in Figures 9.13-14. The new hybrids are allocated within the parametric space not equally spaced between the parent languages but within the proximity to the solutions they graphically share more commonalities. This was also observed and confirmed in the previous chapter where observers had to identify similarities amongst hybrids and existing solutions. Rather than picking examples that consubstantiated style, observers picked examples that shared obvious design similarities. For example the Malagueira hybrid Ab/Cheney house is shown in a more centralized location in the cluster than the existing Bb Malagueira house. This shows that a hybrid might share more design similarities than an original house.

This seems to corroborate the hypothesis that a restricted range of parameters help confining a parametric space of a grammar.

Similarities are showcased in the bi-dimensional reduction analysis charts Figure 9.12-13 by the proximity that each house shows to one another. Not surprisingly houses generated from the same sub-grammar occupy places in the proximity with one another, a phenomenon normally called as clustering.

However what happens when hybrid examples are included within the mix of houses analysed? As discussed in the previous chapter hybrids share commonalities between the parent languages that generated them. So it is expected that these will confirm the clustering effects from their original languages. This was illustrated in figures 9.13 and 9.14.

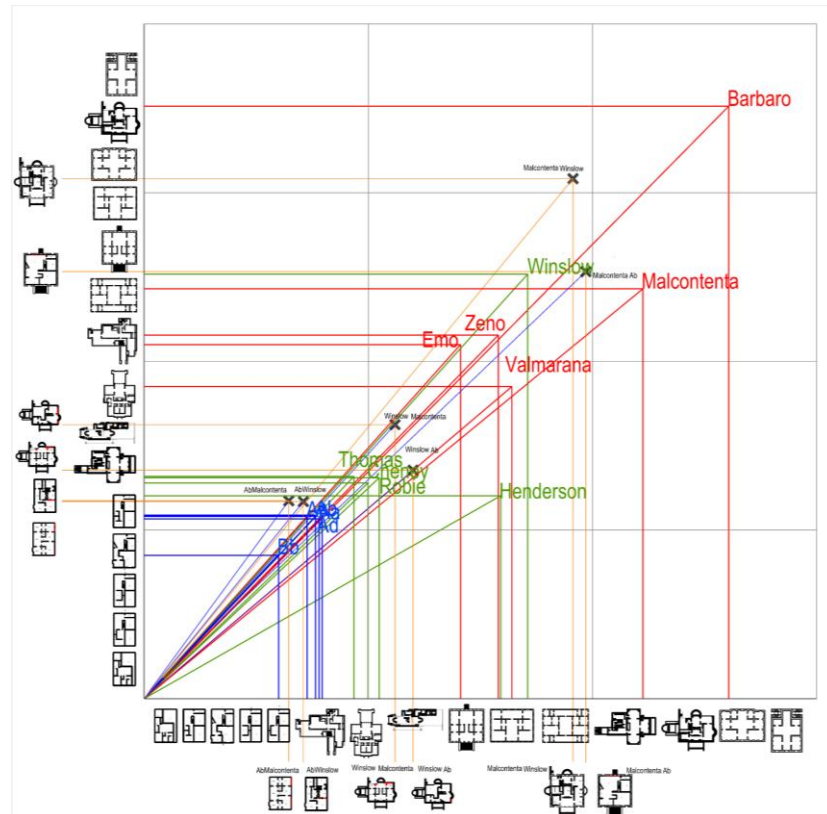


Figure 9.13: B-dimensional reduction analysis chart applied to Malagueira (blue), Prairie houses (green) and Palladian villas (red), the hybrid examples are illustrated in the second row and column

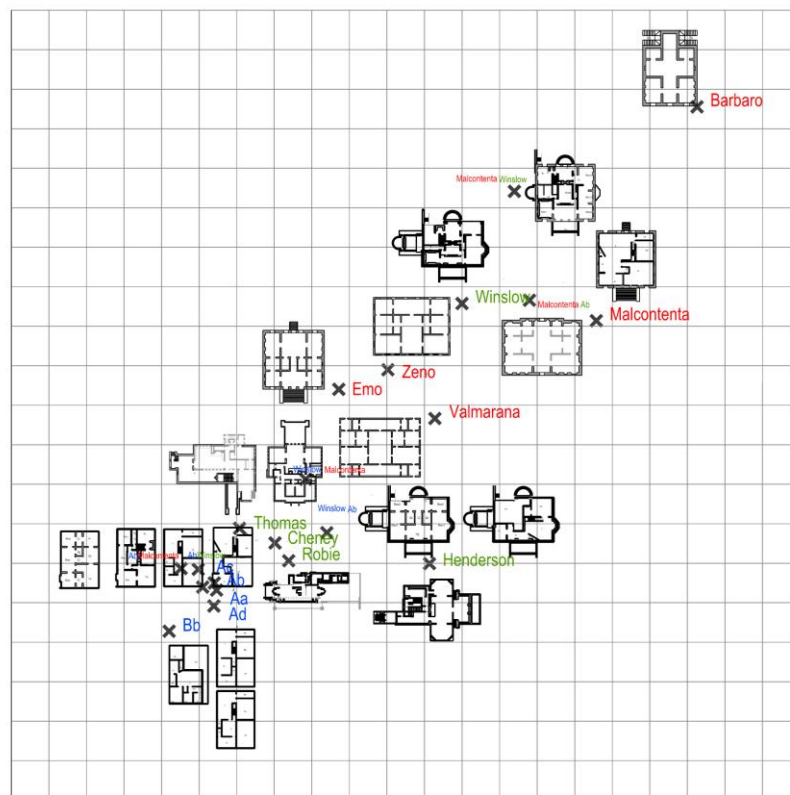


Figure 9.14: Illustrated bi-dimensional reduction analysis chart showing to Malagueira (blue), Prairie houses (green) and Palladian villas (red), and hybrid examples

The bi-dimensional reduction analysis hybrid chart confirmed the suspicions and predictions. The hybrids generated using two parent languages sit between the clusters of languages that originated them. If anything, it confirmed the clustering zone of each language. Hybrid examples such as Ab-Malcontenta and Ab-Winslow are located near the Malagueira cluster. Interesting to compare with results attained in the previous chapter and linked with the perceived style features of these hybrid examples. Ab-Winslow was voted by the majority of observers as the most likely example of a real house when compared with an existing Malagueira house. Its 'veracity' is now confirmed by bi-dimensional reduction analysis placing it in the Malagueira cluster despite the fact that also shares commonalities with the Prairie house cluster. Whilst Ab-Malcontenta was voted by the majority of participants as the house with greatest aesthetic value when confronted by a real Malagueira house (see Table 8.2). Now sits within the Malagueira cluster. The range of parameters that generated these solutions allowed for a parametric space of results where relevant/similar hybrids are also part. This is attested by both quantitative methods and though empirical evaluation using questionnaires.

Participants also voted with absolute majority (81%) electing Ab-Winslow as the house more likely to be related with the Malagueira style. Now its style affinity is also confirmed using quantitative methods and illustrated in the chart (see table 8.3).

Other examples see the Winslow-Malcontenta and Winslow-Ab within the Prairie houses cluster and rightly so. The style similarities are evident as their graphical representation showcases. Winslow Ab was selected as the house with greatest aesthetic value and is also illustrated within the Prairie houses cluster.

The same happens to other hybrids such as Malcontenta-Winslow and Malcontenta-Ab which invariably fall next to the Palladian cluster. As before when questioned about which villas was the most likely to be the existing example, Malcontenta-Winslow was selected amongst real examples of Palladian villas (Table 8.2). The qualitative methods have confirmed the quantitative methods by placing this hybrid next to the Palladian cluster. Nevertheless, due to the unique character of the original Winslow Prairie house design the hybrid also inherited a style particularity and despite being still within the radius of the clustering zone is a satellite example near the limit area.

Figure 9.14 illustrates graphically where each house sits in the chart and their area of proximity and neighbouring houses. Malagueira is shown in blue, Prairie houses in green and Palladian villas in red. This type of representation aids the visual understanding of the relationships between these houses. The hybrids fall harmoniously amongst the examples that created them and neighbour houses share perceived graphical similarities.

This confirms that the qualitative methods used in chapter 8 to assess the perception of design language and the quantitative methods used in this chapter 9 to assess the measurable aspects of the design language and its range of parameters are in line. Both methods confirm that style can be perceived and objectively described. In particular style can be expressed by a formulation, and then parameterized with a range of values. This range of values is particular to

each style, can be formulated and later read and assessed by the occurrence of clustering effects amongst the corpus of solutions.

The clustering illustrated in the chart helps to delineate a space of design language where the corpus of solutions seems to fall creating a defined boundary.

The cluster helps understand the boundaries, proximities and neighbouring relationships amongst design languages.

10. Conclusion		367
10.1	Objectives, research hypothesis and methodology used	367
10.2	Findings and analysis of results	374
10.3	Conclusions and evaluation of results	378
10.4	Final reflections	381

10. Conclusion

The current study focused on shape grammar formulation and corpora.

Two research questions were posed:

Question 1: How well does the parametric space of a shape grammar coincide with the design space expressed by different corpora?

Question 2: How well does the parametric space of a parametric grammar correspond with our intuition of similarities and differences of design?

These research questions allowed the delineation of two hypothesis:

Hypothesis 1: Design families share similarities defined by a generic shape grammar. Design solutions within a corpus parametrically generated use a restricted range of parameters that can be specified within the grammar

Hypothesis 2: Design solutions from the same design family share graphic commonalities that are intuitively perceived by non-expert observers

The first hypothesis aimed at explaining how similarities can be illustrated by the parametric space formed by a shape grammar. Whilst the second hypothesis aimed at understanding the correlation between that parametric space where similarities occur and the observer's perception of family commonalities.

In summary, the study observes that shape grammars, and in particular generic shape grammars allow for a well-defined parametric space where design languages are expressed and these are intuitively perceived by graphical representations of style. This is empirically tested using observers from a group of both laymen and expert professionals.

10.1 Objectives, research hypothesis and methodology used

Shape grammars as generative formalisms were originally created with the goal to recreate design languages. Several shape grammars were developed along the last decades to generate existing and new corpus of solutions based in several design examples. This was described extensively in the state of the art and literature review chapter 2 and in the case study selected as exposed in chapter 3.

Both chapters discuss inferred shape grammars based on architectural examples such as the Palladian villas (Stiny and Mitchell 1978), Prairie houses (Koning & Eizenberg 1981) and Malagueira houses grammar (Duarte 2001).

From the study of these grammars several conclusions could be extracted:

- the grammars replicated existing houses successfully
- all grammars allowed for a new corpus of consistent solutions
- most grammars were criticized for trying to substitute the designer by a formalism

This has been discussed since the first Palladian villas grammar was published in the 70's in several publications, many also exposed previously in this work.

However, two pieces of evidence were not yet discussed and can be assumed from literature:

Assumption 1) One corpus corresponds to only one grammar and

Assumption 2) One grammar corresponds to only one corpus

These assumptions were not expressed in the literature, to the extent of our knowledge, however it sounds reasonable to assume it since to this day no two shape grammar formalisms were inferred to represent the same corpus of solutions.

In the literature these assumptions were never clearly stated, which is why they are termed assumptions and not accepted theories. In linguistics, a language is described by a specific and exclusive grammar. However, the opposite claim i.e. 'more than one grammar for the same style' was never published either. In shape grammar theory, different shape rules are able to produce the same results, as tested many times in shape grammar classes (where students are often challenged to come up with formulations for the same solutions), although these rules can often be rationalised into similar formulations or parameterisations of the same rule. A grammar, however, is a more complex structured formulation and to our knowledge no two grammars have been produced with the same results.

Additionally, when analysing the three case studies seemed that the use of a singular grammar type would allow a more direct result.

The second claim expressed in this study 'Only one grammar corresponds to one corpus' is stated as common assumption too. In fact, a shape rule is applied restrictively to one type of geometry described by the rule. Moreover, each grammar proposes specific shape rules to obtain specific geometric results. Our analysis and cross comparison found analogies among the grammars. That triggered the idea of using the same rule with different parameterization for the three house types (i.e. the division rule, vertical and horizontal.) This is widely explained and illustrated in the house derivations chapter. By generating three types of houses with the same shape rule formulation this assumption was refuted.

To attest this an extensive list of bibliography was consulted in the shape grammar field and several shape grammars studied. The grammars encompassed architectural grammars and other disciplines such as design, art and product design as shown in Table 1.1 and further studied in chapter 2. This chapter was also responsible for exposing Noam Chomsky's syntactic structures and universal grammar which prompted George Stiny's grammar theory, shape rule

formulation and schemas. Shape grammars structure is achieved through a shape rule system. The literature review further attests that grammars are catered to specific languages through a univocal relation. Once a language is covered and inferred grammarians try to pursue a new language to study and infer. Likewise two or more languages are not yet possible to be recreated by the same formalism, so no two corpus can be generated by the same grammar.

The case study chapter describes three paradigmatic shape grammars that represent three distinct type of shape grammar formulation, the addition, subdivision and grid grammar. Additionally they also represent different strategies from the top-down to bottom-up, covering all grammar typologies. This allowed a proper comparison and analysis of previous examples and allowed to draw some preliminary conclusions with regards to grammar formulation. The grammar classification and typology patent revealed a common pattern. All different grammars shared a similar structure, similar rule formulation, despite describing different languages and styles. The shape rules across the three typologies shared commonalities and many of the shape rules were graphically represented using similar illustration. An optimization of the rule system seemed achievable. This occurs for several rules despite the grammar typology which leads to the idea that the formulation could be made in order to 'one case fits all'.

The two formulated hypothesis advocated that a specific range of parameters defined a parametric space that corresponded to a design language and that this design language could be perceived intuitively by users using the graphical representation.

An alternative hypothesis could be formulated, can one grammar describe more than one language? To do so two requirements have to be fulfilled:

1. The grammar structure has to allow for flexibility
2. The shape rule set has to encompass parametric rules with generic formulation and specific parameterization targeted to each language

All case study shape rules are laid out and the consistently recurring examples are selected to formulate generic shape rules. This is shown in chapter 4 entitled generic shape rules. Four basic rules are identified as recurrent from the existing examples:

- 1) shape addition
- 2) offset or parallel copy
- 3) subtraction
- 4) concatenation

In addition specific parametrizations are extrapolated for each language. The generic shape rules are represented by schemas and by parameterized generic expressions with specific values for each style. It is believed that these 4 generic rules could be responsible for the generation of the main features of architectural design and used for the basic house layout as they were originally for the case study observed. Other generic rules are also formulated but deemed accessory. Both their graphical representation and their parametrization was expressed in chapter 4.

Once the generic rules were formulated their grammatical structure had to be implemented.

It was still unclear if two grammars could develop the same corpus of solutions. To this point no two grammar were available to illustrate the same corpus. It seemed reasonable to do so, and to achieve it the most appropriate grammar would be the first and most controversial architectural grammar - the Palladian villas grammar. Chapter 6 is entitled the alternative grammar for the Palladian villas grammar. The chapter illustrates not only a set of 'new shape grammar rules', and a new formalism but a new top-down approach to attain the existing corpus of the Palladian villas.

The grammar is illustrated both graphically and parametrically using a set of values that allow the successful recreation of:

1. The existing corpus of Palladian villas
2. The proposed corpus of Palladian villas as created by the original Palladian grammar
3. The existing corpus of villas not originally covered by Stiny and Mitchell grammar such as villa Rotonda
4. A new set of corpus that follow the design criteria but were not yet generated

The alternative grammar attested that more than one grammar could generate the same corpus of solutions, that there were more than one way to replicate design and most importantly that a given formulation could generically describe a large corpus of solutions by playing with parametrization. As a matter of fact, the formulation used in the alternative grammar does not differ much from the Malagueira grammar method. The alternative grammar used a top-down sub-division method much alike the one developed by Duarte (Duarte 2001), similar rules but with different parametrizations. This prompts the idea that generic rules with specific parametrizations could generate generic grammar formalisms.

More so, the alternative grammar allowed to prove false the Assumption 1:

“One corpus corresponds to only one grammar.”

The knowledge attained with this experiment allowed for the generic formulation as shown in chapter 7. This chapter was responsible for the presentation of the generic grammar formulation containing the sections: methodology, formulation, generation of solutions and the derivation process for Palladian Villas, Prairie Houses and Malagueira single-family houses. Using a similar grammar strategy and methodology as used for the alternative grammar a generic grammar structure was proposed with a grammar syntax, structure and specific parameterization targeted to each language. The grammar was divided into two sections: the generic grammar formulation and the specific formulation. Within the first section the overall house floorplan was generated and its basic functions located. The section was divided into 4 steps:

1. from the overall outline
2. to the initial spatial division
3. to spatial merging
4. followed by the definition of walls

These four steps proposed the same articulation for all languages and specific parametrization for each style. This was demonstrated using the case study examples which were shown with the set of allowed values for each example.

These four steps also allowed the basics of house function and layout. The following steps were used for detailing such as inclusion of doors and windows openings, allocation of terraces and external stairs and any other specific features of design which are style specific. Examples of the specific shape rules were illustrated as generic rules as well as their parametrizations for the

examples of the case study. The generic grammar formulation is then formally illustrated in table 7.1 and allows to determine Assumption 2:

“One grammar corresponds to only one corpus”.

The generic grammar allowed for a conglomeration of languages to be prompted using the same formulation. This was made possible using different parametrizations within the same set of shape rules. However this poses a relevant question:

1. How well does the parametric space of a shape grammar coincide with the design space expressed by different corpora?

Given that the generic grammar allowed for a diverse set of examples including existing solutions, new solutions and even hybrid solutions spanning across design languages the following question deemed relevant:

2. How well does the parametric space of a parametric grammar correspond with our intuition of similarities and differences of design?

To answer both questions two hypotheses were formulated:

Hypothesis 1:

Design families share similarities defined by a generic shape grammar. Design solutions within a corpus parametrically generated use a restricted range of parameters that can be specified within the grammar.

Hypothesis 2:

Design solutions from the same design family share graphic commonalities that are intuitively perceived by non-expert observers.

Two sets of experiments were developed to attest the proposed hypothesis. A quantitative approach was used to test the restricted range of parameters within a grammar for each style. Chapter 8: entitled ‘Validation of the generic grammar’ proposed a quantitative method of evaluation for the generic grammar. Using Hanna’s method of comparing style archetypes (Hanna 2008) the generated examples were plotted in a chart. Their relative proximity allowed the quantification of the proximities of style. The method comprised:

- 1) recreating existing designs using the grammar
- 2) generating new solutions using the same similarities with original family
- 3) analysing the corpus of results using a quantitative method to test a set of numeric values for different house variables measured bi-dimensionally on the floorplan
- 4) Statistical processes are used such as bi-dimensional reduction analysis to plot the results with Cartesian representation, reducing high dimensional data

The second hypothesis would be attested by Chapter 8 entitled: “Hybrid forms”. This experiment envisaged the use of qualitative methods. To test the perception of design familiarities hybrid formulations were created using two parent languages. This hybrid grammar is presented as a sub-grammar of the generic housing shape grammar. In fact it presents the same formulation of

the generic grammar but the parametrization used is a cross-over between styles. The particular parameterization was only suitable for the hybrid style but derived from the parent languages. Several hybrid examples were generated amongst the three parent study cases examples and graphical representation of its floorplan drawn. A set of questionnaires were prepared where real existing examples were compared with hybrid examples. A diverse array of users were asked to avail on its aesthetic characteristics, its design style and language proximity and design language.

Independent subjects were asked to identify language similarities in a range of corpus solutions. Most users identified family similarities. The hybrid study pointed towards the boundaries of design language and the implications amongst different languages.

10.2 Findings and analysis of results

The quantitative method allowed for an objective analysis of the examples generated by the grammar. Houses generated by the grammar were then selected for a case study of 15 houses, 5 from the Palladian, 5 from the Prairie and 5 from the Malagueira corpus. The case study houses were then compared and analysed for specific features (overall boundary or envelope, interior spatial divisions, number of rooms, etc.) and its values recorded into a large set of data such as coordinates/values or vector components. The extent of components allowed an accurate comparison, but also increased the complexity of the graphical representation in a multi-dimensional space. Quantitative methods were used to reduce the dimensionality and represent the results using Cartesian axis charts. The method used to reduce the data into a bi-dimensional chart representation was first used in an architectural context by Hanna in his Style Archetypes study (Hanna 2008). The method allowed to rationalize the data obtained from the case study and plot it into a chart. The goal was to measure the relative distances between examples of each house language. A true assessment of style proximity could be pragmatically measured by the principle of clustering. A well established and defined language would showcase a cluster effect amongst its houses. This was observed amongst the examples generated by the grammar and analysed with quantitative methods.

Respectively Malagueira, Palladian villas and Prairie houses showcased clustering effects (ordered by decreasing levels of definition). Malagueira without a doubt was proved as the style more consistent amongst examples. This is shown if not only by the floorplan similarities but also for the consistent relative distance between the 5 Malagueira houses which at times seem to overlap within the graph. The well-defined Malagueira cluster even casts a shadow into the other languages which in comparison seem to spread across the chart, despite the fact that defined clustering can also be identified for Palladian and Prairie houses. Other findings can be extracted, some more peripheral houses in the aforementioned styles seem to share style similarities. Despite the divergences Winslow prairie house shares with Palladian villas symmetry (very uncommon amongst Prairie houses) and Palladian Villa Barbaro showcases a cross-shaped diagram very much like prairie houses examples. Even the peripheral irregularities seem to consubstantiate the clustering effects within the chart.

The experiment seems to verify the hypothesis that design families share commonalities that can be measurable and quantifiable. These commonalities can be reproduced by specific sets of parameters that define ultimately the style (rather than form only). These can be quantifiable and expressed. The generic grammar formulation seems to be validated by this experiment.

Three arguments can be extracted from the study presented:

- 1) shape grammar formalism is an efficient method to describe and replicate design concepts
- 2) shape grammars are efficient methods of recreating a design language
- 3) a specific shape grammar corresponds to a specific style or language

Design results obtained from the use of shape grammars proved to fall within the restricted area of a parametric space, in addition they were deemed as similar by observers. This was then furthermore illustrated by the use of a generic grammar that contained sub-grammars. Although different examples were generated by the same formalism, their structure, design intent and graphical representation remained faithful to the original design language, occupying the same cluster within the parametric space of corpora (see chapter 9, and Figures 9.11-14)

Parametric space of corpora seemed vindicated by the proximity of the graphical representation of design solutions. The clustering effect is explained by the relative proximity of family related solutions. Within this proximity are also included hybrid examples that despite dissimilarities show clear familiarities with the parent languages. A clear proximity can be drawn between design solutions showcasing a well-defined parametric space for each design language.

This is even more significant if we realize that all three tested design languages were generated from the same formalism. The great innovation of this generic formalism is the ability to confer specificity in language by a set or defined range of design parameters that are then responsible for the parametric space of each design language.

This corroborates the first hypothesis that design languages share similarities defined by a generic shape grammar.

The perception of design familiarities posed an experiment using qualitative methods.

6 hybrids designs were generated using two parent languages from the Palladian, Prairie and Malagueira Houses study case. This hybrid corpus of designs resulted from the generic housing shape grammar, showcasing the same set of rules but with particular parameterization suitable for the hybrid style.

From the 6 examples 2 used the Palladian houses as parent language, 2 used the Prairie houses and 2 used the Malagueira. The results shared 2 parent languages where the parameters were shared amongst the allowed values for each parent language. The results often resembled one design style rather than the other, but all showcased consistent and possible houses solutions that worked both stylistically and programmatically. The sense of consistency was conferred by the efficient generic grammar structure.

The 6 hybrid examples were then mixed with real houses from the case study and presented to a group of observers from a wide range of backgrounds. Amongst the universe of observers there were also designers and architects but a good proportion was not directly linked to the design field.

Several analyses were undertaken, from the aesthetic quality of the examples to the design consistency and feasibility.

Considering that all houses selected were well-known and respected master pieces of architecture one would expect these to be the selected examples in the different categories (aesthetics, consistency and selection of the existing examples). However, the results were surprising:

When confronted with Malagueira house examples and hybrids only 22% identified the real existing plan whilst 62% thought that the prairie/Malagueira hybrid was the real house. In addition, 55% thought that the Palladio/Malagueira hybrid held a higher aesthetic value when compared with real examples.

The same group when confronted with Palladian villas and hybrids only 31% identified the real existing villa versus 56% thought that the Palladio/Prairie hybrid was the real house, and the majority of 54% thought that the Palladio/Prairie hybrid held a higher aesthetic value when compared with other hybrids and existing Palladian villas examples. In this example, even designers with knowledge of Palladio architectural history seemed to make the same choice.

Finally, the group was faced with Frank Lloyd Wright Prairie houses, a much loved house typology even amongst laymen, 56% identified the real prairie house whilst 55% thought that the Prairie/Malagueira hybrid held a higher aesthetic value when compared with other hybrids and real examples.

Other conclusions included a preference by most people which seemed to select the Prairie houses more often based on the internal layout whilst another preference was the Malagueira typical envelope.

Overall the hybrids were often selected when compared with different existing solutions.

In addition, the users seem to always agree in the identification of style similarities amongst real examples and parent related hybrids.

This allows us to conclude that there is a correlation between design language family and their graphical representation, and this can be graphically perceived. A language space formulated by its corpus is perceived by subjects despite their familiarity with design. In addition the generic grammar seemed efficient conveying the language of each style through the shape rules and by the style specific formulation which can be materialized into a successful graphical representation of a house floorplan.

Design solutions within a design family share graphic commonalities that are therefore intuitively perceived.

Two empirical studies were conducted using questionnaires. The first questionnaire enquired about the perception between consistency of the design of both hybrids and existing solutions.

The second questionnaire tested the perception of design similarities within a language. Participants were asked to select houses with similar representation. The observers consistently chose examples that shared visual communalities in detriment of feasibility or architectural value. This showed that even hybrid solutions carried genetic data that informed the graphic representation of each house.

This seems to consubstantiate the hypothesis 2 which claimed that design families share graphic commonalities that are intuitively perceived.

The methodology followed in this study was threefold:

- understanding the differences and similarities between independent shape grammars
- capturing the style and replicating it in a shape grammar formalism
- identifying the process for expressing a generic shape grammar.

The pursue of this methodology resulted into the following findings:

- independent grammars share similar formulations
- it is possible to recreate more than one shape grammar for the same family of results
- grammars of different typologies share similar shape rules and grammatical structures

10.3 Conclusions and evaluation of results

“Language of designs is a set of descriptions of individual designs. Usually, the language of designs does not contain all possible designs, but rather designs with very special structure and properties; intuitively it provides designs in a particular style for use in design contexts” (Styny & March, 1981). The language of design was replicated by a generic grammar. This generic grammar aimed at consolidating the two hypotheses formulated in chapter 1. The generic grammar allowed for sub-grammars to be inherent part of it enabling more than one language to be described by the same formulation. No longer to one grammar corresponded one corpus and vice versa as shown in chapter 6 and 7.

Hypothesis 1 was discussed throughout chapter 9, where the parametric space of each language was analysed and measured, whilst hypothesis 2 explored the correlation between the perception of design similarities and the same parametric space as discussed in chapter 8 and its empirical methods.

The purpose of a generic shape grammar is twofold: firstly, is a powerful tool that encodes knowhow decoded from previous languages of design. This tool can be used and manipulated for design exploration and design analysis. Secondly, this is an advance in the shape grammar research creating a new paradigm which up to now focused on the recreation of style using univocal principles.

The inference process is normally prompted by the extensive analysis of a corpus of pre-existing solutions, the identification of a vocabulary of shapes, the analysis of the existing spatial relationships between the vocabulary of shapes, and the delineation of additive and subtractive shape rules. This was identified by Stiny as a methodology to develop a shape grammar from scratch. However, it also constitutes a methodology to recreate a language of designs. (Stiny 1980)

Another relevant issue is the utility of an alternative grammar that can reproduce a similar set of results or even a generic grammar that can reproduce more than one family of solutions. Each grammar does not fully explore or describe each style, however if that grammar is successful in generating the same corpus of designs, it excludes the need to recreate more. The only benefit of alternative versions is to enrich the description and assist in the generation process. Additionally, the more elegant and economic the grammar the more efficient.

On the other hand, Knight argues that a language of designs can be developed by transforming one language into a new one. (Knight 1983a; Knight 1983b; Knight 1983c)

By manipulating the set of additive and subtractive rules, changing the given spatial relations and alter the sequence of rule application one gets a new grammar with new generative potential. This was tested using the Prairie houses from Koning and Eizenberg’s grammar (Koning & Eizenberg 1981) to generate the Usonian houses, which formulated an efficient grammar developed and published by Knight. (Knight 1994)

This was not the approach tested in this study, although there are some resemblances.

Another approach was used by Duarte for a grammar developed from an existing set of designs, subsequently enlarging the scope of the design solutions while respecting the nature and design concept of the language. (Duarte 2001)

Other studies, not necessarily within the shape grammar field, approached the typologies of housing sets self-inscribed in a rectangular boundary. This study proved efficient in identifying a corpus of feasible solutions that integrated a subdivision like system and integrated functions. (Mitchell, Steadman 1976)

Others have dedicated their research to specific stylistic features of the Palladian villas and proposed a rule-based system inspired by musical harmonies to encode and recreate the style. This system used subdivision methods that in part resemble the grammar proposed here. However, this system fails to represent some of the most paradigmatic examples of the style (such as Villa Rotonda, as previously demonstrated) and in other cases the best results achieved were rough approximations. (Hersey 1992)

Whereas Knight's proposal transforms one design language into a different language, this study proposes that different languages can be generated from the same formulation. As with the study on rectangular floor plans, a corpus of new/possible solutions is also provided.

To the extent of our knowledge this was not attempted before constituting a novelty.

The direct application of this generic grammar could be to create original designs, used in education for style exploration and learning, and ultimately it can be used as an analysis tool for the important design families explored.

In original designs, a generic grammar would have implications on both education and practice. Even though raising potential conflicts, shape grammars are powerful learning tools for designers systematizing design and prompting new solutions for diverse problems. They also allow for design exploration promoting creativity.

Grammars can also be analytic with great descriptive capabilities. This constitutes a great tool for art and architecture historians or anyone who wishes to grasp a specific language. Lastly, this generic grammar hopefully captures not a specific or several specific styles but a process to design single housing. This is achieved by encoding a successful method to develop this housing typology. The process is well described in the rule system, its sequence and its parameterization. Within the parameterization repetitive patterns arise. Is noticeable Palladio's interest in square ratios or in rectangular golden ratios, only substituted by the most common approximation the 2:3. Siza also expresses its favour for this ratio in the Malagueira houses, and this is not a coincidence. This tells us something about harmonic proportions related to living, the generic grammar illustrates this fact in its set of rules.

The novelty within the generic grammar lies in the generation of design families can be triggered by the use of a restricted set of parameters exclusive to a specific language. The parameter space revealed the expected similarities and differences between examples. The quantitative method provided a tool to visualise the 'cluster' and 'space of results' amongst a design family.

Individual design solutions tended to cluster amongst a space of results according to the parameters used to generate them. Hybrids fell within intermediate spaces of the families that generated them. Similarities were parametrically and graphically illustrated. Style features and commonalities were objectively measured and compared. Similarities were also empirically perceived

In summary, this study proposes the use of shape grammars as the formalism for developing a generic system for design housing grammar through the systematization of specific rule sets and the definition of a proper rule application sequence for generating designs.

10.4 Final reflections

The work described above presents a generic shape grammar that allows for the generation of not one, but several signature styles. Unlike previous work, this is not a typical shape grammar, but a generic formulation that allows for the replication of more than one design style, which is believed to be a contribution to shape grammar research. To this end, the generic grammar uses shape grammar structure and shape rules. The rules are formulated as parametric and can be manipulated to generate a particular design. A case study composed of three types of grammars, namely the Palladian Villa, Prairie House and Malagueira House grammars, was selected to illustrate the scope of this generic grammar. The aim was twofold: firstly, to produce an alternative grammar that allowed for the alternative generation of a previously developed grammar, and, secondly, to use this new grammar as a generic grammar capable of producing more than one design style. The methodology started with a cross comparison of the grammars previously inferred and a study of their underlying styles. Each grammar was decomposed and its structure analysed. The complex sets of rules for each grammar were also analysed and similar rule formulations were pinpointed. The grammar comparison and knowledge acquired led to the idea of using subdivision grammars to construct the new generic grammar. This choice reflects the ease of use and intuitive nature of this grammar type and the adaptable nature of the subdivision process in comparison with other creative concepts. A new set of rules was developed for this new generic grammar in order to produce stylistically consistent designs. The set of rules incorporated subdivision rules for the required conditions, such as minimum spacing and bilateral symmetry.

To the best of our knowledge, all the previous work on shape grammars proposed a unique grammar to describe a particular corpus of designs. No one has proposed more than one grammar for the same style or a grammar that can describe more than one style. However, we believe that the effort of developing of an alternative grammar which is different from the original grammar developed for that style can tell us more about the essence of the style. We also believe that developing a grammar that can describe more than one style, termed a generic grammar, aids in the understanding the common features of the different styles and the structure of the common underlying type. This thesis therefore presents a grammar for three different styles of single family homes.

The work refutes certain assumptions regarding shape grammars, namely the uniqueness of the design style that one grammar can produce. Given that there is more than one way to reproduce designs, more than one suitable grammar and the fact that one grammar can produce more than one style, many different representations are potentially viable.

This represents a breakthrough in shape grammar methodology and research. Shape grammars are no longer exclusive, but can potentially be manipulated to generate a larger corpus of new designs. This allows for efficiency in exploring shapes and analysing results, thus widening the scope of grammars.

Future work will focus on the effectiveness and implementation of the generic grammar, such as the exploration of a new corpus of designs and the analysis of generated design hybrids. It is expected that the mutation of these design styles or the overlapping of rules will produce new consistent designs with a new hybrid style. This is currently not allowed by the restrictions implemented. Moreover, computerised implementation will represent a positive development, allowing for the exploration of design solutions and even the enumeration of design corpus results. The potential of this generic grammar will be fully tested with a computerised tool, as was the case with previous work developed for housing shape grammars, such as the ABC system and the Haiti Gingerbread House grammar (Stiny & Mitchell 1978; Koning & Eizenberg 1981; Duarte 2001).

11.	Bibliography
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11.2	Bibliographic references for chapter 2
11.3	Bibliographic references for chapter 3
11.4	Bibliographic references for chapter 4
11.5	Bibliographic references for chapter 5
11.6	Bibliographic references for chapter 6
11.7	Bibliographic references for chapter 7
11.8	Bibliographic references for chapter 8
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12.	Image index
12.1	Image index for chapter 1
12.2	Image index for chapter 2
12.3	Image index for chapter 3
12.4	Image index for chapter 4
12.5	Image index for chapter 5
12.6	Image index for chapter 6
12.7	Image index for chapter 7
12.8	Image index for chapter 8
12.9	Image index for chapter 9
12.10	Image index for chapter 10

12.1 Image index for chapter 1**12.2 Image index for chapter 2:**

Figure 2. 1: Universal grammar tree diagram	29
Figure 2. 2: Deep grammar x-bar theory	30
Figure 2. 3: Universal grammar N-bar theory	31
Figure 2. 4: Grammatical shape rules	33
Figure 2. 5: Derivation	33
Figure 2. 6: Corpus of shape grammar solutions	33
Figure 2. 7: Ice-ray shape rules	39
Figure 2. 8: Ice-ray shape derivation	39
Figure 2. 9: Ice-ray shape corpus of solutions (Stiny 1977)	39
Figure 2. 10: Subdivision rules for the Ice-Ray grammar	48
Figure 2. 11: Grid cell rules for the Palladian grammar (Stiny & Mitchell 1978) (adapted)	49
Figure 2. 12: Embedded square	51
Figure 2. 13: The Queen Anne House grammar: an example of an additive grammar (Flemming 1987)	56
Figure 2. 14: Wren's City Church grammar: an example of a grid grammar (Buelinckx 1993)	57
Figure 2. 15: The Buffalo Bungalow grammar: an example of a subdivision grammar (Downing & Flemming 1981)	60

12.3 Image index for chapter 3:

Figure 3. 1: Nine existing Palladian villas with 3x3 and 5x3 floor plan grids	
(G Stiny & W J Mitchell 1978)	79
Figure 3. 2: The 'Malcontenta' Villa 5x3 floor plan grid and elevation	
(G Stiny & W J Mitchell 1978)	79
Figure 3. 3: Partial derivation of the 'Malcontenta' villa using the Palladian Grammar	
(G Stiny & W J Mitchell 1978)	80
Figure 3. 4: Stiny and Mitchell 1978 structure of the Palladian Villa grammar	83
Figure 3. 5: Cell rules 1-10 for the Palladian grammar, adapted from	
(Stiny & Mitchell 1978)	84
Figure 3. 6: Application of the Palladian grammar rules to generate a 5x3 grid floor plan	84
Figure 3. 7: Complete Palladian Villa grammar rule set, adapted from	
(Stiny & Mitchell 1978)	87
Figure 3. 8: Corpus of 3x3 grids for Palladian villas generated by the shape grammar	
(G Stiny & W J Mitchell 1978)	87
Figure 3. 9: Prairie House grammar rule structure (Koning & Eizenberg, 1981)	91
Figure 3. 10: Original Prairie House grammar rule structure	91
Figure 3. 11: Rule system for the Prairie House grammar, adapted from	
(Koning & Eizenberg 1981)	93
Figure 3. 12: New Prairie grammar houses: Stiny house derivation	
(Koning & Eizenberg, 1981)	96
Figure 3. 13: New Prairie grammar houses: Stiny, Mitchell and March	
(Koning & Eizenberg, 1981)	97
Figure 3. 14: Malagueira house, type Ab: a three-bedroom, semi-detached	

house with courtyard, plans and elevation from (Duarte, 2001)	98
Figure 3. 15: Malagueira subdivision patterns from (Duarte, 2001)	98
Figure 3. 16: Original Malagueira House grammar structure	101
Figure 3. 17: Summary of the Malagueira subdivision rules (Duarte 2005)	102
Figure 3. 18: Malagueira grammar derivation for house type Ab (Duarte 2001)	106
Figure 3. 19: Comparison of all case study grammars	109
12.4 Image index for chapter 4:	
Figure 4.1: Rule comparison among the Palladian, Prairie and Malagueira Houses	
Grammar Addition of an initial rectangle (rule 1 in the Palladian	
Grammar, rule 1 in the prairie house grammar and rule 0 in the	
Malagueira grammar)	118
Figure 4.2: Offset Rule comparison among the Palladian, Prairie and Malagueira	
Houses Grammar	119
Figure 4.3: Subtractive Rule comparison among the Palladian (rule 27), Prairie	
(rule 49) and Malagueira Houses Grammar (rule 62)	119
Figure 4.4: Concatenation Rule comparison among the Palladian, Prairie	
and Malagueira Houses Grammar	120
Figure 4.5: Design Rule - Shape Addition	121
Figure 4.6: Design Rule - Shape Parallel copy or Offset	123
Figure 4.7: Design Rule – Shape Subtraction	125
Figure 4.8: Design Rule – Shape Concatenation	127
Figure 4.9: Generic shape rule – Copy	129
Figure 4.10: Generic shape rule – Move	131
Figure 4.11: Generic shape rule – Scale	133

Figure 4.12: Generic shape rule – Rotate	135
Figure 4.13: Design Rule – Shape Subdivision	136
12.5 Image index for chapter 5:	
Figure 5. 1: Palladian villa Malcontenta Adjacency and Functional Derivation Key: A1 to C5	
Grid cells numbered by column and labelled alphabetically by row: Rm1- drawing room, Rm2- study, Rm3- formal dining, Rm4, Circulation/ante-chamber, St- stair, Lv- Living space, En- Ent	144
Figure 5. 2: Palladian villas (from top right to bottom left): Caldogno, Poiana, Rotonda, Valmarana, Sarraceno, Barbaro, Emo, Cornaro, Malcontenta and Pisani	150
Figure 5. 3: Winslow houses Functional Diagram Key: Fp-Fireplace, R-Room, C-Circulation, Dn-dining, Te-Terrace, St-Stair, Ha-Hall, Ki-Kitchen, Pc-Porte-Coucherre or Porch	151
Figure 5. 4: Robie house Functional Diagram Key: Fp-Fireplace, R-Room, C-Circulation, Dn-Dining, Te-Terrace, St-Stair, Ha-Hall, Ki-Kitchen, Pc-Porte-coucherre or Porch	151
Figure 5. 5: Malagueira functional tree diagram adapted from (Duarte 2001a)	154
Figure 5. 6: Malagueira house Ab Adjacency graph Key: Lt- Lot, Out-Outside, In-Inside, Se- Service, Ya- Yard, Co-Corridor, Cl- Closet, Li- Living, Sl- Sleeping, Be- Bedroom, Pa- Pantry, Ba-Bathroom, St- Stair	155
Figure 5. 7: Malagueira house Ab functional Derivation graph Key: Lt- Lot, Out-Outside, In-Inside, Se- Service, Ya- Yard,	

Co- Corridor, Cl- Closet, Li- Living, Sl- Sleeping,

Be- Bedroom, Pa- Pantry, Ba-Bathroom, St- Stair 155

Figure 5. 8: Malagueira house Bb Adjacency graph Key: Lt- Lot, Out-Outsid,

In-Inside, Se- Service, Ya- Yard, Co-Corridor, Cl- Closet,

Li- Living, Sl- Sleeping, Be- Bedroom, Pa- Pantry,

Ba-Bathroom, St- Stair 156

Figure 5. 9: Malagueira house Bb functional Derivation graph Key: Lt- Lot,

Out-Outside, In-Inside, Se- Service, Ya- Yard, Co- Corridor,

Cl- Closet, Li- Living, Sl- Sleeping, Be- Bedroom, Pa- Pantry,

Ba-Bathroom, St- Stair 156

Figure 5. 10: Palladian rooms' ratios according to Hersey (Hersey 1992) 160

Figure 5. 11 Palladian X, I, T, rooms illustrated as shape rules from the

Palladian grammar (Stiny & Mitchell 1978) 161

Figure 5. 12: Palladian villas Functional Diagram 163

Figure 5. 13: Prairie houses Functional Diagram 164

Figure 5. 14: Malagueira house Ab functional Derivation 167

Figure 5. 15: Malagueira house Bb functional Derivation Key: Lt- Lot, Out-Outside,

In-Inside, Se- Service, Ya-Yard, Co- Corridor, Cl- Closet,

Li- Living, Sl- Sleeping, Be- Bedroom, Pa- Pantry, Ba-Bathroom,

St- Stair 167

Figure 5. 16: Malagueira house Bb functional Derivation Key: Lt- Lot, Out-Outside,

In-Inside, Se- Service, Ya-Yard, Co- Corridor, Cl- Closet,

Li- Living, Sl- Sleeping, Be- Bedroom, Pa- Pantry, Ba-Bathroom,

St- Stair 167

Figure 5. 17: Malagueira houses Functional Diagram	168
Figure 5. 18: Malagueira tree diagram with functions (Duarte 2001a)	169
12.6 Image index for chapter 6:	
Figure 6. 1: Palladian villas as pictured in 'Il Quattro libri' (Palladio 1997)	178
Figure 6. 2: Ceramic tiles pattern as provided to Team A and B	182
Figure 6. 3: One of the teams inferring shape rules and its graphical representation	182
Figure 6. 4: Team 1 inferred shape grammar	183
Figure 6. 5: Team 2 inferred shape grammar	183
Figure 6. 6: Alternative set of rules for generating a Palladian grammar	184
Figure 6. 7: Original Palladian Grammar Derivation Tree	187
Figure 6. 8: Proposed Palladian Grammar Derivation Tree	188
Figure 6. 9: Alternative Palladian grammar Derivation for Villa Angarano, Poiana, Pisani and Sarraceno	191
Figure 6. 10: The Palladian grammar universe of solutions for grids 3x3 in (G Stiny & W J Mitchell 1978)	191
Figure 6. 11: Original Palladian Villas and their internal proportional ratios	193
Figure 6. 12: Proposed Shape Rules	195
Figure 6. 13: Cell rules 1-10 for the Palladian grammar adapted from (Stiny & Mitchell 1978)	195
Figure 6. 14: Application of the Palladian grammar rules to generate a 5x3 grid floorplan	195
Figure 6. 15: Alternative Palladian grammar wall placement shape rules	197
Figure 6. 16: Door openings alignment (Illustrated on the left villa Emo overlapped with the door alignment axes in dashed, on the right rules 19 and rules 20 of the alternative grammar illustrating how the entrance	

and rear entrance door is introduced aligned with the symmetry axis and the alignment of internal doors in a mid-bay centred within the rooms, respectively)	199
Figure 6. 17: Sequence of window openings design (Illustrated on the left villa Emo overlapped with the window alignment axes in dashed, on the right rules 29 and 30 of the alternative grammar illustrating the introduction of windows in the main facades (north and south) centrally aligned with the internal doors, and windows in the west and east façade following the same principle, respectively)	200
Figure 6. 18: Functional derivation of Palladian grammar	206
Figure 6. 19: Partial functional derivation for Palladian villas	208
Figure 6. 20: Spatial function rules for Palladian villas based on existing examples observed	208
Figure 6. 21: Partial derivation of the Malcontenta Villa using Stiny and Mitchell's grammar	223
Figure 6. 22: Derivation of the Malcontenta Villa using the alternative grammar	224
Figure 6. 23: New corpus of solutions of the alternative grammar with 3x2 and 3x4 bays	227
Figure 6. 24: Villa Rotonda al Capri derivation using Palladian grammar	231
Figure 6. 25: Villa Rotonda al Capri derivation using Alternative Palladian grammar	231
Figure 6. 26: Villa Rotonda al Capri approximation generated by Plan-maker design system developed by Hersey and Freedman in (Hersey 1992)	232
Figure 6. 27: New villa solution generated with the alternative Palladian grammar	233
Figure 6. 28: Computer implementation	235

12.7 Image index for chapter 7:

Figure 7. 1: Self-contained shape/bounding box diagram	242
Figure 7. 2: Adjacency diagrams for the three grammars (Palladian, Prairie and Malagueira)	245
Figure 7. 3: Top-down and bottom-up approach diagram	246
Figure 7. 4: Generic grammar tree diagram illustrating the derivation of Palladio's Villa La Malcontenta	249
Figure 7. 5: Generic grammar tree diagram	250
Figure 7. 6: Generic grammar derivation – Palladio's Villa La Malcontenta	254
Figure 7. 7: Generic grammar derivation – Prairie Robie house	256
Figure 7. 8: Generic grammar derivation – Prairie Winslow House	257
Figure 7. 9: Generic grammar derivation – Prairie Henderson House	260
Figure 7. 10: Generic grammar derivation – Malagueira house type Ab	263
Figure 7. 11: Generic grammar derivation – Malagueira house type Bb	264
Figure 7. 12: Generic grammar structure	269
Figure 7. 13: Generic grammar diagram	284
Figure 7. 14: Generic grammar corpus of solutions for 1, 2 and 3 divisions.	285
Figure 7. 15: Generic grammar corpus of solutions for 4 divisions.	286
Figure 7. 16: Generic grammar corpus of solutions for 5 divisions (part 1).	287
Figure 7. 17: Generic grammar corpus of solutions for 5 divisions (continued part 2).	288
Figure 7. 18: Acceptable Palladian villa patterns	290
Figure 7. 19: Acceptable Malagueira house patterns	290

12.8 Image index for chapter 8:

Figure 8. 1: Representation of villa Poiana from Palladio (Palladio 2002)	
and Prairie Winslow House from Frank Lloyd Wright were	
some visible similarities can be drawn (Koning & Eizenberg 1981)	288
Figure 8. 2: Buffalo bungalow A, B and C as illustrated in the	
Buffalo bungalow's shape grammar (Downing & Flemming 1981)	290
Figure 8. 3: Malagueira supported patterns (Duarte 2001)	292
Figure 8. 4: Buffalo bungalow A derivation using the generic grammar formulation	293
Figure 8. 5: Buffalo bungalow B derivation using the generic grammar formulation	294
Figure 8. 6: Buffalo bungalow C derivation using the generic grammar formulation	295
Figure 8. 7: Buffalo bungalows supported patterns	297
Figure 8. 8: Malagueira supported patterns (Duarte 2001)	297
Figure 8. 9: Buffalo bungalow A hybrid - A derivation using the generic	
grammar formulation (pattern 1 from Malagueira typical	
patterns can be used to design Buffalo Bungalow A as illustrated	
in Figure 8.7)	299
Figure 8. 10: Buffalo bungalow B hybrid –	
derivation using the generic grammar formulation	300
Figure 8. 11: Buffalo bungalow C hybrid - derivation using the	
generic grammar formulation (a typical Malagueira pattern	
'8' can be used to representate Buffalo bungalow B as	
shown in Figure 8.7)	301
Figure 8. 12: Hybrid design comparison	303
Figure 8. 13:	304

Figure 8. 14: Hybrid design comparison (top to bottom:	
Buffalo bungalow A, hybrid type 1,	306
Figure 8. 15: Hybrid design comparison (left to right, top to bottom:	
Buffalo bungalow A, hybrid type 1, Malagueira house Ab	
(Duarte 2005) and hybrid type 2)	307
Figure 8. 16: Hybrid design comparison	309
Figure 8. 17: Hybrid design comparison (top to bottom: Palladian Villa Emo,	
hybrid design Emo x Winslow, and hybrid Emo x Ab)	312
Figure 8. 18: Hybrid design comparison (top to bottom:	
Winslow prairie house, hybrid Winslow x Emo, and hybrid Winslow x Ab,)	313
Figure 8. 19: Hybrid design comparison (top to bottom: Malagueira house Ab,	
hybrid Ab x Emo, and hybrid Ab x Winslow)	314

12.9 Image index for chapter 9:

Figure 9. 1: Case study set of 5 Palladian villas, 5 prairie houses and 5 Malagueira	330
Figure 9. 2: 24 twentieth century buildings plotted into feature space (Hanna 2007)	334
Figure 9. 3: Chart with the application of rule 1 with tags	337
Figure 9. 4: Chart with the application of rule 1 without tags	337
Figure 9. 5: Chart with the application of rule with tags	339
Figure 9. 6: Chart with the application of rule without tags	339
Figure 9. 7: Chart with the application of rule 3 with tags Figure 9. 8: Chart with the application of rule 3 without tags	340
Figure 9. 9: Chart with the application of rule 1 with tags Figure 9. 10: Chart with the application of rule 1	343
Figure 9. 11: PCA analysis plotting the results of the villas studied	344
Figure 9. 12: Graphical representation of the villas studied plotted into a chart	345
Figure 9. 13: PCA chart applied to Malagueira (blue), Prairie houses (green) and Palladian villas (red), the hybrid examples are illustrated in the second row and column	349
Figure 9. 14: Illustrated PCA chart showing to Malagueira (blue), Prairie houses (green) and Palladian villas (red), and hybrid examples	349

12. Table index

12.1	Table index for chapter 1
12.2	Table index for chapter 2
12.3	Table index for chapter 3
12.4	Table index for chapter 4
12.5	Table index for chapter 5
12.6	Table index for chapter 6
12.7	Table index for chapter 7
12.8	Table index for chapter 8
12.9	Table index for chapter 9
12.10	Table index for chapter 10

12.1 Table index for chapter 1

12.2 Table index for chapter 2

Table 2. 1: Lattice schemas, adapted from (Stiny 2011)	37
Table 2. 2: Rule, inverse rule and alternate form schemas, adapted from (Stiny 2011)	38
Table 2. 3: Lattice schemas, adapted from (Stiny 2011)	39
Table 2. 4: Lattice schemas , adapted from (Stiny 2011)	41
Table 2. 5: Previously inferred shape grammars, organised by field.....	53

12.3 Table index for chapter 3

12.4 Table index for chapter 4

12.5 Table index for chapter 5

Table 5. 1: Palladian villas floorplan information by grid, number of rooms and main room typology	117
Table 5. 2 : Grammar functional generation by stages in Prairie houses grammar.....	137
Table 5. 2: Functional definition by stages in Malagueira houses - adapted from (Duarte 2001b))	141

12.6	Table index for chapter 6	
	Table 6. 1: Set of alternative grammar shape rules	189
12.7	Table index for chapter 7	
	Table 7. 1: Generic grammar shape rules	231
12.8	Table index for chapter 8	
	Table 8. 1: Hybrid corpus questionnaire	327
	Table 8. 2: Second questionnaire on hybrid families.....	333
12.9	Table index for chapter 9	
	Table 9. 1: House boundary dimension following the application of rule 1	275
	Table 9. 2: House value of horizontal division following the application of rule 2	277
	Table 9. 3: House value of horizontal division following the application of rule 3	278
	Table 9. 4: House number of divisions versus horizontal divisions	280
12.10	Table index for chapter 10	